SCHOOL OF COMPUTING
Faculty of Engineering

ASSIGNMENT 3

DISCRETE STRUCTURE (SECI 1013)
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Lecturer: Dr. Haswadi Bin Hasan

Group: 7	
Name	Matric
AHMEDALLA HANEA HAMZA HAFEZ HANOURA	A19EC4045
RADIN NAZHAN BIN RADIN JAMZULKOMAR	A20EC0136
MD. TAWFIQUZZAMAN	A20EC4036

Section: 07

Q1)

Question -1

(a) here,

$A = \{1, 2, 3, 4, 5, 6, 7, 8\}$

$B = \{2, 5, 9\}$

$C = \{a, b\}$

(i) $A - B$

$= \{1, 2, 3, 4, 5, 6, 7, 8\} - \{2, 5, 9\}$

$= \{1, 3, 4, 6, 7, 8\}$ (Ans).

(ii) $(A \cap B) \cup C$

$(A \cap B) = \{1, 2, 3, 4, 5, 6, 7, 8\} \cap \{2, 5, 9\}$

$= \{2, 5\}$

so, $(A \cap B) \cup C$

$= \{2, 5\} \cup \{a, b\}$

$= \{2, 5, a, b\}$ (Ans)

(iii) $A \cap B \cap C$

$= \{1, 2, 3, 4, 5, 6, 7, 8\} \cap \{2, 5, 9\} \cap \{a, b\}$

$= \phi$ (Ans.)

(iv) $B \times C$

$= \{2, 5, 9\} \times \{a, b\}$

~~= {2, c},~~

$= \{(2, a), (2, b), (5, a), (5, b), (9, a), (9, b)\}$ (Ans.)

(v) $P(C)$

$= P(a, b)$

~~= {a,~~

$= \{\{a\}, \{b\}, \{a, b\}, \phi\}$ (Ans.)

(b)

$$L.S = (P \cap ((P' \cup Q)')) \cup (P \cap Q)$$

$$= (P \cap ((P')' \cap Q')) \cup (P \cap Q)$$

$$= (P \cap (P \cap Q')) \cup (P \cap Q)$$

$$= ((P \cap P) \cap Q') \cup (P \cap Q)$$

$$= (P \cap Q') \cup (P \cap Q)$$

$$= P \cap (Q \cup Q')$$

$$= P \cap U$$

$$= P$$

(c) $A = (\neg p \vee q) \leftrightarrow (q \rightarrow p)$

Truth table is given below—

p	q	$\neg p$	$\neg p \vee q$	$q \rightarrow p$	$(\neg p \vee q) \leftrightarrow (q \rightarrow p)$
T	T	F	T	T	T
T	F	F	F	T	F
F	T	T	T	F	F
F	F	T	T	T	T

(d) for all integer x, if x is an odd, then $(x+2)^2$ is odd.

so ~~we~~ let,

$P(x)$ = x is odd integer

and $Q(x)$ = $(x+2)^2$ is odd integer also.

for, $\forall_x \; P(x) \rightarrow Q(x)$

now,

$$x = 2a + 1$$

so, $(x+2)^2 = (2a+1+2)^2$

$$\Rightarrow \quad = (2a+3)^2$$

$$= 4a^2 + 12a + 9$$

$$= 2(2a^2 + 6a + 4) + 1$$

so $(x+2)^2 = 2b+1$ where $b = 2a^2+6a+4$

so, $(x+2)^2$ is an odd integer.

so, for all integers if x is odd then $(x+2)^2$ is also an odd integer.

(e)

here,

$P(x,y)$ be the propositional function $x \geqslant y$.

x and y is the set of all positive intigers.

(i) $\exists_x \exists_y P(x,y)$ is true when, $x \geqslant y$.

example $\rightarrow$ $x = 2,\ y = 2$, $x = 3,\ y = 2$

$x \geqslant y$ $x > y$

and false when, $x < y$.

example $\rightarrow$ $x = 1,\ y = 2$,

$x < y$ [false]

(ii) $\forall_x \forall_y P(x,y)$ is false because for all x

all y can't be '$x \geqslant y$'.

Ans.

Q2)

a) (i) $R = \{(1,1), (1,2), (2,2), (3,1)\}$

Domain $= \{1,2,3\}$

Range $= \{1,2\}$

(ii) Since $(1,1), (2,2) \in R$

Therefore, relation R is not irreflexive

For all $(a,b) \in R$ and $a \neq b$, $(b,a) \notin R$

Therefore, relation R is antisymmetric

b) (i) $S = \{(4,5), (5,4), (5,5)\}$

(ii)

$$M_R = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 1 \end{bmatrix}$$

Since all main diagonal matrix element are not 1

Therefore, relation S is not reflexive

For all $(a,b) \in X$, if $(a,b) \in S$, then $(b,a) \in S$

Therefore, relation S is symmetric

$$\begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 1 \end{bmatrix} \otimes \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 1 \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 1 & 1 \end{bmatrix}$$

Since $M_R \otimes M_R \neq M_R$

Therefore, relation S is not transitive

Since relation S is not reflexive, symmetric and not transitive

Therefore, relation S is not an equivalence relation

c) (i) $f = \{(1,1), (2,2), (3,3)\}$

f is one-to-one but not onto because 4 in codomain has no object in domain

(ii) $g = \{(1,1), (2,1), (3,2)\}$

g is onto but not one-to-one because $g(1) = g(2) = 1$ and all image in codomain has at least one object in domain

(iii) $h = \{(1,1), (2,1), (3,2)\}$

h is neither one-to-one nor onto because $h(1) = h(2) = 1$ and 3 in codomain has no object in domain

d) (i) let $m(x) = y$

$$m(x) = 4x + 3$$
$$y = 4x + 3$$
$$y - 3 = 4x$$
$$x = \frac{y-3}{4}$$

$$m^{-1}(y) = \frac{y-3}{4}$$

(ii)
$$\begin{aligned}(n \circ m)(x) &= n(m(x)) \\ &= n(4x+3) \\ &= 2(4x+3) - 4 \\ &= 8x + 6 - 4 \\ &= 8x + 2\end{aligned}$$

Q3)

a) (i) $a_1 = 1$

$a_2 = a_{2-1} + 2(2) = a_1 + 4 = 1 + 4 = 5$

$a_3 = a_{3-1} + 2(3) = a_2 + 6 = 5 + 6 = 11$

The first three terms are

1, 5, 11

(ii) Input : k

Output : a(k)

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a(k) {
    if (k=1)
        return 1
    return a(k-1) + 2k
}
```

b) $r_k = 2r_{k-1}$, $k \geqslant 2$ with initial condition $r_1 = 7$

recurrence relation when $k=2$

$$r_2 = 2r_1$$

obtain r_3 by substituting r_2

$$\begin{aligned} r_3 &= 2r_2 \\ &= 2(2r_1) \\ &= 2^2 \times r_1 \end{aligned}$$

obtain r_4 by substituting r_3

$$\begin{aligned} r_4 &= 2r_3 \\ &= 2(2^2 \times r_1) \\ &= 2^3 \times r_1 \end{aligned}$$

The recurrence relation is

$r_k = 2^{k-1} \times r_1$, where $k \geqslant 2$ with initial condition $r_1 = 7$

c) Trace $S(4)$

$S(4)$

$n = 4$
Because $n \neq 1$
return $5 * S(3)$

$S(3)$

$n = 3$
Because $n \neq 1$
return $5 * S(2)$

$S(2)$

$n = 2$
Because $n \neq 1$
return $5 * S(1)$

$S(1)$

$n = 1$
Because $n = 1$
return 5

$S(4) = 625$

Return $5 * 125$

$S(3) = 125$

Return $5 * 25$

$S(2) = 25$

Return $5 * 5$

$S(1) = 5$

Return 5

Answer:
$S(4) = 625$

Q4)

Q4)

a) numbers of which begin with any of digits :

From 3 to B $\Rightarrow$ 9 Possibilities

From 5 to F $\Rightarrow$ 11 possibilities

For 4 digits long will be:

$9 \times 16 \times 16 \times 11 = 25344$ ways

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b) The number of license plates begin with A and end with 0.

A $\Rightarrow$ 1 Possibility

2nd place $\Rightarrow$ 26 Possibilities , 5th place $\Rightarrow$ 10 Possibilities

3rd place $\Rightarrow$ 26 Possibilities , 6th place $\Rightarrow$ 10 Possibilities

4th place $\Rightarrow$ 26 Possibilities , 0 $\Rightarrow$ 1 possibility

So,

$1 \times 26 \times 26 \times 26 \times 10 \times 10 \times 1 = 1757600$ plates

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c) For being arrenged in a row with no more than 3 letters using COMPUTER word

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Words with one letter $\Rightarrow$ 8
(Single letter)

Two letter words $\Rightarrow 8P2 = 8 \times 7 = 56$

Three letter words $\Rightarrow 8P3 = 8 \times 7 \times 6 = 336$

The total number of words $= 8 + 56 + 336 = 396$

Q4)

d) 7 women , 6 men

If group is consist of 4 women & 3 men .

$$\binom{7}{4}\binom{6}{3}$$

$$\binom{7}{4}\binom{6}{3} = \frac{7!}{4!(7-4)!} \times \frac{6!}{(6-3)! \times 3!} = \frac{7!}{4! \times 3!} \times \frac{6!}{3! \times 3!}$$

$$= 35 \times 20 = 700$$

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e) Word PROBABILITY can be arrenged by :

letters $\Longrightarrow$ P, R, O, A, L, T, Y occur only once.

letters $\Longrightarrow$ B, I occur twice.

So, $\frac{11!}{2! \times 2!} = 9979200$ ways

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f) 6 different kind of pastry , different selections of 10 pastries .

we can say that :

$x1 + x2 + x3 + x4 + x5 + x6 = 10$ and, $x_1 + x_2 + \dots + x_k \leq n$

$C(n+k-1, k-1)$

So, $C(10+6-1, 6-1) = C(15, 5) = 3003$

Question-5

(a) here,

Different 1st and last name combinations

are $= 3 \times 2$

$= 6$

there are total 18 person who ~~has~~ got ~~the~~

first names Ali, Bahar and earlie.

so, $\left\lceil \frac{18}{6} \right\rceil = 3$ [Showed]

so, there are at least three persons who got the same first and last names.

(b) From ~~one~~ 1 to 20 there are,

10 odd numbers

and 10 even numbers

For the worst case we can get

10 even numbers.

so, to get a 1 odd number we

can take = 10+1 = 11 numbers (Ans).

(c) from 1 to 100 there are 20

numbers which is ~~divise~~ divisible by

5. The rest , 80 numbers.

The worst case that we can take

numbers (80 +1) = 81 , so, at least

one of them (between 81) is a

divisible of 5.