UTM UNIVERSITI TEKNOLOGI MALAYSIA

SCHOOL OF COMPUTING
Faculty of Engineering

ASSIGNMENT 2

DISCRETE STRUCTURE (SECI 1013)
SEMESTER I, SESSION 2020/2021

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Section: 07

Q1)

consider 3-digit number whose digit are either

2, 3, 4, 5, 6, 7.

~~(a)~~ here,

the total digits ~~are~~ = 6

(a)

here,

the first position of the number can be selected ~~in~~ by '6' ways.

the second position of the number can be selected by '6' ways.

the third position of the number can be selected by '6' ways.

so, the total ~~num~~ number is $= 6 \times 6 \times 6$

$= 216$

(Ans.)

(b) if the digits are distict then,

for the first position the numbers can be selected by '6' ways.

for the second position the numbers can be selected by '5' ways.

for the third position the numbers can be selected by '4' ways.

so,

if the digits are distict then the numbers can be selected in $= 6 \times 5 \times 4$

$= 120$ ways (Ans)

(c)

In case of ~~only~~ taking numbers ~~whose~~ which are odd numbers and between 300 to 700,

for the first place only (3, 4, 5, 6) are allowed

so, the ~~fir~~ first place can be select in '4' ways.

for the second place ~~only~~ all digits are allowed.

so, the second place can be select in '6' ways.

and for the third place ~~on~~ we can take only odd numbers (3, 5, 7)

so, it can be select in '3' ways.

so, the total number is $= 4 \times 6 \times 3$

$= 72$ ~~so~~ (Ans)

Q2)

a) When men sit next to each other

= (mens arrenge) (circular arrange and men as a single element)

mens arrenge = $5! = 120$ ways

Circular Permutation (mens as a single element) = $(6-1)! = 5! = 120$ ways

men sett next to each other = $120 \cdot 120 = 14400$

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b) When couple sit next to each other

= (couples arrange) (circular arrange)

Couples arrange = $2! = 2$ ways

Circular Permutation = $(9-1)! = 8! = 40320$ ways

Comple set next to each other = $(2) \cdot (40320) = 80640$ way

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c) When men & women set in alternative seats

= (men sit in alternate seat) (Women fill the rest seats)

men sit in alternate seat = $(5-1)! = 4! = 24$

women fill the rest seats = $5! = 120$

men & women set in alternate seats = $(24) \cdot (120) = 2880$ ways

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d) When Anita & her husband stand next to each other

= (arrenge Anita & her husband) (arrenge people in line with Anita & her husband together)

arrange Anita & her husband = $2! = 2$

arrange people in line = $11! = 39916800$ ways

Anita & her husband stand next to each other = $(2) \cdot (39916800) = 79833600$ ways

Question 3

a) $5!$

$= 120$ ways

b) ${}^5C_2 \times 4!$

$= 10 \times 24$

$= 240$ ways

c) ${}^5C_2 \times {}^3C_2 \times 3!$

$= 10 \times 3 \times 6$

$= 180$ ways

Question 4

a) $n=6$, $r=12$

$$C(n+r-1, r)$$
$$= C(6+12-1, 12)$$
$$= C(17, 12)$$
$$= \frac{17!}{12!(17-12)!}$$
$$= \frac{17!}{12!5!}$$
$$= 6188 \text{ ways}$$

b) Let us first select 2 of each kind, which are 12 croissants in total. Then we still need to select the remaining 12 croissants.

$n=6$, $r=12$

$$C(n+r-1, r)$$
$$= C(6+12-1, 12)$$
$$= C(17, 12)$$
$$= \frac{17!}{12!(17-12)!}$$
$$= \frac{17!}{12!5!}$$
$$= 6188 \text{ ways}$$

c) Let us first select 5 chocolate croissants and 3 almond croissant. Then we still need to select the remaining $24-5-3=16$ croissants

$n=6, \ r=16$

$C(n+r-1, r)$

$= C(6+16-1, 16)$

$= C(21, 16)$

$= \dfrac{21!}{16!(21-16)!}$

$= \dfrac{21!}{16!5!}$

$= 20\ 349$ ways

Question 5

a) 2 wins among 4 games : $C(4,2)$

$$= \frac{4!}{2!(4-2)!}$$

$$= \frac{4!}{2!2!}$$

$$= 6$$

1 win among 3 games : $C(3,1)$

$$= \frac{3!}{1!(3-1)!}$$

$$= \frac{3!}{1!2!}$$

$$= 3$$

2 wins and 1 ties/wins : $C(4,2) \cdot C(3,1) \cdot 2$

$$= 36$$

1 wins and 3 ties/wins : $C(3,1) \cdot C(4,3) \cdot 2^3$

$$= 96$$

Number of scenarios $= 2 \cdot (36 + 96)$

$$= 2 \cdot 132$$

$$= 264$$

(b) If 10 penalty kicks are played:

$2^{10} = 1024$ possible outcome

Number of scenarios result in the game not being finished in the first round of 10 penalty kicks:

$1024 - 264$
$= 760$ scenarios

First round : 760 scenarios
Second round : 264 scenarios

Number of scenarios $= 760 \times 264$
$= 200\ 640$ scenarios

(c) First round : 760 scenarios
Second round : 760 scenarios
Sudden death : $2 + 2 + 2 + 2 + 2 = 10$ scenarios

Number of scenarios $= 760 \times 760 \times 10$
$= 5\ 776\ 000$

Question 6

The number of different answer sheet that are possible :

$$4^{10} = 1\,048\,576$$

Minimum number of student that guarantee at least 3 answer sheets must be identical :

$$2 \times 1048\,576 + 1$$

$$= 2\,097\,153 \text{ students}$$

Q7)

Let's suppose that H refers to student Passed in History event, and M for Mathematics. Here we can see that:

$P(H) = 0.75$, $P(M) = 0.65$

$P(H \cap M) = 0.50$, $n(H^c \cap M^c) = 35$

Here, $P(H \cup M) = P(H) + P(M) - P(H \cap M)$

$= 0.75 + 0.65 - 0.50 = 0.90$

and, $\dfrac{n(H \cup M)}{N} = 0.90$

N = number of candidates

$N = \dfrac{n(H \cup M)}{0.90}$

Since,

$n(H \cup M) = N - n(H^c \cap M^c) \Longrightarrow \cancel{N} - 35$

$N = \dfrac{N - 35}{0.90}$

$N(0.90) = N - 35 \Longrightarrow (0.10)N = 35 \Longrightarrow N = \dfrac{35}{0.10} = 350$

$\boxed{N = 350}$

Number of candidates sitted for the exam is 350

Q8)

An integer from 300 through 780,

so the Possible outcomes ~~are~~ is

$= (780-299)$

$= 481$

we need to find the ~~p~~ right numbers.

we need to find 1 in,

digit 3, digit 2, digit 1

1 in digit 3 → it's '0' ~~b~~ because it can't possible to take 1 as the first digit as ~~7~~ the condition is (300 − 780).

~~ti~~ 1 in digit 2 → it's possible

= 311, ~~2~~ 411, 511, 611, 711.

so, '5' ways we can tak this

1 in digit 1 → It's Possible

= 301, 310, 312, 313, 314, 315, 316, 317, 318, 319,

321, 331, 341, 351, 361, ~~371~~ 371, 381, 391

here, 18 possible ways.

again → 401, 410, 412, 413, 414, 415, 416, 417, 418, 419,

421, 431, 441, 451, 461, 471, 481, 491

here, 18 possible ways.

again → 501, 510, 512, 513, 514, 515, 516, 517, 518, 519,

521, ~~53~~ 531, 541, 551, 561, 571, 581, 591

here, 18 possible ways.

again → 601, 610, 612, 613, 614, 615, 616, 617, 618, 619, 621,

631, ~~65~~ 641, 651, 661, 671, 681, 691

here, 18 possible ways.

again → 701, 710, 712, 713, 714, 715, 716, 717, 718, 719, 721,

~~2~~ 731, 741, 751, 761, 771,

here, 16 possible ways.

so the total outcomes = 0+5+ 18+ 18+18+ 18+16

= 93

so the probability = $\frac{93}{481}$ = 0.19335 (Ans)

Q9)

a)

$$P_B = \frac{P(10,2)}{2!} = 45 \text{ ways to park Blue cars}$$

$$P_y = \frac{P(8,4)}{4!} = 70 \text{ ways to park yellow cars}$$

for all cars we can use:

$$P = (P_B)\cdot(P_y) = (45)\cdot(70) = \underline{3150} \text{ ways to park all cars}$$

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b) In this case we need to consider that all cars are parked side by side to each other.

- For 6 cars we need 6 slots as an entity

$$P_s = C(5,1) = 5$$

- for Blue cars:

$$P_B = \frac{P(6,2)}{2!} = 15 \text{ ways to park}$$

- for Yellow cars:

$$P_y = \frac{P(4,4)}{4!} = 1 \Rightarrow \text{only one way to park}$$

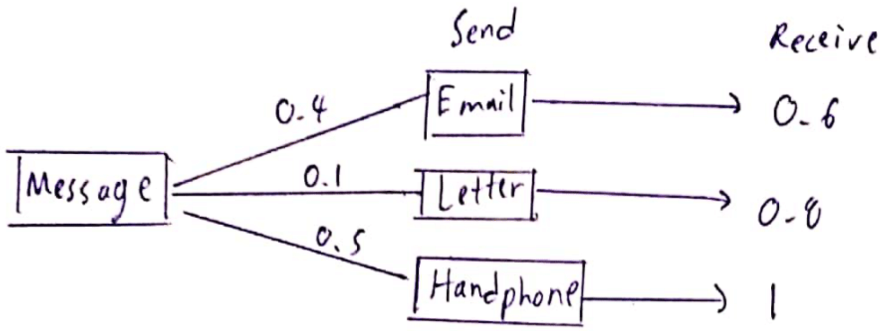
- for Parking slots or lots to be next to each other:

$$P = P_s \cdot P_B \cdot P_y = (5)\cdot(15)\cdot(1) = \underline{75} \text{ ways}$$

- Probability of lots being next to each other:

$$P = \frac{C(5,1)}{C(10,6)} = \frac{1}{42} = 0.023$$

Question 10



(a) Let A = receive the message

$$P(A) = (0.4 \times 0.6) + (0.1 \times 0.8) + (0.5 \times 1)$$
$$= 0.24 + 0.08 + 0.5$$
$$= 0.82$$

(b) Let B = via email

$$P(B \mid A) = \frac{P(B \cap A)}{P(A)}$$
$$= \frac{0.4 \times 0.6}{0.82}$$
$$= \frac{12}{41}$$

Q11)

Events:

A = light truck, A' = Cars, B = Fatal accident, B' = Non fatal accident

Givens:

$P(B|A') = \frac{20}{10000}$ and $P(B|A) = \frac{25}{100000} \Longrightarrow P(A) = 0.4$

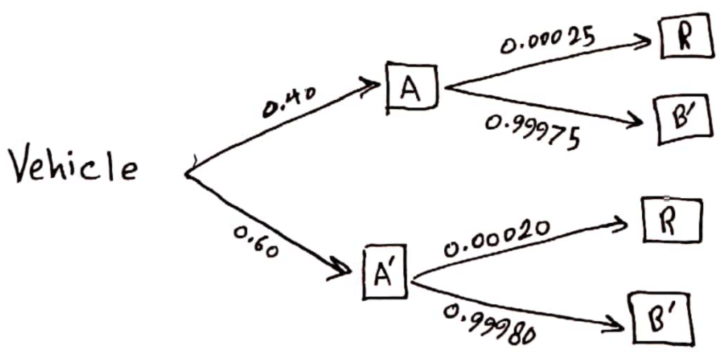
$P(A') = 1 - P(A) = 0.6$

We need to find $P(A|B)$

$$P(A|B) = \frac{P(B|A)P(A)}{P(B|A)P(A) + P(B|A')P(A')} = \frac{(0.00025)(0.4)}{(0.00025)(0.4) + P(0.00020)(0.6)} = 0.4545$$

$= 45.45\%$

using reverse tree method,



Question 12

Number of possible ways, without restriction is

$$4^9 = 262144 \text{ ways}$$

Number of disallowed ways

$$(4 \times 3^9) - (4 \times 1^9)$$

$$= 78732 - 4$$

$$= 78728 \text{ ways}$$

Number of ways to place these 9 letters into the 4 boxes such that each box contain at least 1 letter.

$$262144 - 78728$$

$$= 183416 \text{ ways}$$