SCHOOL OF COMPUTING
Faculty of Engineering

ASSIGNMENT 1

DISCRETE STRUCTURE (SECI 1013)
SEMESTER I, SESSION 2020/2021

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Section: 07

Q1)

1) $A = \{x \in R \mid 0 < x \leq 2\}$, $B = \{x \in R \mid 1 \leq x < 4\}$,

$C = \{x \in R \mid 3 \leq x < 9\}$

a) $A \cup C = \{x \in R \mid 0 < x \leq 2 \text{ or } 3 \leq x < 9\}$

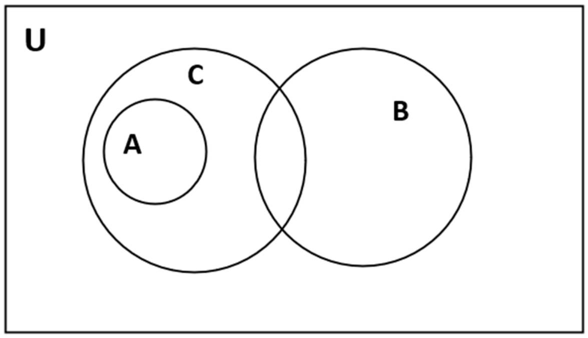
b) $(A \cup B)' = \{x \in R \mid \neg(0 < x < 4)\} = \{x \in R \mid x \leq 0 \text{ or } x \geq 4\}$

c) $A' \cup B' = \{x \in R \mid 0 \geq x \text{ or } 2 < x\} \cup \{x \in R \mid 1 > x \text{ or } 4 \leq x\}$

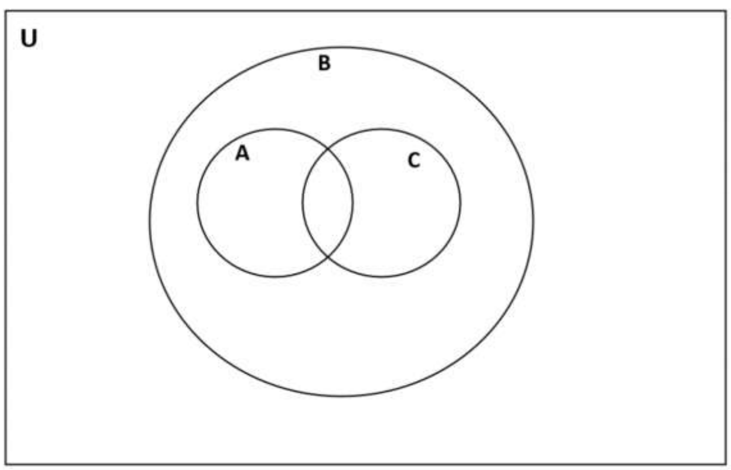
$= \{x \in R \mid x < 1 \text{ or } 2 < x\}$

Q2)

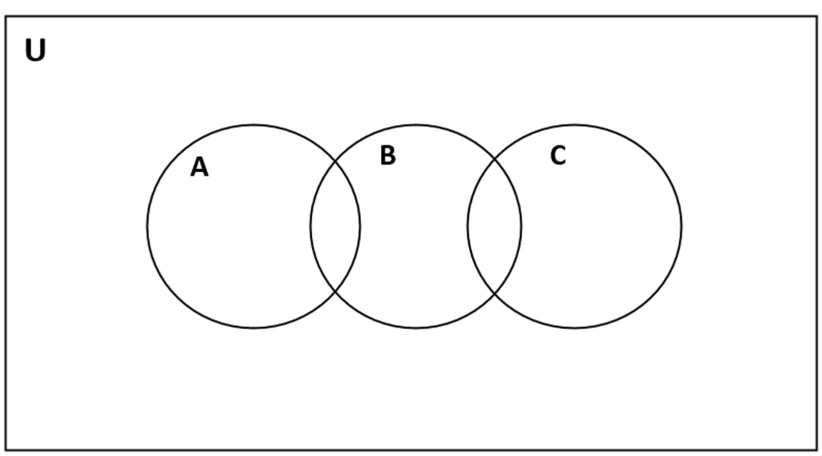
a) $A \cap B = \varnothing,\ A \subseteq C, C \cap B \neq \varnothing$



b) $A \subseteq B, \mathrm{C} \subseteq B, \mathrm{A} \cap C \neq \varnothing$



c) $A \cap B \neq \varnothing, B \cap C \neq \varnothing, A \cap C = \varnothing,\ A \not\subset B,\ C \not\subset B$



Q3)

3) $A = \{-1, 1, 2, 4\}$ and $B = \{1, 2\}$

$A \times B = \{(-1,1), (-1,2), (1,1), (1,2), (2,1), (2,2), (4,1), (4,2)\}$

for all $(x,y) \in A \times B$, $x S y \leftrightarrow |x| = |y|$

So, S relation between $A \times B$: $|-1| = 1$, $|-1| \neq |2|$, $|1| = |1|$, $|1| \neq |2|$,

$|2| \neq |1|$, $|2| = |2|$, $|4| \neq |1|$, $|4| \neq |2|$.

Thus, $S = \{(-1,1), (1,1), (2,2)\}$

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for all $(x,y) \in A \times B$, $x T y \leftrightarrow x - y =$ even

So, T relation between $A \times B$: $(-1)-1 =$ even, $(-1)-2 \neq$ even, $1-1 =$ even,

$1-2 \neq$ even, $2-1 \neq$ even, $2-2 =$ even, $4-1 \neq$ even, $4-2 =$ even.

Thus, $T = \{(-1,1), (1,1), (2,2), (4,2)\}$

$S \cap T = \{(x,y) \in A \times B \mid (x,y) \in S \text{ and } (x,y) \in T\}$
$= \{(-1,1), (1,1), (2,2)\}$

$S \cup T = \{(x,y) \in A \times B \mid (x,y) \in S \text{ or } (x,y) \in T\}$

$= \{(-1,1), (1,1), (2,2), (4,2)\}$

Q4)

4. $\neg\big((\neg p \wedge q) \vee (\neg p \wedge \neg q)\big) \vee (p \wedge q)$

$= \neg(\neg p \wedge q) \wedge \neg(\neg p \wedge \neg q) \vee (p \wedge q)$ [De Morgan's Laws]

$= (p \vee \neg q) \wedge (p \vee q) \vee (p \wedge q)$ [De Morgan's Laws]

$= p \vee (\neg q \wedge q) \vee (p \wedge q)$ [Distributive Laws]

$= p \vee \varnothing \vee (p \wedge q)$ [Complement Laws]

$= p \vee (p \wedge q)$ [Identity Laws]

$= p$ [Absorption Laws]

Q5)

a) $A_1 = \begin{bmatrix} 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 0 \\ 1 & 1 & 1 & 0 & 0 \\ 1 & 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 \end{bmatrix}$

b) $A_2 = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 \\ 1 & 1 & 0 & 0 & 0 \\ 1 & 1 & 1 & 0 & 0 \\ 1 & 1 & 1 & 1 & 0 \end{bmatrix}$

c) Since the main diagonal element of matrix A_1 is not all consist of 1's, therefore R_1 is not reflexive.

$$A_1 \text{ Transpose} = \begin{bmatrix} 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 0 \\ 1 & 1 & 1 & 0 & 0 \\ 1 & 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 \end{bmatrix}$$

Since matrix A_1 is equal to matrix A_1 Transpose, therefore R_1 is symmetric.

$$\begin{bmatrix} 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 0 \\ 1 & 1 & 1 & 0 & 0 \\ 1 & 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 \end{bmatrix} \otimes \begin{bmatrix} 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 0 \\ 1 & 1 & 1 & 0 & 0 \\ 1 & 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 \end{bmatrix}$$

Since $A_1 \otimes A_1 \neq A_1$, therefore R_1 is not transitive.

Since R_1 is not reflexive and not transitive, therefore R_1 is not equivalence relation.

d) Since the main diagonal element of matrix A_2 is not all consist of 1's, therefore R_2 is not reflexive.

Since for all $(y,z) \in R_2$ and $y \neq z$, $(z,y) \notin R_2$, therefore R_2 is antisymmetric.

$$\begin{bmatrix} 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 \\ 1 & 1 & 0 & 0 & 0 \\ 1 & 1 & 1 & 0 & 0 \\ 1 & 1 & 1 & 1 & 0 \end{bmatrix} \otimes \begin{bmatrix} 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 \\ 1 & 1 & 0 & 0 & 0 \\ 1 & 1 & 1 & 0 & 0 \\ 1 & 1 & 1 & 1 & 0 \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 \\ 1 & 1 & 0 & 0 & 0 \\ 1 & 1 & 1 & 0 & 0 \end{bmatrix}$$

Since $A_2 \otimes A_2 \neq A_2$, therefore R_2 is not transitive.

Since R_2 is not reflexive and not transitive, therefore R_2 is not partial order relation.

Q6)

6) $R_1 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 1 \\ 1 & 0 & 1 \end{bmatrix}$, $R_2 = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 1 & 0 \\ 1 & 0 & 1 \end{bmatrix}$

a) $R_1 \cup R_2 = \begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & 1 \\ 1 & 0 & 1 \end{bmatrix}$, b) $R_1 \cap R_2 = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 1 & 0 \\ 1 & 0 & 1 \end{bmatrix}$

Q7)

7) We need to prove that $\forall x, y \in R \ (f+g)(x) = (f+g)(y) \rightarrow x = y$

So, using indirect proof $\Rightarrow \neg q \rightarrow \neg p$

$x \neq y \rightarrow (f+g)(x) \neq (f+g)(y)$

$x \neq y \rightarrow f(x) \neq f(y)$ and $g(x) \neq g(y)$

We can see that f and g is in one-to-one relation

Both using one-to-one. Thus, f+g is one-to-one also.

Q8)

When n = 1
The different ways to climb the staircase :
(1)
The number of different ways to climb the staircase, $c_1 = 1$

When n = 2
The different ways to climb the staircase :
(1,1), (2)
The number of different ways to climb the staircase, $c_2 = 2$

When n = 3
The different ways to climb the staircase :
(1,1,1), (1,2), (2,1)
The number of different ways to climb the staircase, $c_3 = 3$

When n = 4
The different ways to climb the staircase :
(1,1,1,1), (1,1,2), (1,2,1), (2,1,1), (2,2)
The number of different ways to climb the staircase, $c_4 = 5$

When n = 5
The different ways to climb the staircase :
(1,1,1,1,1), (1,1,1,2), (1,1,2,1), (1,2,1,1), (2,1,1,1), (1,2,2), (2,1,2), (2,2,1)
The number of different ways to climb the staircase, $c_5 = 8$

The sequence of c_n is 1, 2, 3, 5, 8, …..
The recurrence relation for $c_1, c_2, \ldots, c_n$ is
$c_n = c_{n-1} + c_{n-2}$, $n \geq 3$ with $c_1 = 1$ and $c_2 = 2$

Q9)

a) $t_3 = t_2 + t_1 + t_0 = 1 + 1 + 0 = 2$

$t_4 = t_3 + t_2 + t_1 = 2 + 1 + 1 = 4$

$t_5 = t_4 + t_3 + t_2 = 4 + 2 + 1 = 7$

$t_6 = t_5 + t_4 + t_3 = 7 + 4 + 2 = 13$

$t_7 = t_6 + t_5 + t_4 = 13 + 7 + 4 = 24$

b) input : n

Output : t (n)

```
t(n) {
    if (n = 0)
    return 0
    if (n=1 or n=2)
    return 1
    return t(n-1) + t(n-2) + t(n-3)
    }
```