UNIVERSITI TEKNOLOGI MALAYSIA

TUTORIAL 1

GROUP NAME : T5

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SECTION : SECI1013 – 09

1. a) $A \cup C = \{x \in \mathbb{R} \mid 0 < x < 9\}$

b) $(A \cup B) = \{x \in \mathbb{R} \mid 0 < x < 4\}$

$(A \cup B)' = \{x \in \mathbb{R} \mid x \leq 0, x \geq 4\}$

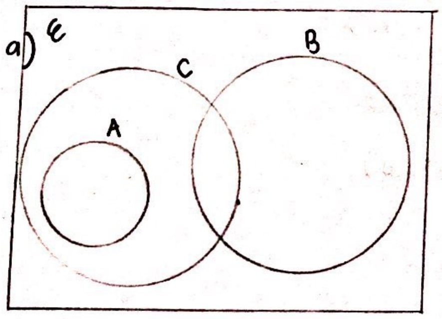
c) $A' \cup B' = (A \cap B)'$

$(A \cap B) = \{x \in \mathbb{R} \mid 1 \leq x \leq 2\}$

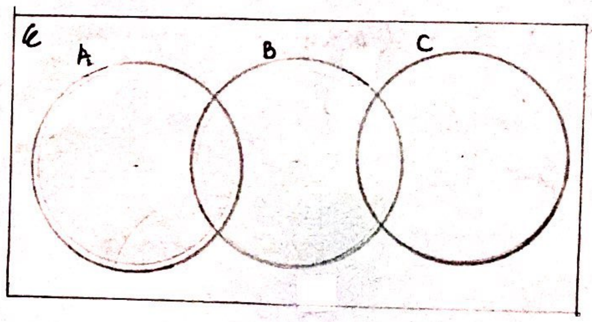
$(A \cap B)' = \{x \in \mathbb{R} \mid x \leq 0, x \geq 3\}$

$A' \cup B' = \{x \in \mathbb{R} \mid x \leq 0, x \geq 3\}$

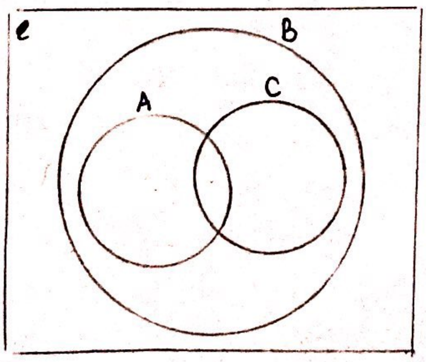
2. a)



c)



b)



3. $A \times B = \{(-1,1),(1,1),(2,1),(4,1),(-1,2),(1,2),(2,2),(4,2)\}$

$S = \{(1,1),(2,2),(-1,1)\}$

$T = \{(1,1),(2,2),(4,2),(-1,1)\}$

$S \cap T = \{(1,1),(2,2),(-1,1)\}$

$S \cup T = \{(1,1),(2,2),(4,2),(-1,1)\}$

4. $\neg((\neg p \wedge q) \vee (\neg p \wedge \neg q) \vee (p \wedge q) \equiv p$

$\equiv \neg(\neg p \wedge (q \vee \neg q)) \vee (p \wedge q)$ (Distributive laws)

$\equiv \neg(\neg p) \vee \neg(q \vee \neg q) \vee (p \wedge q)$ (De Morgan's Laws)

$\equiv \neg(\neg p) \vee (\neg q \wedge \neg\neg q) \vee (p \wedge q)$ (De Morgan's Laws)

$\equiv p \vee (\neg q \wedge q) \vee (p \wedge q)$ (Double negation laws)

$\equiv p \vee F \vee (p \wedge q)$ (contradiction)

$\equiv p \vee (p \wedge q)$

$\equiv p$ (absorption law)

5. $R_1 = \{(1,1),(1,2),(1,3),(1,4),(1,5),(2,1),(2,2),(2,3),(2,4),(3,1),(3,2),(3,3),(4,1),(4,3),(5,1)\}$

$R_2 = \{(2,1),(3,1),(3,2),(4,1),(4,2),(4,3),(5,1),(5,2),(5,3),(5,4)\}$

a) $A_1 = \begin{array}{c c} & \begin{array}{ccccc} 1 & 2 & 3 & 4 & 5 \end{array} \\ \begin{array}{c} 1 \\ 2 \\ 3 \\ 4 \\ 5 \end{array} & \begin{bmatrix} 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 0 \\ 1 & 1 & 1 & 0 & 0 \\ 1 & 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 \end{bmatrix} \end{array}$

b) $A_2 = \begin{array}{c c} & \begin{array}{ccccc} 1 & 2 & 3 & 4 & 5 \end{array} \\ \begin{array}{c} 1 \\ 2 \\ 3 \\ 4 \\ 5 \end{array} & \begin{bmatrix} 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 \\ 1 & 1 & 0 & 0 & 0 \\ 1 & 1 & 1 & 0 & 0 \\ 1 & 1 & 1 & 1 & 0 \end{bmatrix} \end{array}$

c) - R_1 is not reflexive since $(1,1)\ (2,2)\ (3,3) \in R$ but $(4,4)\ (5,5) \notin R_1$

$A_1^T = \begin{bmatrix} 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 0 \\ 1 & 1 & 1 & 0 & 0 \\ 1 & 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 \end{bmatrix}$

- R_1 is symmetric since $A_1 = A_1^T$

$A_1 \otimes A_1 =$

$$\begin{bmatrix} 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 0 \\ 1 & 1 & 1 & 0 & 0 \\ 1 & 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 \end{bmatrix} \otimes \begin{bmatrix} 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 0 \\ 1 & 1 & 1 & 0 & 0 \\ 1 & 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 \end{bmatrix}$$

- R_1 is not transitive since $A_1 \otimes A_1 \neq A_1$.

- R_1 is not equivalence relation since R_1 is not reflexive, and not transitive relation.

d) - R_2 is not reflexive since $(1,1)\ (2,2)\ (3,3)\ (4,4)\ (5,5) \notin R_2$.

- R_2 is anti-symmetric since $(2,1) \in R_2$ but $(1,2) \notin R_2$
$(3,1) \in R_2$ but $(1,3) \notin R_2$
$(3,2) \in R_2$ but $(2,3) \notin R_2$
$(4,1) \in R_2$ but $(1,4) \notin R_2$
$(4,2) \in R_2$ but $(2,4) \notin R_2$
$(4,3) \in R_2$ but $(3,4) \notin R_2$
$(5,1) \in R_2$ but $(1,5) \notin R_2$
$(5,2) \in R_2$ but $(2,5) \notin R_2$
$(5,3) \in R_2$ but $(3,5) \notin R_2$
$(5,4) \in R_2$ but $(4,5) \notin R_2$

$A_2 \otimes A_2 =$

$$\begin{bmatrix} 0&0&0&0&0 \\ 1&0&0&0&0 \\ 1&1&0&0&0 \\ 1&1&1&0&0 \\ 1&1&1&1&0 \end{bmatrix} \otimes \begin{bmatrix} 0&0&0&0&0 \\ 1&0&0&0&0 \\ 1&1&0&0&0 \\ 1&1&1&0&0 \\ 1&1&1&1&0 \end{bmatrix} = \begin{bmatrix} 0&0&0&0&0 \\ 0&0&0&0&0 \\ 1&0&0&0&0 \\ 1&1&0&0&0 \\ 1&1&1&0&0 \end{bmatrix}$$

- R_2 is not transitive since $A_2 \otimes A_2 \neq A_2$

- R_2 is not partial order since R_2 is not reflexive and not transitive.

6 a) $R_1 \cup R_2 = \{(1,1), (1,2), (2,2), (2,3), (3,1), (3,3)\}$

$$\begin{array}{c} \\ 1 \\ 2 \\ 3 \end{array} \begin{array}{c} \begin{array}{ccc} 1 & 2 & 3 \end{array} \\ \begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & 1 \\ 1 & 0 & 1 \end{bmatrix} \end{array}$$

b) $R_1 \cap R_2 = \{(2,2), (3,1), (3,3)\}$

$$\begin{array}{c} \\ 1 \\ 2 \\ 3 \end{array} \begin{array}{c} \begin{array}{ccc} 1 & 2 & 3 \end{array} \\ \begin{bmatrix} 0 & 0 & 0 \\ 0 & 1 & 0 \\ 1 & 0 & 1 \end{bmatrix} \end{array}$$

7. Let $f(x) = x$

$g(x) = -x$

$\therefore (f+g)(x) = f(x) + g(x)$

$= x + (-x)$

$= 0$

Based on the case above, $f(x)$ and $g(x)$ are both one-to-one function. However, the result of $(f+g)(x)$ for $\forall x$ is 0. Therefore, $f+g$ is not guarantee to be an one-to-one function although f and g are both one-to-one function.

8. n = number of stairs

C_n = number of different ways to climb the stairs.

When $n = 1$, $C_n = 1$

when $n = 2$, $C_n = 2$

When $n \geq 3$, staircase has more than 2 stairs and combination of 1-stair and 2 stairs increment required.

If the last move is 1-stairs, there are C_{n-1} ways to climb

If the last move is 2-stairs, there are C_{n-2} ways to climb

$\therefore$ The reccurence relation for $C_1, C_2, \dots, C_n$ is

$$C_n = C_{n-1} + C_{n-2}, \quad n \geq 3, \; C_1 = 1, \; C_2 = 2$$

9\. a) $t_7 = t_{7-1} + t_{7-2} + t_{7-3}$

$= t_6 + t_5 + t_4$

$= 13 + 7 + 4$

$= 24$

$t_3 = t_2 + t_1 + t_0$

$= 1 + 1 + 0$

$= 2$

$t_4 = t_3 + t_2 + t_1$

$= 2 + 1 + 1$

$= 4$

$t_5 = t_{5-1} + t_{5-2} + t_{5-3}$

$= t_4 + t_3 + t_2$

$= 4 + 2 + 1$

$= 7$

$t_6 = t_{6-1} + t_{6-2} + t_{6-3}$

$= t_5 + t_4 + t_3$

$= 7 + 4 + 2$

$= 13$

b)
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t(n)
{   if (n = 0)
       return 0
    if (n = 1 or n = 2)
       return 1

   return t(n-1) + t(n-2) + t(n-3)

}
```