Submitted by
Toya Lazmin Khan (A20EC0284)
Nafis Ahmed (A20EC4044)
Shah Sajid (A20EC4050)

Question-1

a) i) $A-B$

$\{1,2,3,4,5,6,7,8\} - \{2,5,9\}$

$= \{1,3,4,6,7,8\}$

ii) $(A \cap B) \cup C$

$= A \cap B$

$= \{1,2,3,4,5,6,7,8\} \cap \{2,5,9\}$

$= \{2,5\}$

$(A \cap B) \cup C$

$= \{2,5\} \cup \{a,b\}$

$= \{2,5,a,b\}$

iii) $A \cap B \cap C$

$= \{1,2,3,4,5,6,7,8\} \cap \{2,5,9\} \cap \{a,b\}$

$= \{2,5\} \cap \{a,b\}$

$= \emptyset$

iv) $B \times C$

$= \{2,5,9\} \times \{a,b\}$

$= \{(2,a),(2,b),(5,a),(5,b),(9,a),(9,b)\}$

v) $P(C)$

$C = \{a,b\}$

$P(C) = \{\{a\},\{b\},\{a,b\},\emptyset\}$

b) $(P\cap((P'\cup Q'))\cup(P\cap Q)=P$

= Left-hand side

$= (P\cap(P'\cup Q'))\cup(P\cap Q)$

$= (P\cap \cancel{(P\cap Q)'})\,\cancel{\cup (P\cap Q)}$ ~~(De morgan's law)~~

$= P\cap \cancel{((P\cap Q)'\cup Q)}$

$= (P\cap P')\cup(P\cap Q')\cup(P\cap Q)$ (Distributive laws)

$= \emptyset \cup (P\cap Q')\cup(P\cap Q)$ (complement laws)

$= (P\cap Q')\cup(P\cap Q)$ (identity laws)

$= P\cap(Q'\cup Q)$ (distributive laws)

$= P\cap U$ (complement laws)

$= P$ (identity laws)

c) $A = (\neg p \vee q) \leftrightarrow (q \rightarrow p)$

p	q	$\neg p$	$\neg p \vee q$	$q \rightarrow p$	$(\neg p \vee q) \leftrightarrow (q \rightarrow p)$
T	T	F	T	T	T
T	F	F	F	T	F
F	T	T	T	F	F
F	F	T	T	T	T

d) Direct Proof

if x is odd, $(x+2)^2$ is odd.

$x = 2n+1$

$x+2 = 2n+1+2$

$x+2 = 2n+3$

$(x+2)^2 = (2n+3)^2$

$\Rightarrow 4n^2 + 12n + 9$

$\Rightarrow 4n^2 + 12n + 8 + 1$

$\Rightarrow 2(2n^2 + 6n + 4) + 1$ (Let, $k = 2n^2 + 6n + 4$)

$\Rightarrow 2k+1$

$\Rightarrow$ proved

if x is odd, $(x+2)^2$ is also odd.

e) i) $\exists x \exists y P(x,y)$

This statement is TRUE

because if $x = 12$ and $y = 10$

then, $x \geq y$ is TRUE

as x is ~~greathe~~ greater than y, ~~there exists an~~ x

ii) $\forall x \forall y P(x,y)$

This statement is FALSE

as every x is not greater than every y.

For example,

if $x = 1$ and $y = 2$,

$x \geq y$ is FALSE.

$x \geq y$ condition, not satisfied.

Question 2

a) i)

$$\begin{array}{c|ccc} & 1 & 2 & 3 \\ \hline 1 & 1 & 1 & 0 \\ 2 & 0 & 1 & 0 \\ 3 & 1 & 0 & 0 \end{array}$$

~~Domain = {(1,1), (1,2), (2,2), (3,1)}~~

Domain = {1, 2, 3}

Range = {1, 2}

ii)

$$\begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{bmatrix}$$

the relation is not irreflexive as the diagonal ~~has a 1~~ is not all zero,

the relation is antisymmetric as the relation has (1,2) but no (2,1) it has (3,1) but no (1,3) and it also has (1,1) and (2,2)

b) i) $X = \{2, 3, 4, 5\}$ $\{2,3,4,5\}$

$S = \{(4,5), (5,4), (5,5)\}$

ii) M_2 matrix for S

$$M = \begin{array}{c|cccc} & 2 & 3 & 4 & 5 \\ \hline 2 & 0 & 0 & 0 & 0 \\ 3 & 0 & 0 & 0 & 0 \\ 4 & 0 & 0 & 0 & 1 \\ 5 & 0 & 0 & 1 & 1 \end{array}$$

the relation is not reflexive because the diagonal does not have all 1 in it.

the relation is symmetric because if we transpose, the matrixs are the same. shown below

$$M_2 \begin{vmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 1 \end{vmatrix}, M_T = \begin{vmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 1 \end{vmatrix}$$

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the relation is not transitive as the product of matrix of S is not the same as the matrix of S . Shown below

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$$M \times M = \begin{vmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 1 \end{vmatrix} \times \begin{vmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 1 \end{vmatrix} = \begin{vmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 1 & 1 \end{vmatrix}$$

the relation is not equivalence as the relation is not reflexive, not transitive .

c) i) $f : X \to Y$

$$f = \{(1,1), (2,2), (3,3)\}$$

ii) $g : X \to Z$

$$\cancel{g = \{(1,1), (2,2), (3,3), (3,4)\}}$$

$$g = \{(1,1), (2,2), (3,2)\}$$

iii) $h : X \to X$

$$h = \{(1,1), (2,1), (3,2)\}$$

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d) i) $m(x) = 4x+3$

Let, $y = m(x)$

$y = 4x+3$

$y-3 = 4x$

$x = \frac{y-3}{4}$

So, $f^{-1}(y) = \frac{y-3}{4}$

d ii) $n \circ m$

$m(x) = 4x+3$, $n(x) = 2x-4$

$n \circ m$

$= 2(4x+3) - 4$

$= 8x+6-4$

$= 8x+2$

Question 3

a) i) $a_k = a_{k-1} + 2k$, $k > 2$, $a_1 = 1$

~~$a_2 = a_1$~~

$a_2 = a_1 + 2k \Rightarrow 1 + 2\times2 \Rightarrow 5$

$a_3 = a_2 + 2k \Rightarrow 5 + 2\times3 \Rightarrow 11$

$a_4 = a_3 + 2k \Rightarrow 11 + 2\times4 \Rightarrow 19$

1st three terms = 5, 11, 19

ii) Input: k

Output: a(k)

```
a(k) { if (k=1)
          return 1
       return a_{k-1} + 2k }
```

b) O_k = number of operations

So, $O_1 = 7$ $n_1 = 7$

So, for $k \geq 2$,

$n_k = 2n_{k-1}$

$O_k = 2O_{k-1}$

b) r_k = number of operations

So,

$r_1 = 7$

So, for $r \geq 2$

$r_k = 2r_{k-1}$

recursive relation $\Rightarrow r_k = 2r_{k-1}$

b) r_k = number of operations

So, $\cancel{r_1 = 7}$ $r_1 = 1$, $r_2 = 7$

So, for $r \geq 2$,

$\cancel{r_k = 2r_k - 1}$ $r_k = 2r_{k-1}$

So, value for $r_2 = 2r_1 = 14$

recursive relation $\Rightarrow r_k = 2r_{k-1}$

e) Trace S(4)

$5 * S(n-1)$

$5 * S(3)$

$S(3) = 5 * S(2)$

$S(2) = 5 * S(1)$

$= 5 * 5$

$= 25$

So, $S(4) = 25 * 5 * 5$

$= 625$

Question 4

a) $\underset{\to 1st}{\underline{9P_1}}$ $\underset{\to 2nd}{\underline{16P_1}}$ $\underset{\to 3rd}{\underline{16P_1}}$ $\underset{\to 4th}{\underline{11P_1}}$

$= 9P_1 \times 16P_1 \times 16P_1 \times 11P_1$

$= 25344$

b) automobiles have 7 4 letter first and 3 digits later

$\underset{1P_1}{\underline{A}}$ $\underbrace{\_\ \_\ \_}_{26^3}$ $\underbrace{\_\ \_}_{10^2}$ $\underset{1P_1}{\underline{O}}$

~~$= 1P_1 \times 26P_3 \times 10P_2 \times 1P_1$~~ $= 1P_1 \times 26^3 \times 10^2 \times 1P_1 = 1757600$

c) COMPUTER = 8 letters

for 3 letters = $\overline{}_{8P_1}\ \overline{}_{7P_1}\ \overline{}_{6P_1}$

$= 8P_3$

for 2 letters = $\overline{}_{8P_1}\ \overline{}_{7P_1}$

$= 8P_2$

for 1 letter = $\overline{}_{8P_1}$

so, arrangements = $8P_3 + 8P_2 + 8P_1$

$= 336 + 56 + 8$

$= 400$

d) groups of seven ⇒ 7C

4 women chosen = $7C_4$

3 men " = $6C_3$

groups of seven formed = $7C_4 \times 6C_3$

$= 35 \times 20$

$= 700.$

e) PROBABILITY → 11 letters

arrangements = $\frac{11!}{2!\,2!}$ = 9979200

b) $n = 6,\ r = 10$

$C(6+10-1, 10)$

$C(15, 10)$

$^{15}C_{10}$

$= 3003$

→ when, where, who

Question 5

a) Pigeons, N → 18

Pigeons holes, K → combinations of cards name → $3 \times 2 = 6$.

$\left\lceil \frac{N}{K} \right\rceil = \left\lceil \frac{18}{6} \right\rceil = 3$ = shown.

b) even number = 10

odd number = 10

So, for worst take we get 10 even numbers

So, for getting atleast 1 odd we take $= 10 + 1$

$= 11$ numbers (Answer)

c) numbers divisible by 5 ~~between~~ ~~the~~ from 1 to 100

$= 20$

numbers not divisible by 5 $= 80$

so, for getting one integer divisible by 5 we pick $= 81$ numbers (Answer)