

1. $|v-w| \leq 3$

$$1 = \{(1,1), (1,2), (1,3), (1,4)\}$$

$$2 = \{(2,2), (2,3), (2,4), (2,5)\}$$

$$3 = \{(3,3), (3,4), (3,5), (3,6)\}$$

$$4 = \{(4,4), (4,5), (4,6), (4,7)\}$$

$$5 = \{(5,5), (5,6), (5,7), (5,8)\}$$

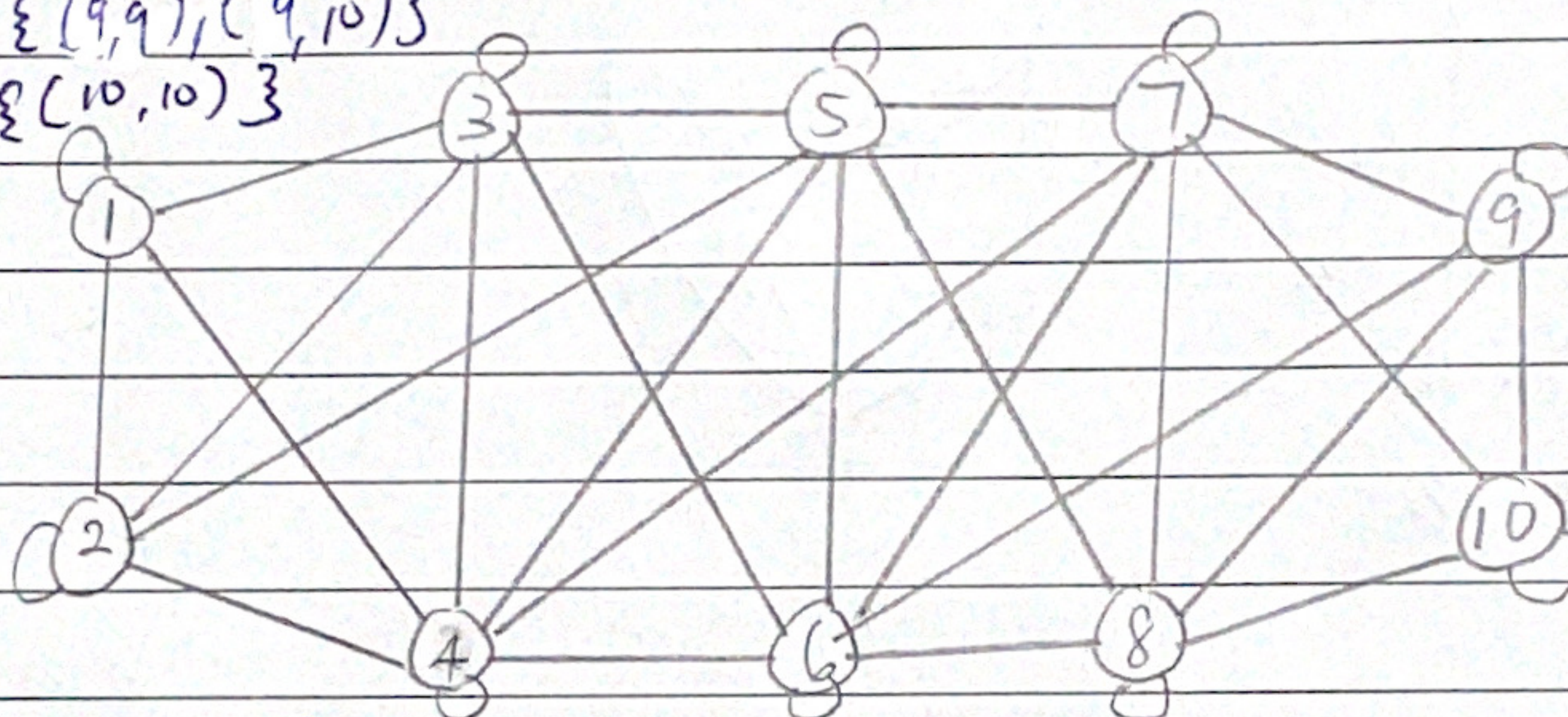
$$6 = \{(6,6), (6,7), (6,8), (6,9)\}$$

$$7 = \{(7,7), (7,8), (7,9), (7,10)\}$$

$$8 = \{(8,8), (8,9), (8,10)\}$$

$$9 = \{(9,9), (9,10)\}$$

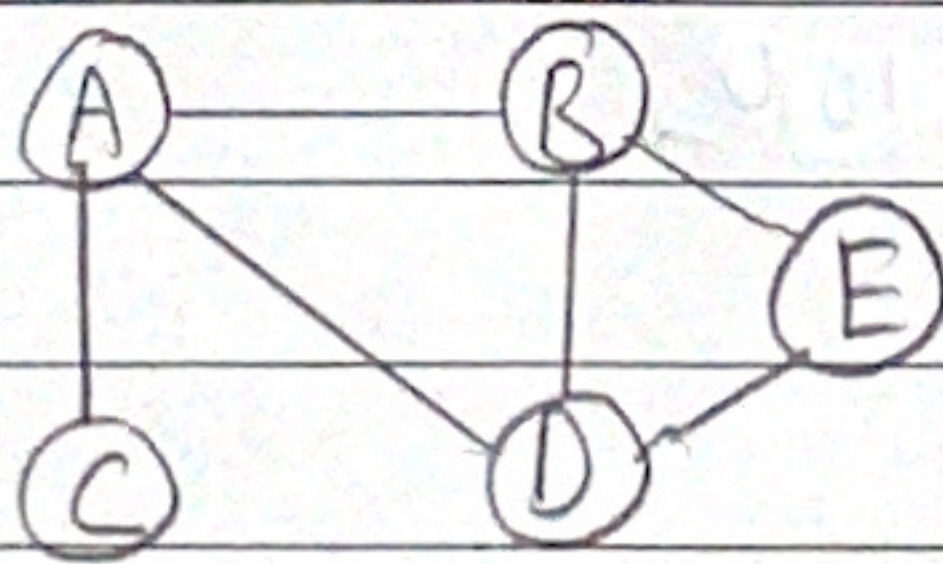
$$10 = \{(10,10)\}$$



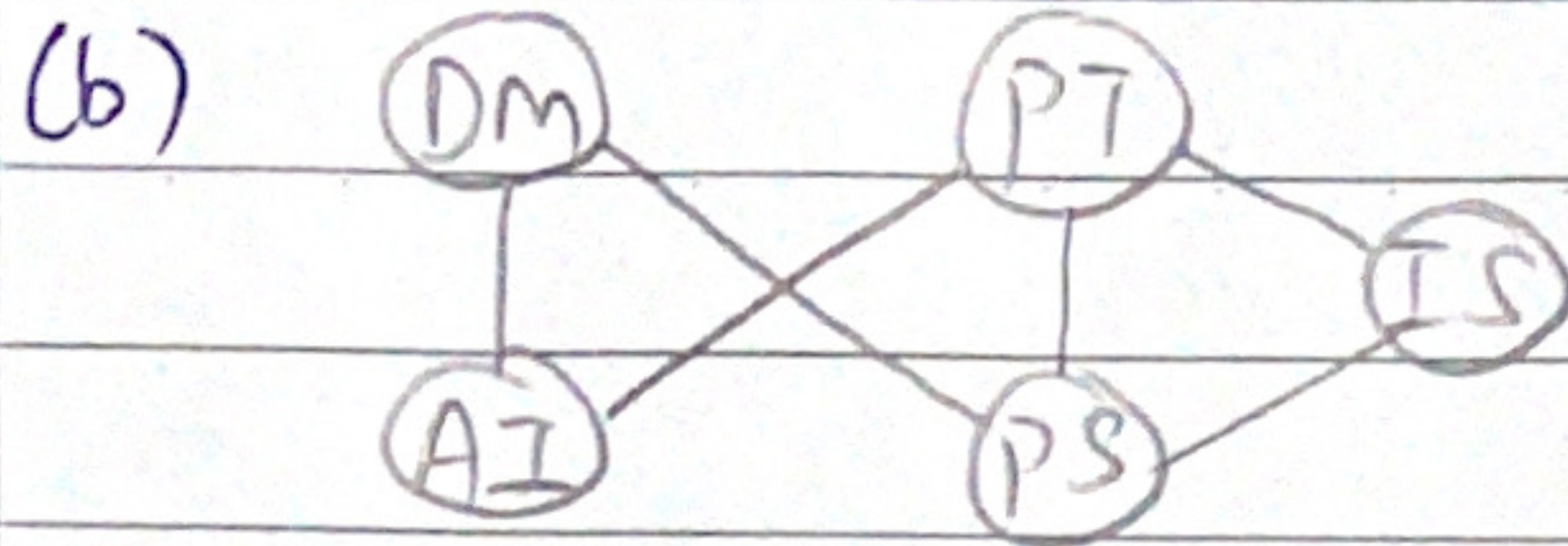
$$e(G) = 4 + 4 + 4 + 4 + 4 + 4 + 4 + 3 + 2 + 1$$

$$= 34$$

2 (a) A = Ahmad ; B = Babri ; C = Chong ; D = David ; E = Ehsan

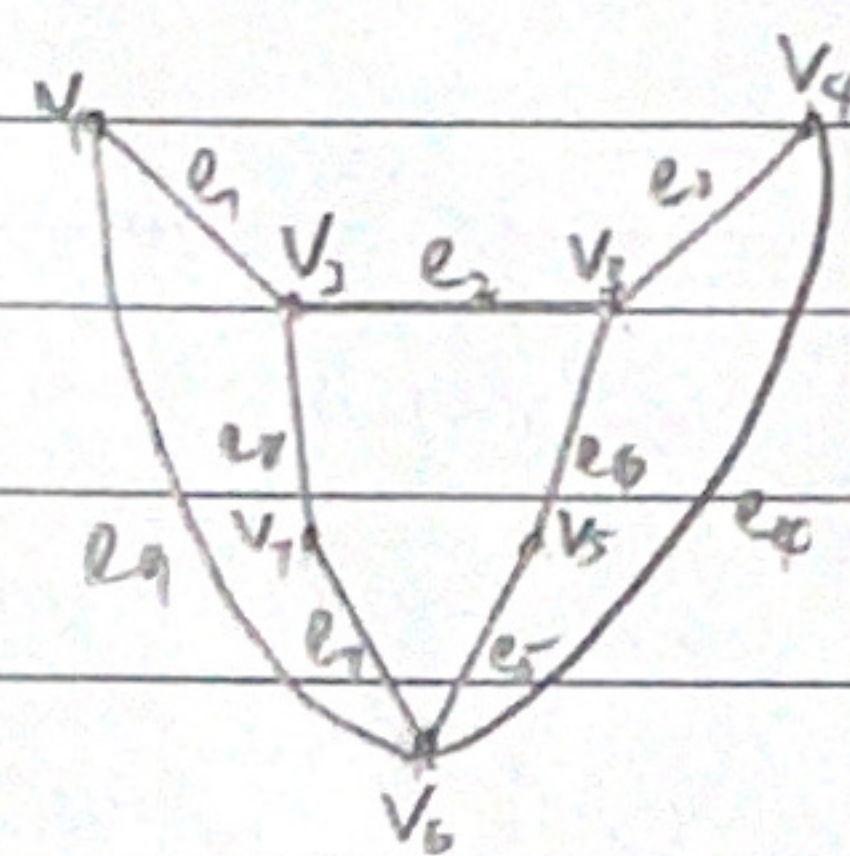
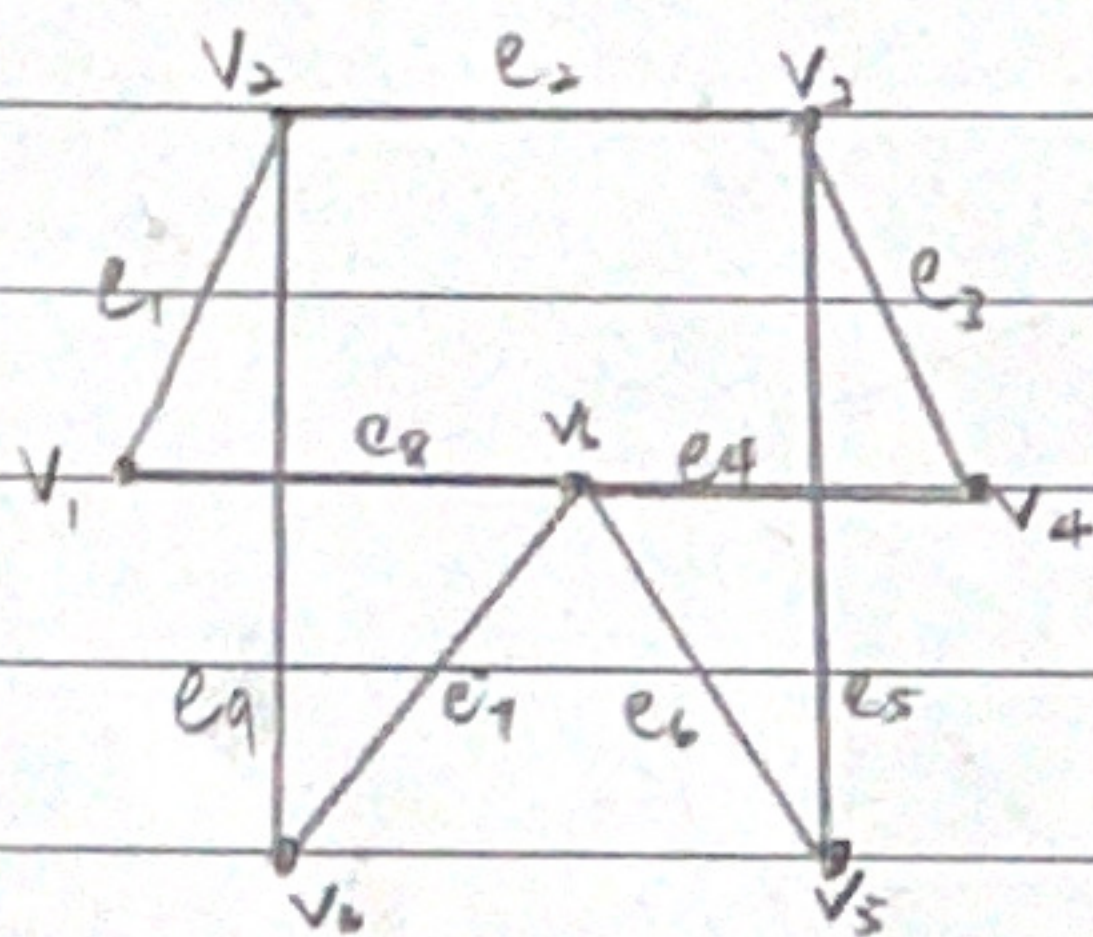


$$A_G = \begin{matrix} & \begin{matrix} A & B & C & D & E \end{matrix} \\ \begin{matrix} A \\ B \\ C \\ D \\ E \end{matrix} & \begin{bmatrix} 0 & 1 & 1 & 1 & 0 \\ 1 & 0 & 0 & 1 & 1 \\ 1 & 0 & 0 & 0 & 0 \\ 1 & 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 1 & 0 \end{bmatrix} \end{matrix}$$



	DM	PT	AI	PS	IS
DM	0	0	1	1	0
PT	0	0	1	1	1
AI	1	1	0	0	0
PS	1	1	0	0	1
IS	0	1	0	1	0

3.



4. G:

adjacency matrix

	V1	V2	V3	V4
V1	1	0	2	0
V2	0	0	1	0
V3	2	1	0	2
V4	0	0	2	0

incidence matrix

	e1	e2	e3	e4	e5	e6
V1	2	1	1	0	0	0
V2	0	0	0	1	0	0
V3	0	1	1	1	1	1
V4	0	0	0	0	1	1

H:

adjacency matrix

	V1	V2	V3	V4
V1	1	0	0	0
V2	0	1	1	2
V3	0	1	1	0
V4	0	2	0	0

incidence matrix

	e1	e2	e3	e4	e5	e6
V1	2	0	0	0	0	0
V2	0	2	1	1	1	0
V3	0	0	0	0	1	2
V4	0	0	1	1	0	0

5. (a) G_1 and G_2 both have 5 vertices and 5 edges.

G_1 and G_2 both have 3 vertices with 2 degrees.

G_1 and G_2 both have 1 vertex with 3 degrees.

G_1 and G_2 both have 1 vertex with 1 degree.

adjacency matrix for G_1 :

	v_1	v_2	v_3	v_4	v_5
v_1	0	1	0	1	0
v_2	1	0	1	0	0
v_3	0	1	0	1	1
v_4	1	0	1	0	0
v_5	0	0	1	0	0

adjacency matrix for G_2 :

	u_5	u_4	u_3	u_2	u_1
u_5	0	1	0	1	0
u_4	1	0	1	0	0
u_3	0	1	0	1	1
u_2	1	0	1	0	0
u_1	0	0	1	0	0

Both G_1 and G_2 adjacency matrix is the same.

Therefore, G_1 and G_2 are isomorphic.

(b) H_1 and H_2 both have 5 vertices and 6 edges.

H_1 has 2 vertices with 1 degree, 1 vertex with 5 degrees, 1 vertex with 3 degrees and 1 vertex with 2 degrees.

H_2 has 3 vertices with 3 degrees, 1 vertex with 2 degrees and 1 vertex with 1 degree.

Therefore, H_1 and H_2 are not isomorphic as the degree of vertices are different.

6. (a) trails

(b) walk

(c) simple circuits

(d) cycles

(e) closed walks

(f) paths

7. (a) 3 paths

(b) $3 + 3! = 9$ trails

(c) infinite walk

8. Graph (a) have Euler circuit. This is because (a) contains every vertex with positive even degree: 2 vertices with 2 degrees, 3 vertices with 4 degrees.
Euler circuit: $V_2 e_1 V_1 e_2 V_4 e_5 V_3 e_4 V_2 e_2 V_5 e_7 V_4 e_6 V_5 e_3 V_2$.

Graph (b) does not have Euler circuit. This is because (b) does not contain every vertex with positive even degree: 5 vertices with 2 degrees, 3 vertices with 3 degrees, 1 vertices with 5 degrees and 1 vertices with 4 degrees.

9. There is a Euler path in (a). (a) consist of 2 vertex with odd degree while others all are positive degree. Euler path: $U, V_1, V_0, V_1, U, V_2, V_3, V_4, V_2, V_0, V_4, W, V_0, V_5, W$.

There is no Euler path in (b). This is because (b) contains 4 vertices with odd degree.

10. There is Hamiltonian circuit in graph (a) while graph (b) does not have Hamiltonian circuit.

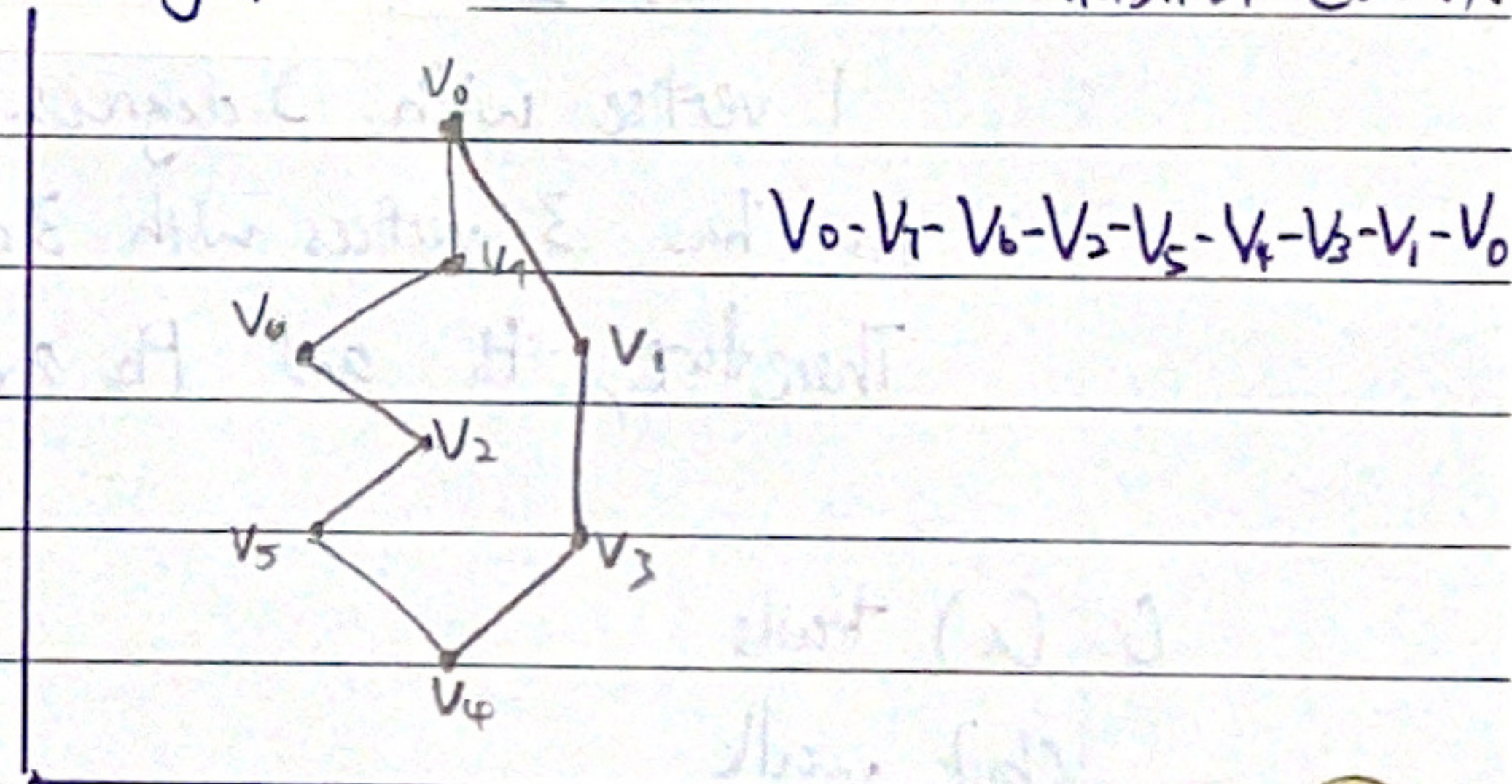
11. $n = 100, m = 3$

$$L = n - \frac{(n-1)}{m}$$

$$= 100 - \frac{(100-1)}{3}$$

$$= 67$$

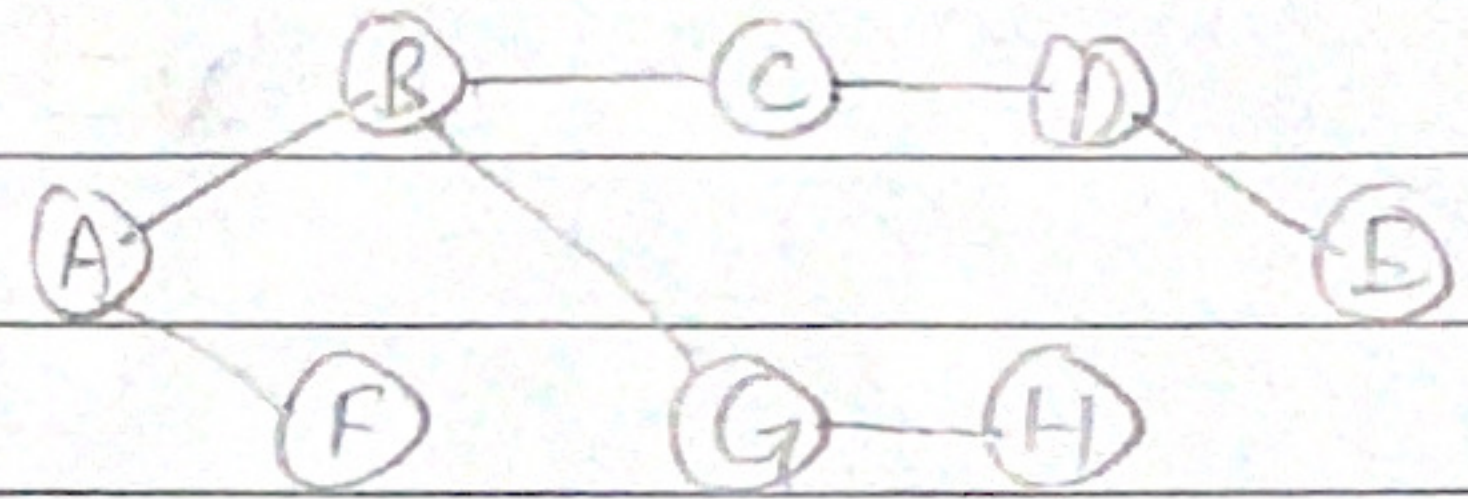
The tree have 67 leaves.



12. (a) a, b, d, e, g, h, j, n
 (b) a, b, d, e, g, h, j, n
 (c) c, f, i, k, l, m, o, p, q, r, s
 (d) r, s
 (e) b
 (f) l, m
 (g) a, d, j, q
 (h) b, e, f, g, k, l, m, n, r, s

13. preorder : a, b, e, k, l, m, f, g, n, r, s, c, d, h, o, i, j, p, q
 inorder : k, e, l, n, b, f, r, n, s, g, a, c, o, h, d, i, p, j, q
 postorder : k, l, m, e, f, r, s, n, g, b, c, o, h, i, p, q, d, d, a

14. AB 1 ∴ AB 1
 GH 1 GH 1
 BC 2 BC 2
 AF 3 AF 3
 BF 4 CD 4
 CD 4 BG 5
 BG 5 DE 7
 CG 6
 DH 6
 DG 7
 DE 7
 FG 8
 EH 8



$$\text{Total weight} = 1 + 1 + 2 + 3 + 4 + 5 + 7 = 23$$

15. Iteration	S	N	L(M)	L(N)	L(O)	L(P)	L(Q)	L(R)	L(S)	L(T)
1.	{3}	{M, N, O, P, Q, R, S, T}	0	∞	∞	∞	∞	∞	∞	∞
2.	{M}	{N, O, P, Q, R, S, T}	0	4	∞	2	∞	5	∞	∞
3.	{M, P}	{N, O, Q, R, S, T}	0	4	∞	2	6	5	5	∞
4.	{M, P, N}	{O, Q, R, S, T}	0	4	10	2	6	5	5	∞
5.	{M, P, N, R}	{O, Q, S, T}	0	4	7	2	6	5	5	6
6.	{M, P, N, R, S}	{O, Q, T}	0	4	7	2	6	5	5	6
7.	{M, P, N, R, S, T}	{O, Q}	0	4	7	2	6	5	5	6

shortest path from M to T is 6, M-R-T.