SECI 1013

DISCRETE STRUCTURE

TUTORIAL 3

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1a.) $A = \{1,2,3,4,5,6,7,8\}$

$B = \{2,5,9\}$

$C = \{a,b\}$

i.) $A - B = \{1,3,4,6,7,8\}$

ii.) $(A \cap B) \cup C = \{2,5\} \cup \{a,b\}$

$= \{2,5,a,b\}$

iii.) $A \cap B \cap C = \emptyset$

iv.) $B \times C = \{2,5,9\} \times \{a,b\}$

$= \{(2,a),(5,a),(9,a),(2,b),(5,b),(9,b)\}$

v.) $P(C) = \{\emptyset, \{a\}, \{b\}, \{a,b\}\}$

b.) $(P \cap ((P' \cup Q)')) \cup (P \cap Q) = (P \cap (P \cap Q')) \cup (P \cap Q)$ De Morgan's Law

$= ((P \cap P) \cap Q') \cup (P \cap Q)$ Associative law

$= (P \cap Q') \cup (P \cap Q)$ idempotent law

$= P \cap (Q \cup Q')$ distributive law

$= P \cap (U)$ Complement law

$= P$ # Properties of universal set

c)

p	q	$\neg p$	$\neg p \vee q$	$q \to p$	$(\neg p \vee q) \leftrightarrow (q \to p)$
T	T	F	T	T	T
T	F	F	F	T	F
F	T	T	T	F	F
F	F	T	T	T	T

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d.) For all integer x, if x is odd, then $(x+2)^2$ is odd.

let $P(x)$: x is odd.

$Q(x)$: $(x+2)^2$ is odd

$\forall x\ (P(x) \to Q(x))$

x is odd, $x = 2n+1$

$(x+2)^2$ is odd, $(x+2)^2 = 2m+1$

$$(2n+1+2)^2 = (2n+3)(2n+3)$$
$$= 4n^2 + 6n + 6n + 9$$
$$= 4n^2 + 12n + 8 + 1$$
$$= 2(2n^2 + 6n + 4) + 1$$
$$= 2m + 1$$

$\therefore (x+2)^2$ is odd is proved

e.) i.) $\exists x \exists y\ P(x,y)$ = True

The statement is true when $x=2, y=1$.

ii) $\forall x \forall y\ P(x,y)$ = False

The statement is false when $x=2, y=1$.

2a.) $R = \{(1,1), (1,2), (2,2), (3,1)\}$

Domain = $\{1, 2, 3\}$

Range = $\{1, 2\}$

ii.) The relation is not irreflexive, it is not reflexive as it included $(1,1)$ and $(2,2)$ $\in R$, but $(3,3) \notin R$.

The relation is antisymmetric.

$(1,2) \in R$, $(2,1) \notin R$

$(3,1) \in R$, $(1,3) \notin R$

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b.) i.) $S = \{(4,5),(5,4),(5,5)\}$

ii.)
$$S = \begin{array}{c|cccc} & 2 & 3 & 4 & 5 \\ \hline 2 & 0 & 0 & 0 & 0 \\ 3 & 0 & 0 & 0 & 0 \\ 4 & 0 & 0 & 0 & 1 \\ 5 & 0 & 0 & 1 & 1 \end{array}$$

S is not reflexive as $(2,2),(3,3),(4,4) \notin S$

$$\begin{array}{c|cccc} & 2 & 3 & 4 & 5 \\ \hline 2 & 0 & 0 & 0 & 0 \\ 3 & 0 & 0 & 0 & 0 \\ 4 & 0 & 0 & 0 & 1 \\ 5 & 0 & 0 & 1 & 1 \end{array} = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 1 \end{bmatrix}$$

S is symmetric as $M_S = M_{S^T}$

$$\begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 1 \end{bmatrix} \otimes \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 1 \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 1 & 1 \end{bmatrix}$$

S is not transitive as $M_S \otimes M_S \neq M_S$

S is not equivalence relation as it is not reflexive and not transitive.

c.) i.) $f = \{(1,2),(2,3),(3,4)\}$

ii.) $f = \{(1,1),(1,2),(2,1),(3,2)\}$

iii.) $f = \{(1,1),(1,2),(2,1),(2,2),(3,2)\}$

d.) i.) let $y = 4x+3$

$y-3 = 4x$

$\frac{y-3}{4} = x$

$f^{-1}(x) = \frac{x-3}{4}$

ii.) $nom = 2(4x+3)-4$

$= 8x+6-4$

$= 8x+2$

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3.) a.) i.) $a_1 = 1$

$a_2 = a_{2-1} + 2(2)$

$= a_1 + 4$

$= 1 + 4$

$= 5$

$a_3 = a_{3-1} + 2(3)$

$= a_2 + 6$

$= 5 + 6$

$= 11$

∴ The first three terms is 1, 5, 11

ii.) a(k)

{ if k = 1

return 1

return a(k-1) + 2k }

b.) $r_1 = 7$

$r_2 = 2a_{k-1}$ when $k \geq 2$

c.) $S(1) = 5$

$S(2) = 5 * S(2-1)$

$= 5 * S(1)$

$= 5 * 5$

$= 25$

$S(3) = 5 * S(3-1)$

$= 5 * S(2)$

$= 5 * 25$

$= 125$

$S(4) = 5 * S(4-1)$

$= 5 * S(3)$

$= 5 * 125$

$= 625$

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4. a.) Digit 3 through B = {3, 4, 5, 6, 7, 8, 9, A, B}
= 9

Digit 5 through F = {5, 6, 7, 8, 9, A, B, C, D, E, F}
= 11

Total hexadecimal numbers can form = $9 \times 16 \times 16 \times 11$
= 25344

b.) A to Z = 26
0 to 9 = 10
Total = $1 \times 26 \times 26 \times 26 \times 10 \times 10 \times 1$
= 1757600

c.) 1 letter = 8
2 letters = 8×7
= 56
3 letters = $8 \times 7 \times 6$
= 336
Total arrangement = 8 + 56 + 336
= 400

d.) ${}^7C_4 \times {}^6C_3 = 35 \times 20$
= 700

e.) $\frac{11!}{2!2!} = 9979200$

f.) $\frac{(6+10-1)!}{10!5!} = 3003$

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5. a.) pigeon, $n = 18$ people

3 first name and 2 last name $= 3 \times 2$

$= 6$

pigeonhole, $k = 6$

$\left\lceil \frac{18}{6} \right\rceil = 3$

b.) odd integer $= \{1, 3, 5, 7, 9, 11, 13, 15, 17, 19\}$

$= 10$

even integer $= \{2, 4, 6, 8, 10, 12, 14, 16, 18, 20\}$

$= 10$

The person may take all even number first, so if wanted to get at least 1 odd number, the person must pick at least 11 integers.

c.) integers divisible by $5 = \{5, 10, 15, 20, 25, 30, 35, 40, 45, 50, 55, 60, 65, 70, 75, 80, 85, 90, 95, 100\}$

$= 20$

integers not divisible by $5 = 100 - 20$

$= 80$

The person may pick all integers that are not divisible by 5 first, so if wanted to get at least 1 integer divisible by 5, the person must pick 81 integers.