



SECI 1013
DISCRETE STRUCTURE

TUTORIAL 1

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1. $A = \{x \in \mathbb{R} \mid 0 < x \leq 2\}$

$B = \{x \in \mathbb{R} \mid 1 \leq x < 4\}$

$C = \{x \in \mathbb{R} \mid 3 \leq x < 9\}$

a) $A \cup C = \{x \in \mathbb{R} \mid 0 < x \leq 2 \text{ or } 3 \leq x < 9\}$

$= \{x \in \mathbb{R} \mid 0 < x < 9\}$

b) $(A \cup B)' = \{x \in \mathbb{R} \mid \neg(0 < x \leq 2 \text{ or } 1 \leq x < 4)\}$

$= \{x \in \mathbb{R} \mid \neg(0 < x < 4)\}$

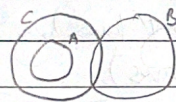
$= \{x \in \mathbb{R} \mid x \leq 0 \text{ or } x \geq 4\}$

c) $A' \cup B' = \{x \in \mathbb{R} \mid \neg(0 < x \leq 2) \text{ or } \neg(1 \leq x < 4)\}$

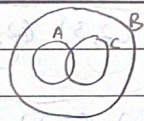
$= \{x \in \mathbb{R} \mid (x \leq 0 \text{ or } x \geq 2) \text{ or } (x < 1 \text{ or } x \geq 4)\}$

$= \{x \in \mathbb{R} \mid x < 1 \text{ or } x \geq 2\}$

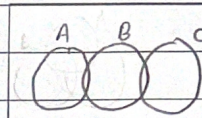
2. a.)



b.)



c.)



3. $S = \{(-1, 1), (1, 1), (2, 2)\}$

$T = \{(-1, 1), (1, 1), (2, 2), (4, 2)\}$

$A = \{-1, 1, 2, 4\}$

$B = \{1, 2\}$

$A \times B = \{(-1, 1), (-1, 2), (1, 1), (1, 2), (2, 1), (2, 2), (4, 1), (4, 2)\}$

$S \cap T = \{(-1, 1), (1, 1), (2, 2)\}$

$S \cup T = \{(-1, 1), (1, 1), (2, 2), (4, 2)\}$

4. $\neg((\neg p \wedge q) \vee (\neg p \wedge \neg q)) \vee (p \wedge q)$

$= \neg((\neg p \wedge (q \vee \neg q)) \vee (p \wedge q))$ (Distributive laws)

$= \neg((\neg p \wedge U)) \vee (p \wedge q)$ (Complement laws)

$= \neg(\neg p) \vee (p \wedge q)$ (Properties of Universal set)

$= p \vee (p \wedge q)$ (Double complement laws)

$= p$ (Absorption laws)

5. $R_1 = \{(1, 1), (1, 2), (1, 3), (1, 4), (1, 5), (2, 1), (2, 2), (2, 3), (2, 4), (3, 1), (3, 2), (3, 3), (4, 1), (4, 2), (5, 1)\}$

$R_2 = \{(2, 1), (3, 1), (3, 2), (4, 1), (4, 2), (4, 3), (5, 1), (5, 2), (5, 3), (5, 4)\}$

a.)

	1	2	3	4	5
1	1	1	1	1	1
2	1	1	1	1	0
3	1	1	1	0	0
4	1	1	0	0	0
5	1	0	0	0	0

b.)

	1	2	3	4
1	1	0	0	0
2	1	1	0	0
3	1	1	1	0
4	1	1	1	1

c) R_1 is not reflexive as $(4,4), (5,5) \notin R_1$.

R_1 is symmetric because it is same with its transpose matrix.

$$R_1 = \begin{bmatrix} 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 0 \\ 1 & 1 & 1 & 0 & 0 \\ 1 & 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 \end{bmatrix} = R_1^T = \begin{bmatrix} 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 0 \\ 1 & 1 & 1 & 0 & 0 \\ 1 & 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 \end{bmatrix}$$

R_1 is not transitive as $R_1 \otimes R_1 \neq R_1$.

$$\begin{bmatrix} 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 0 \\ 1 & 1 & 1 & 0 & 0 \\ 1 & 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 \end{bmatrix} \otimes \begin{bmatrix} 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 0 \\ 1 & 1 & 1 & 0 & 0 \\ 1 & 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 0 \\ 1 & 1 & 1 & 1 & 0 \\ 1 & 1 & 1 & 1 & 0 \\ 1 & 1 & 1 & 1 & 0 \\ 1 & 1 & 1 & 1 & 0 \\ 1 & 1 & 1 & 1 & 0 \\ 1 & 1 & 1 & 1 & 0 \\ 1 & 1 & 1 & 1 & 0 \\ 1 & 1 & 1 & 1 & 0 \end{bmatrix}$$

Therefore, R_1 is not equivalence relation.

d) R_2 is not reflexive as $(2,2), (3,3), (4,4), (5,5) \notin R_2$.

R_2 is antisymmetric because it is not same as its transpose matrix.

$$R_2 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 1 & 1 & 0 & 0 \\ 1 & 1 & 1 & 0 \\ 1 & 1 & 1 & 1 \end{bmatrix} \neq R_2^T = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 0 & 1 & 1 & 1 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

R_2 is transitive as $R_2 \otimes R_2 = R_2$.

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 1 & 1 & 0 & 0 \\ 1 & 1 & 1 & 0 \\ 1 & 1 & 1 & 1 \end{bmatrix} \otimes \begin{bmatrix} 1 & 0 & 0 & 0 \\ 1 & 1 & 0 & 0 \\ 1 & 1 & 1 & 0 \\ 1 & 1 & 1 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 1 & 1 & 0 & 0 \\ 1 & 1 & 1 & 0 \\ 1 & 1 & 1 & 1 \end{bmatrix}$$

Therefore, R_2 is not a partial order relation.

$$6. R_1 = \{(1,1), (2,2), (2,3), (3,1), (3,3)\}$$

$$R_2 = \{(1,2), (2,2), (3,1), (3,3)\}$$

$$a.) R_1 \cup R_2 = \{(1,1), (1,2), (2,2), (2,3), (3,1), (3,3)\}$$

$$b.) R_1 \cap R_2 = \{(2,2), (3,1), (3,3)\}$$

7. $f(x) = x$ is an one-to-one function, $g(x) = -x$ also is an one-to-one function.

For example, $f(1) = 1$, $g(1) = -1$

$$f(3) = 3, g(3) = -3$$

$$f(5) = 5, g(5) = -5$$

$$\text{Then, } f(1) + g(1) = 1 + (-1) = 0$$

$$f(3) + g(3) = 3 + (-3) = 0$$

$$f(5) + g(5) = 5 + (-5) = 0$$

$$\text{Therefore, } f(1) + g(1) = f(3) + g(3) = f(5) + g(5) = 0.$$

Hence, we can conclude that $f(x)$ and $g(x)$ is one-to-one function while $f(x) + g(x)$ is not an one-to-one function.

8. C_n = Number of different ways to climb staircase with n stairs.

When $n = 1$, the staircase only contains 1 stair.

$$C_1 = 1$$

When $n = 2$, the staircase only contains 2 stairs.

$$C_2 = 2$$

When $n \geq 3$, the staircase contains more than 2 stairs and needed combination of 1 and 2 stairs.

If the last move be a 1-stair step, then there were a_{n-1} ways to arrive at previous stair.

If the last move be a 2-stair step, then there were a_{n-2} ways to arrive at previous stair.

So, the total number of ways is the sum of last move being 1-stair step and 2-stair step.

$$C_n = C_{n-1} + C_{n-2}$$

$$C_1 = 1$$

$$C_2 = 2$$

$$C_n = C_{n-1} + C_{n-2} \text{ when } n \geq 3$$

9. $t_0 = 0, t_1 = t_2 = 1, t_n = t_{n-1} + t_{n-2} + t_{n-3}$ for $n \geq 3$

a) $t_3 = t_{3-1} + t_{3-2} + t_{3-3}$
 $= t_2 + t_1 + t_0$
 $= 1 + 1 + 0$
 $= 2$

$t_4 = t_{4-1} + t_{4-2} + t_{4-3}$
 $= t_3 + t_2 + t_1$
 $= 2 + 1 + 1$
 $= 4$

$t_5 = t_{5-1} + t_{5-2} + t_{5-3}$
 $= t_4 + t_3 + t_2$
 $= 4 + 2 + 1$
 $= 7$

$t_6 = t_{6-1} + t_{6-2} + t_{6-3}$
 $= t_5 + t_4 + t_3$
 $= 7 + 4 + 2$
 $= 13$

$t_7 = t_{7-1} + t_{7-2} + t_{7-3}$
 $= t_6 + t_5 + t_4$
 $= 13 + 7 + 4$
 $= 24$

b) $t(n)$
 $\{$
 if $(n = 0)$
 return 0
 return $t(n-1) + t(n-2) + t(n-3)$
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