SECI1013-06

DISCRETE STRUCTURE

Assignment

Group 13

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QUESTION 1

a) Let A= {1, 2, 3, 4, 5, 6, 7, 8}, B= {2, 5, 9}, and C= {a, b}. Find each of the following:

i) **A–B**

$A-B = \{1, 2, 3, 4, 5, 6, 7, 8\}-\{2, 5, 9\}$
$= \{1, 3, 4, 6, 7, 8\}$

ii) **(A∩B) ∪C**

$(A\cap B) = \{1, 2, 3, 4, 5, 6, 7, 8\} \cap \{2, 5, 9\}$
$= \{2, 5\}$
$(A\cap B) \cup C = \{2, 5\} \cup \{a, b\}$
$= \{2, 5, a, b\}$

iii) **A∩B∩C**

$A\cap B = \{1, 2, 3, 4, 5, 6, 7, 8\} \cap \{2, 5, 9\}$
$= \{2, 5\}$
$A\cap B\cap C = \{2, 5\} \cap \{a, b\}$
$= \emptyset$

iv) **B×C**

$B\times C = \{(2, a), (2, b), (5, a), (5, b), (9, a), (9, b)\}$

v) **P(C)**

$P(C) = \{\emptyset, \{a\}, \{b\}, \{a,b\}\}$

b) By referring to the properties of set operations, show that:

$(P\cap ((P'\cup Q)')) \cup (P\cap Q) = P$

$(P\cap ((P'\cup Q)')) \cup (P\cap Q)$

$= (P\cap (P\cap Q')) \cup (P\cap Q)$	De Morgan's laws
$= ((P\cap P)\cap Q') \cup (P\cap Q)$	Associative laws
$= (P\cap Q')) \cup (P\cap Q)$	Idempotent laws
$= P\cap(Q'\cup Q)$	Distributive laws
$= P\cap U$	Complement laws
$= P$ (shown)	Properties of universal set

c) Construct the truth table for, A = $(\neg p \lor q) \leftrightarrow (q \rightarrow p)$.

p	q	¬p	¬p ∨ q	q → p	(¬p ∨ q) ↔ (q → p)
T	T	F	T	T	T
T	F	F	F	T	F
F	T	T	T	F	F
F	F	T	T	T	T

d) Proof the following statement using direct proof

"For all integer x, if x is odd, then (x+2)2 is odd"

Let P(x) = x is odd.
Q(x) = $(x+2)^2$ is odd.

$\forall x\ (P(x) \rightarrow Q(x))$

Since x is odd, let x = 2n+1

$x = 2n + 1$
$x + 2 = 2n + 1 + 2$
$(x + 2)^2 = (2n + 3)^2$
$(x + 2)^2 = 4n^2 + 12n + 9$
$(x + 2)^2 = 2(2n^2 + 6n + 4) + 1$
$(x + 2)^2 = 2m + 1$

Since $(x+2)^2 = 2m + 1$, no matter what the value of m is, 2m + 1 is always equal to an odd number, this proved that $(x+2)^2$ is odd for all x which is odd .

e) Let P(x,y) be the propositional function $x \geq y$. The domain of discourse for x and y is the set of all positive integers. Determine the truth value of the following statements. Give the value of x and y that make the statement TRUE or FALSE.

i. $\boldsymbol{\exists x\, \exists y\, P(x, y)}$
True when $x \geq y$. For example, when x = 2 and y = 1, the statement is true. When x = 0, y = 0, the statement is true. However, when x = 1, y = 2, this statement also consider as true since there exist other values that make the statement true.

ii. $\boldsymbol{\forall x\, \forall y\, P(x, y)}$
False when $x < y$. For example, when x = 1 and y = 2, the statement is false.

QUESTION 2

a) Suppose that the matrix of relation R on {1, 2, 3} is $\begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{bmatrix}$ relative to the ordering 1, 2, 3.

R={(1, 1), (1, 2), (2, 2), (3, 1)}

i. **Find the domain and the range of R.**

Domain of R = {1, 2, 3}
Range of R = {1, 2}

ii. **Determine whether the relation is irreflexive and/or antisymmetric. Justify your answer.**

The relation is not irreflexive because it contains xRx which is (1, 1) and (2, 2).
The relation is antisymmetric because x ≠ y and (x, y) ∈ R, then (y,x) ∉ R for all x, y ∈ R. For example, the relation has (1, 2) ∈ R, but (2, 1) ∉ R.

b) Let S={(x,y)| x+y ≥9}is a relation on X={2, 3, 4, 5}. Find:

i. **The elements of the set S.**

S = {(4, 5), (5, 4), (5, 5)}

ii. **Is S reflexive, symmetric, transitive, and/or an equivalence relation? Justify your answer.**

- S is not reflexive because not every x ∈ X, (x, x) ∈ S. In S does not have (2, 2), (3, 3) and (4, 4).
- S is symmetric because all x, y ∈ X, if (x, y) ∈ S then (y, x) ∈ S. In S contain (4, 5) and (5, 4).
- S is not transitive because because $M_S \otimes M_S \neq M_S$. (4, 5) ∈ S, (5, 4) ∈ S, but (4, 4) ∉ S

$$\begin{matrix} & 2 & 3 & 4 & 5 \\ 2 & 0 & 0 & 0 & 0 \\ 3 & 0 & 0 & 0 & 0 \\ 4 & 0 & 0 & 0 & 1 \\ 5 & 0 & 0 & 1 & 1 \end{matrix} \otimes \begin{matrix} & 2 & 3 & 4 & 5 \\ 2 & 0 & 0 & 0 & 0 \\ 3 & 0 & 0 & 0 & 0 \\ 4 & 0 & 0 & 0 & 1 \\ 5 & 0 & 0 & 1 & 1 \end{matrix} = \begin{matrix} & 2 & 3 & 4 & 5 \\ 2 & 0 & 0 & 0 & 0 \\ 3 & 0 & 0 & 0 & 0 \\ 4 & 0 & 0 & 1 & 1 \\ 5 & 0 & 0 & 1 & 1 \end{matrix}$$

- S is not an equivalence relation because it is not reflexive, symmetric and not transitive.

c) Let X={1, 2, 3}, Y={1, 2, 3, 4}, and Z={1, 2}.

i. **Define a function f: X→Y that is one-to-one but not onto.**

f = {(1, 1), (2, 2), (3, 3)}

ii. **Define a function g: X→Z that is onto but not one-to-one.**

f = {(1, 1), (2, 2), (3, 1)}

iii. **Define a function h: X→X that is neither one-to-one nor onto.**

f = {(1, 1), (2, 2), (3, 1)}

d) Let m and n be functions from the positive integers to the positive integers defined by the equations:

m(x) = 4x+3, n(x) = 2x–4

i. **Find the inverse of m.**

Let $m^{-1}(x) = y$

m(y) = x

$$m(y) = x$$
$$4y + 3 = x$$
$$4y = x - 3$$
$$y = \frac{x-3}{4}$$
$$m^{-1}(x) = \frac{x-3}{4}$$

ii. **Find the compositions of n o m.**

n o m = nm(x)

= n (4x + 3)

= 2 (4x + 3) – 4

= 8x + 6 – 4

=8x + 2

QUESTION 3

a) Given the recursively defined sequence.

$a_k = a_{k-1} + 2k$, for all integers $k \geq 2$, $a_1 = 1$

i. **Find the first three terms.**

$a_1 = 1$
$a_2 = a_{2-1} + 2(2)$
$= a_1 + 4$
$= 1 + 4$
$= 5$
$a_3 = a_{3-1} + 2(3)$
$= a_2 + 6$
$= 5 + 6$
$= 11$

ii. **Write the recursive algorithm.**

```
a(k)
{ if(k=1)
        return 1
  return a(k-1) + 2
}
```

b) A certain computer algorithm executes twice as many operations when it is run with an input of size k as it is run with an input of size k–1 (where k is an integer that is greater than 1). When the algorithm is run with an input of size 1, it executes seven operations. Let r_k = the number of executes with an input size k. Find a recurrence relation for $r_1, r_2, \ldots. r_k$.

Since computer algorithm executes twice as many operations let the sequence be:

a, 2a, 2(2a)

k is the input of size.

When the input of size is 1 which mean k=1, it executes seven operations, therefore

$r_1 = 1$

Based on the sequence we know that the term after, r_k is twice of the term before, r_{k-1}, therefore we can write it as:

$r_k = 2 * (r_{k-1})$

c) Given the recursive algorithm:

```
Input: n
Output: S (n)
S(n) {
    if (n=1)
        return 5
    return 5*S(n–1)
    }
```

Trace S(4)

n = 1, S(1) = 5

n = 2, S(2) = 5 * S(2-1)

= 5 * S(1)

= 5 * 5

= 25

n = 3, S(3) = 5 * S(3-1)

= 5 * S(2)

= 5 * 25

= 125

n = 4, S(4) = 5 * S(4-1)

= 5 * S(3)

=5 * 125

=625

QUESTION 4

a) Hexadecimal numbers are made using the sixteen digits 0, 1, 2, 3, 4, 5, 6, 7, 8, 9, A, B, C, D, E, F. They are denoted by the subscript 16. How many hexadecimal numbers begin with one of the digits 3 through B, end with one of the digits 5 through F and are 4 digits long?

For a 4 digits long hexadecimal number:

First digit (start with digit 3 through B)(3, 4, 5, 6, 7, 8, 9, A, B) = 9 digits
Second digit (no restrict) = 16 digits
Third digit (no restrict) = 16 digits
Fourth digit (end with digit 5 through F)(5, 6, 7, 8, 9, A, B, C, D, E, F) = 11 digits

By using the multiplication rule:
9 x 16 x 16 x 11 = 25344

25344 hexadecimal numbers begin with one of the digit 3 through B, end with one of the digits 5 through F.

b) Suppose that in a certain state, all automobile license plates have four letters followed by three digits. How many license plates could begin with A and end in 0?

Since the letters and digits can be repeated in the license plates. Therefore 26 letters which is from A to Z can be arranged in the second, third and fourth column. While in the fifth and sixth column, 10 whole numbers which is 0 to 9 can be arranged. While in the first column, A is fixed in it, and in the last column, 0 is fixed in it.

By using the multiplication rule and the permutation formula with repetition:

A						**0**

1 x 26 x 26 x 26 x 10 x 10 x 1 = 1757600

1757600 license plates could begin with A and end in 0.

c) How many arrangements in a row of no more than three letters can be formed using the letters of word COMPUTER (with no repetitions allowed)?

To arrange no more that three letters, we can have arrangement of one letter, two letters or three letters.

To arrange one letter:
8 arrangements can be done since in the word COMPUTER contain 8 letters.

To arrange two letters:

$$8P2 = 56$$

To arrange three letters:

$$8P3 = 336$$

Since arrangement of one letter, two letters or three letters, addition rule is used since or is used. Therefore

$$\text{Total number of arrangements} = 8 + 56 + 336$$
$$= 400$$

d) A computer programming team has 13 members. Suppose seven team members are women and six are men. How many groups of seven can be chosen that contain four women and three men?

To choose four women from seven women = 7C4
To choose three men from six men = 6C3

By using multiplication rule:

$$7C4 \text{ X } 6C3 = 700$$

There are 700 groups of seven that contain four women and three men can be formed.

e) How many distinguishable ways can the letters of the word PROBABILITY be arranged?

In the word PROBABILITY, consist of 1P, 1R, 1O, 2B, 1A, 2I, 1L, 1T and 1Y.

By using the formula:

$$P(11) = \frac{11!}{1!\,1!\,1!\,2!\,1!\,2!\,1!\,1!\,1!}$$
$$= 9979200$$

9979200 distinguishable ways can the letters of the word PROBABILITY be arranged.

f) A bakery produces six different kinds of pastry. How many different selections of ten pastries are there?

Combination repetition, $\binom{n+r-1}{r} = \binom{6+10-1}{10}$

$$= \binom{15}{10}$$

By using formula, $\left(\frac{(n+r-1)!}{r!\,(n-1)!}\right) = \left(\frac{6+10-1}{10!\,(6-1)!}\right)$

$$= \left(\frac{15!}{10!\,(6-1)!}\right)$$

$$= \left(\frac{15!}{10!\,5!}\right)$$

$$= 3003$$

3003 different selections of ten pastries are there.

QUESTION 5

a) Eighteen persons have first names Ali, Bahar, and Carlie and last names Daud and Elyas. Show that at least three persons have the same first and last names.

Pigeonhole, m = 3C1 * 2C1(combination of the first name and last name)
pigeon,n = 18
k = n / m
= 18/6
= 3

b) How many integers from 1 through 20 must you pick in order to be sure of getting at least one that is odd?

Based on the pigeonhole principle, the principle states that if n is the number of pigeonhole and n+1 is the number of pigeons that fly into n pigeonholes, then there is at least one pigeon containing two or more pigeons.

From 1 through 20, there are

10 **odd** integers: 1, 3, 5, 7, 9, 11, 13, 15, 17, 19

10 **even** integers: 2, 4, 6, 8, 10, 12, 14, 16, 18, 20

At most, 10 even integers can be picked out of them. If we need to pick another one integer after picking all of 10 even integers, it must be an odd integer (based on pigeonhole principle). So, we need at least one more integer to pick from 1 through 20 to be sure that there is at least one odd integer.

10 + 1 = 11 integers

Therefore, in total, we need to pick at least 11 integers from 1 through 20 to be sure of getting at least one that is odd.

c) How many integers from 1 through 100 must you pick in order to be sure of getting one that is divisible by 5?

Numbers that is divisible by 5 = 20 numbers

5, 10, 15, 20, 25, 30, 35, 40, 45, 50, 55, 60, 65, 70, 75, 80, 85, 90, 95, 100

Numbers that is **not** divisible by 5 = 100 – 20

= 80 numbers

At most, 80 numbers that is not divisible by 5 can be picked out of 100 numbers. If we need to pick another one number after picking all the 80 numbers, it must be a number that is divisible by 5 (based on pigeonhole principle). So, we need at least one more number to pick from 1 through 100 to be sure that there is at least one number that divisible by 5.

80+1= 81 integers

Therefore, in total, we need to pick at least 81 integers from 1 through 100 to be sure of getting at least one that is divisible by 5.