SECI1013-06

DISCRETE STRUCTURE

Assignment

Group 13

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Question 1

Consider 3-digit number whose digit are either 2,3, 4, 5, 6 or 7

a. How many numbers are there?

If there is no restriction, the number can be repeated

$$6^3 = 216$$

b. How many numbers are there if the digits are distinct?

If the number cannot be repeated,

$$P(6,3) = {}^6P_3 = \frac{6!}{(6-3)!} = \frac{6!}{3!} = \frac{720}{6} = 120$$

c. How many numbers between 300 to 700 are only odd digits allowed?

Total number of digits $= 6$
To make an odd digit number, the last digit in the number must be 3, 5 or 7 which is total of 3 digits.

3		

$$6 \times 3 = 18$$

4		

$$6 \times 3 = 18$$

5		

$$6 \times 3 = 18$$

6		

$$6 \times 3 = 18$$

Total number between 300 to 700 are only odd digit allowed $= 18 + 18 + 18 + 18 = 72$

Question 2

Anita invited 10 of her friends, a couple, four men and four women to her anniversary dinner, how many ways Anita can arrange them around a round dinner table

There has a total of 11 people include Anita which is 5 men and 6 women.

a. Men insist to sit next to each other

In order to insist men sit next to each other, 7 groups formed which is 1 group men and 6 women. They should be sit in the circular.

$$P_n = P_7 = (7-1)! = 6!$$

Men group switch places $= 5!$
By using the multiplication rule, $6! \times 5! = 86400$

b. The couple insisted to sit next to each other

For the couple to sit next to each other in round table = 10 groups = (10-1)! = 9!
Couple switch places = 2!
By using multiplication rule, 9! × 2! = 725760

c. Men and women sit in alternate seat

To let men and women sit in alternate seat, the men should be fixed in particular seat and among men should switch places = 5!
For the 6 women sit in the round table = (6-1)! = 5!
By using multiplication rule, 5! × 5! = 14400

d. Before her friend left, Anita wanted to arrange a photoshoot. How many ways can her photographer arrange in a row, if Anita and her husband stand next to each other

Since Anita's husband is not included in 10 friends, therefore the total people become 12.

Anita and her husband stand next to each other in a row, become 11 groups = 11!
Anita and her husband switch places = 2!
By using multiplication rule, 11! × 2! = 79833600

Question 3

In a school sport day, five sprinters are competing in a 100 meter race. How many ways are there for the sprinter to finish

a. If no ties

There are five places, 1^{st}, 2^{nd}, 3^{rd}, 4^{th}, 5^{th}, among 5 sprinters = 5! = 120

b. Two sprinters tie

There are four places since two sprinters tie = 4! = 24

c. Two group of two sprinters tie

There are three places since two groups of two sprinters tie = 3! = 6

Question 4

A croissant shop has plain croissants, cherry croissants, chocolate croissants, almond croissants, apple croissants, and broccoli croissants. How many ways are there to choose

a. a dozen croissants?

In the shop has 6 types of croissants, n=6.
To choose a dozen, n=12

$$\mathrm{C}(\mathrm{n}+\mathrm{r}-1,\mathrm{r}) = \mathrm{C}(6+12-1,12) = \frac{(6+12-1)!}{12!\,(6-1)!} = \frac{17!}{12!\,5!} = 6188$$

b. two dozen croissants with at least two of each kind?

To choose two dozen, n=24
With at least two of each kind, 6 types × 2 = 12
Since 12 croissants chosen, 24 – 12 = 12, 12 croissants with 6 types still need to be chosen, therefore r=12
By using formula,

$$C(n+r-1,r) = C(6+12-1,12) = \frac{(6+12-1)!}{12!\,(6-1)!} = \frac{17!}{12!\,5!} = 6188$$

c. two dozen croissants with at least five chocolate croissants and at least three almond croissants?

To choose two dozen, n=24
Since five chocolate croissants and three almond croissants has chosen, the remaining croissants is, 24 – 8 = 16, 16 croissants with 6 types still need to be chosen therefore r=16
By using formula,

$$C(n+r-1,r) = C(6+16-1,16) = \frac{(6+16-1)!}{16!\,(6-1)!} = \frac{21!}{16!\,5!} = 20349$$

Question 5

This procedure is used to break ties in games in the championship round of the World Cup soccer tournament. Each team selects five players in a prescribed order. Each of these players takes a penalty kick, with a player from the first team followed by a player from the second team and so on, following the order of players specified. If the score is still tied at the end of the 10 penalty kicks, this procedure is repeated. If the score is still tied after 20 penalty kicks, a sudden-death shootout occurs, with the first team scoring an unanswered goal victorious.

a. How many different scoring scenarios are possible if the game is settled in the first round of 10 penalty kicks, where the round ends once it is impossible for a team to equal the number of goals scored by the other team?

Consider that have two situations might end the round which is a team has scored 2 wins and with another 1 win or tie, or a team has scored 1 win and with another 3 win or tie. The order of winning game does not matter during first **n** rounds, so we can conclude that:

2 wins among 4 games: ${}^4C_2 = 6$
1 win among 3 games: ${}^3C_1 = 3$

Since there are total 5 games need to play, for the 2 win condition the team just need to score 1 tie or win from any of the rest of the 3 games, for the 1 win condition the team need another 3 tie or win from any of the rest of the 4 games.

2 win and 1 ties/win: ${}^4C_2 \times {}^3C_1 \times 2 = 36$
1 win and 3 ties/win: ${}^3C_1 \times {}^4C_3 \times 2^3 = 96$

Consider there are two team that can have the win

Number of scenarios = 2 x (36+96)

= 2 x 132

=264

b. How many different scoring scenarios for the first and second groups of penalty kicks are possible if the game is settled in the second round of 10 penalty kicks?

All the possible of 10 penalty kicks are 2 x 2 x 2 …….x 2 = 2^{10}

10 penalty = 2^{10} = 1024

Scenarios that if not settled in the first round of 10 penalty which is 1024 – 264 = 760. The scenarios for if the game is settled in the second round we can use the answer we get at part a.

First round = 1024 - 264

= 760 scenarios

Second round= 264 scenarios

Number of scenarios = 760 x 264

= 200640 scenarios

c. How many scoring scenarios are possible for the full set of penalty kicks if the game is settled with no more than 10 total additional kicks after the two rounds of five kicks for each team?

Scenario if the round cannot be settled is 760 scenarios. If there is another maximum 10 additional kick and it will stop immediately when one team has scored but another team not so the scenarios will be like 2 + 2 + 2 + 2 + 2 = 10.

First round = 760

Second round = 760

Sudden death = 2 + 2 + 2 + 2 + 2

= 10

Number of scenarios = 760 x 760 x 10

= 5776000

Question 6

A professor gives a multiple-choice quiz that has ten questions, each with four possible responses, a, b, c, d. What is the minimum number of students that must be in the professor's class in order to guarantee that at least three answer sheets must be identical? (Assume that no answers are left blank.)

The pigeonhole in this situation is possible answer sheet, $m = 4^{10} = 1048576$

To have at least three answer sheets identical, $k > 2$

$$k > 2$$

$$\frac{n}{m} > 2$$

$$\frac{n}{1048576} > 2$$

$$n > 10485762 \times 2$$

$$n > 2097153$$

To have the third identical answer sheet 1 answer sheet should be added since 2097153 is the number of two identical answer sheet. Therefore, $2097153 + 1 = 2097154$, at least from 2097154 of the answer sheets, we can get at least one group of three identical answer sheets.

Question 7

In a secondary examination, 75% of the students have passed in history and 65% in Mathematics, while 50% passed both in history and mathematics. If 35 candidates failed in both the subjects, what is the number of candidates sit for the exam?

Let
H = History
M = Mathematics

P(H) = 0.75
P(M) = 0.65
P(HM) = 0.5

P(H or M) = P(H) + P(M) - P(HM)
= 0.75 + 0.65 – 0.5
= 0.9

P(H'M') = 1 – P(H or M)
= 1- 0.9
= 0.1

Let the total number be n, given that

$$nP(H'M') = 35$$

$$n(0.1) = 35$$

$$n(0.1) = \frac{35}{0.1}$$

$$n = 350$$

Total number of candidates in exam is 350.

<u>Question 8</u>

An integer from 300 through 780, inclusive is to be chosen at random, find the probability that the number is chosen will have 1 as at least one digit.

Total number from 300 through 780 = 481 (include 300 and 780)

Four numbers which is 3, 4, 5 and 6 can be chosen for the first column and digit 1 as the last digit at the third column, 10 digits which is 0-9 (included 0 and 9) switch in the middle.

		1

$4 \times 10 \times 1 \quad = 40$

Four numbers which is 3, 4, 5 and 6 can be chosen for the first column and digit 1 as second digit at second column, 9 digits which is 0-9 (included 0 and 9) expect 1 switch at the last digit since the number 311, 411, 511 and 611 already counted above.

	1	

$4 \times 1 \times 9 \quad = 36$

7 as the first digit and 1 as the second digit, 10 digits which is 0-9 (included 0 and 9) switch at the last digit.

7	1	

$1 \times 1 \times 10 \quad = 10$

7 as the first digit and 1 as the last digit, 7 digits which is 0-6 (included 0 and 6) switch at the second digit.

7		1

$1 \times 7 \times 1 \quad = 7$

Total number of numbers have 1 as at least one digit $= 40 + 36 + 10 + 7 = 93$

Probability that the number is chosen will have 1 as at least one digit,

$$Probability = \frac{Total\ number\ of\ numbers\ have\ 1\ as\ at\ least\ one\ digit}{Total\ number\ from\ 300\ through\ 780}$$

$$= \frac{40 + 36 + 10 + 7}{481}$$

$$= \frac{93}{481}$$

<u>Question 9</u>

Two blue and four yellow cars are to be parked in a row of 10 parking lots. Assume that cars of the same color are not distinguishable, and the parking lots are chosen at random.

a. In how many ways can the cars be parked in the parking lots?

Since same colour are not distinguishable and no restriction for the lots,

$$\frac{^{10}P_6}{2!\,4!} = \frac{151200}{48} = 3150$$

b. In how many ways can the cars be parked so that the empty lots are next to each one another? Find the probability that the empty lots are next to one another?

For empty lots must next to each other = $^{7}P_6$

$$\frac{^{7}P_6}{2!\,4!} = \frac{5040}{48} = 105$$

$$Probability = \frac{Empty\ slot\ must\ next\ to\ each\ other}{No\ restriction\ for\ lots}$$

$$= \frac{105}{3150}$$

$$= \frac{1}{30}$$

<u>Question 10</u>

A coach wishes to give a message to a trainee. The probabilities that he uses email, letter and handphone are 0.4,0.1 and 0,5 respectively. He uses only one method. The probabilities of the trainee receive the message if the coach uses email, letter or hand phone are 0.6,0.8 and 1 respectively

Let
E = Email
L = Letter
H = Handphone
T = Trainee receive message

P(E) = 0.4
P(L) = 0.1
P(H) = 0.5
P(T|E) = 0.6
P(T|L) = 0.8
P(T|H) = 1

a. Find the probability the trainee receives the message

$$P(T) = P(T|E)P(E) + P(T|L)P(L) + P(T|H)P(H)$$

$$= (0.6)(0.4) + (0.8)(0.1) + (1)(0.5)$$

$$= 0.24 + 0.08 + 0.5$$

$$= 0.82$$

b. Given that the trainee receives the messages, find the conditional probabilities that he receives it via email

$$P(E|T) = \frac{P(T|E)P(E)}{P(T)}$$

$$= \frac{(0.6)(0.4)}{0.82}$$

$$= \frac{0.24}{0.82}$$

$$= \frac{12}{41}$$

<u>Question 11</u>

In recent News, it was reported that light trucks, which include SUV's , pick-up trucks and minivans, accounted for 40% of all personal vehicles on the road in 2012. Assume the rest are cars. Of every 100,000 cars accidents, 20 involve a fatality; of every 100,000 light trucks accidents, 25 involve a fatality. If a fatal accident is chosen at random, what is the probability the accident involved a light truck?

Let
L = Light truck's accident
C = Car's accident
F = Fatality

P(L) = 0.4
P(C) = 0.0002
P(F|C) = 0.0002
P(F|L) = 0.00025

$$P(L|F) = \frac{P(F|L)P(L)}{P(F|L)P(L) + P(F|C)P(C)}$$

$$= \frac{(0.00025)(0.4)}{(0.00025)(0.4) + (0.0002)(0.6)}$$

$$= \frac{0.0001}{0.00022}$$

$$= \frac{5}{11}$$

Question 12

There are 9 letters having different colors (red, orange, yellow, green, blue, indigo, velvet) and 4 boxes of different shapes (tetrahedron, cube, polyhedron, dodecahedron). How many ways are there to place these 9 letters into the 4 boxes such that each box contains at least 1 letter?

By using inclusion-exclusion principle,

|A∪B∪C∪D|=|A|+|B|+|C|+|D|} all singletons
−(|A∩B|+|A∩C|+|A∩D|+|B∩C|+|B∩D|+|C∩D|)} all pairs
+(|A∩B∩C|+|A∩B∩D|+|A∩C∩D|+|B∩C∩D|)} all triples
−|A∩B∩C∩D|} all quadruples

$$^4C_4 4^9 - {}^4C_3 3^9 + {}^4C_2 2^9 - {}^4C_1 1^9$$

$$= 262144 - 78732 + 3072 - 4$$

$$= 186480$$