SECI1013-06

DISCRETE STRUCTURE

Assignment

Group 13

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Question 1

Let the universal set be the set R of all real numbers and let A={x∈R | 0 < x ≤ 2}, B={x∈R | 1 ≤ x < 4} and C={x∈R | 3 ≤ x < 9}. Find each of the following:

a) A ∪ C = {x∈R | 0 < x ≤ 2 or 3 ≤ x < 9}

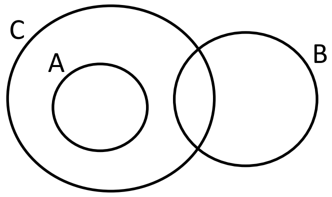
b) (A ∪ B)′ = {x∈R | x ≤ 0 or x ≥4}

c) A′ ∪ B′ = {x∈R | x ≤ 0 or x > 2}

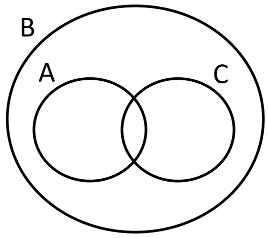
Question 2

Draw Venn diagrams to describe sets A, B, and C that satisfy the given conditions.

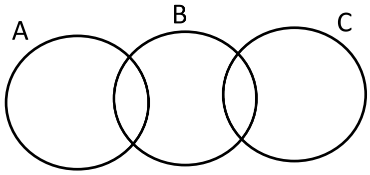
a) A ∩ B = ∅, A ⊆ C, C ∩ B ≠ ∅



b) A ⊆ B, C ⊆ B, A ∩ C ≠ ∅



c) A ∩ B ≠ ∅, B ∩ C ≠ ∅, A ∩ C = ∅, A ⊄ B, C ⊄ B



Question 3

Given two relations S and T from A to B,
$S \cap T = \{(x,y) \in A\times B \mid (x,y) \in S \text{ and } (x,y) \in T\}$
$S \cup T = \{(x,y) \in A\times B \mid (x,y) \in S \text{ or } (x,y) \in T\}$
Let A={−1, 1, 2, 4} and B={1,2} and defined binary relations S and T from A to B as follows:
For all $(x,y) \in A\times B$, $x\,S\,y \leftrightarrow |x| = |y|$
For all $(x,y) \in A\times B$, $x\,T\,y \leftrightarrow x - y$ is even
State explicitly which ordered pairs are in A×B, S, T, S ∩ T, and S ∪ T.

A×B = {(-1,1),(-1,2),(1,1),(1,2),(2,1),(2,2),(4,1),(4,2)}

S = {(-1,1),(1,1),(2,2)}

T = {(-1,1),(1,1),(2,2),(4,2)}

S ∩ T = {(-1,1),(1,1),(2,2)}

S ∪ T = {(-1,1),(1,1),(2,2),(4,2)}

Question 4

Show that $\neg ((\neg p\wedge q) \vee (\neg p\wedge\neg q)) \vee (p\wedge q) \equiv p$. State carefully which of the laws are used at each stage.

$\neg ((\neg p\wedge q) \vee (\neg p\wedge\neg q)) \vee (p\wedge q)$

$= \neg (\neg p\wedge q) \vee \neg (\neg p\wedge\neg q) \vee (p\wedge q)$ - De Morgan's law

$= (p\wedge\neg q) \vee (p\wedge q) \vee (p\wedge q)$ - Double Negation law and De Morgan's law

$= p \vee (\neg q\wedge q) \vee (p\wedge q)$ - Distributive law

$= p \vee \emptyset \vee (p\wedge q)$ - Contradiction and Set Operation Identity

$= p \vee (p\wedge q)$ - Absorption Law

$= p$ (shown)

Question 5

$R_1=\{(x,y) \mid x+y \le 6\}$; R_1 is from X to Y; $R_2=\{(y,z) \mid y>z\}$; R_2 is from Y to Z; ordering of X, Y, and Z: 1, 2, 3, 4, 5. Find:

a) The matrix A_1 of the relation R_1 (relative to the given orderings)

$$A_1 = \begin{bmatrix} 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 0 \\ 1 & 1 & 1 & 0 & 0 \\ 1 & 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 \end{bmatrix}$$

b) The matrix A_2 of the relation R_2 (relative to the given orderings)

$$A_2 = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 \\ 1 & 1 & 0 & 0 & 0 \\ 1 & 1 & 1 & 0 & 0 \\ 1 & 1 & 1 & 1 & 0 \end{bmatrix}$$

c) Is R_1 reflexive, symmetric, transitive, and/or an equivalence relation?

R_1 is not reflexive because it has only three pair of $(x,x)\in R_1$, $\forall x{:}x\in X$

R_1 is symmetric because for all $x,y\in X$, if $(x,y)\in R_1$, then $(y,x)\in R_1$.

R_1 is not transitive because (2,1) and $(1,5)\in R_1$, $(2,5)\in R_1$ but $(2,5)\notin R_1$.Thus R_1^2 is not a subset of R_1.

$$\begin{bmatrix} 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 0 \\ 1 & 1 & 1 & 0 & 0 \\ 1 & 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 \end{bmatrix} \otimes \begin{bmatrix} 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 0 \\ 1 & 1 & 1 & 0 & 0 \\ 1 & 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 \end{bmatrix}$$

R_1 is not an equivalence relation since it is not reflexive, symmetric and not transitive.

d) Is R_2 reflexive, antisymmetric, transitive, and/or a partial order relation?

R_2 is not reflexive because it does not have any pair of $(x,x)\in R_2$, $\forall x{:}x\in X$.

R_2 is antisymmetric because $y\neq z$, whenever yR_2z, then z is not related to y

R_2 is transitive because $R_2^2 \subseteq R_2$, every element of R_2^2 is the element of R_2.

$$\begin{bmatrix} 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 \\ 1 & 1 & 0 & 0 & 0 \\ 1 & 1 & 1 & 0 & 0 \\ 1 & 1 & 1 & 1 & 0 \end{bmatrix} \otimes \begin{bmatrix} 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 \\ 1 & 1 & 0 & 0 & 0 \\ 1 & 1 & 1 & 0 & 0 \\ 1 & 1 & 1 & 1 & 0 \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 \\ 1 & 1 & 0 & 0 & 0 \\ 1 & 1 & 1 & 0 & 0 \end{bmatrix}$$

R_2 is not a partial order relation because it is not reflexive, antisymmetric and not transitive.

Question 6

6. Suppose that the matrix of relation R_1 on {1, 2, 3} is

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 1 \\ 1 & 0 & 1 \end{bmatrix}$$

relative to the ordering 1, 2, 3, and that the matrix of relation R_2 on {1, 2, 3} is

$$\begin{bmatrix} 0 & 1 & 0 \\ 0 & 1 & 0 \\ 1 & 0 & 1 \end{bmatrix}$$

relative to the ordering 1, 2, 3. Find:

a) The matrix of relation $R_1 \cup R_2$

$$\begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & 1 \\ 1 & 0 & 1 \end{bmatrix}$$

b) The matrix of relation $R_1 \cap R_2$

$$\begin{bmatrix} 0 & 0 & 0 \\ 0 & 1 & 0 \\ 1 & 0 & 1 \end{bmatrix}$$

Question 7

If f :**R**→ **R** and g:**R**→ **R** are both one-to-one, is $f + g$ also one-to-one? Justify your answer.

Let f(x) = x, Let g(x) = x

Let $f(x_1) + g(x_1) = f(x_2) + g(x_2)$:

$x_1 + x_1 = x_2 + x_2$

$2x_1 = 2x_2$

$x_1 = x_2$

This show f + g is also one-to-one

Question 8

With each step you take when climbing a staircase, you can move up either one stair or two stairs. As a result, you can climb the entire staircase taking one stair at a time, taking two at a time, or taking a combination of one- or two-stair increments. For each integer n≥1, if the staircase consists of n stairs, let cn be the number of different ways to climb the staircase. Find a recurrence relation for $c1$, $c2$, …., cn.

First few sequence : 1, 2, 3, 5, 8,13

$c_1 = 1$

$c_2 = 2$

$c_3 = 3$

$c_4 = c_3 + c_2 = 3 + 2 = 5$

$c_5 = c_4 + c_3 = 5 + 3 = 8$
Recurrence relation $c_n = c_{n-1} + c_{n-2}, n \geq 3$

Question 9

The Tribonacci sequence (t_n) is defined by the equations,
$t_1 = t_2 = t_3 = 1$, $t_n = t_{n-1} + t_{n-2} + t_{n-3}$ for all $n \geq 4$.
a) Find t_7.
t1 = t2= t3 = 1, t4 = 3, t5 = 5, t6 = 9,
t7 = 9 + 5 + 3
= 17

b) Write a recursive algorithm to compute t_n, $n \geq 1$.

```
input : n
  output : t(n)
  t(n){
     if ( 0 < n < 4)
           return 1
      return t(n-1) + t(n-2) + t(n-3)
  }
```