ASSIGNMENT 2

COURSE NAME: DISCRETE STRUCTURE

COURSE CODE: SECI 1013

SECTION: 03

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GROUP NUMBER: 7

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1. Consider 3-digit number whose digit are either 2,3, 4, 5, 6 or 7
   a) How many numbers are there?
      P (6,3) = 6^3 = 216
   b) How many numbers are there if the digits are instinct?
      ${}^6P_3$ = 120
   c) How many numbers between 300 to 700 is only odd digits allow?
      First place = 2 ways (3,5)
      Second place = 3 ways (3,5,7)
      Third place = 3 ways (3,5,7)
      2 * 3 * 3 = 18 ways

2. Anita invited 10 of her friends, a couple, four men and four women to her anniversary dinner, how many ways Anita can arrange them around a round dinner table
   a) Men insist to sit next to each other
      (6 - 1)! * 5! = 14 400 ways
   b) The couple insisted to sit next to each other
      (9 - 1)! *2! = 80 640 ways
   c) Men and women sit in alternate seat
      .$\frac{(5-1)!}{2!} = 12\ ways$
   d) Before her friend left, Anita wants to arrange a photoshoot. How many ways can her photographer arrange in a row, if Anita and her husband stand next to each other
      2! * 10! * 10 = 72 576 000 ways

3. In a school sport day, five sprinters are competing in a 100-meter race. How many ways are there for the sprinter to finish?
   a) If no ties

      Since there is a total of 5 sprinters and no ties,

      They can only finish in 1st,2nd,3rd,4th, and 5th position.

      $Hence, \quad 5! = 120\ ways$

   b) Two sprinters tie

      Since there are 5 sprinters but only positions to be filled are 1st,2nd,3rd, and 4th and two sprinters must be chosen to tie.

      Number ways if two sprinters tie.

      $No\ of\ ways\ =\ 4!\ \times\ C(5,2)$

      $C(5,2)\ =\ \frac{5!}{2!\,(5-2)!} = 10\ ways$

      $\therefore =\ 4! \times\ 10 = 240\ ways\ if\ two\ sprinters\ tie.$

c) Two group of two sprinters tie

Since there is 5 sprinters and 2 groups of two sprinters will tie only positions to be filled are 1st,2nd, and 3rd, and two groups of 2 sprinters have to be chosen.

$$C(5,2) = \frac{5!}{2!\,(5-2)!} = 10\ ways$$

$$C(3,2) = \frac{3!}{2!\,(3-2)!} = 3\ ways$$

$$3! = 6\ ways$$

Therefore, $10 \times 3 \times 6 = 180\ ways$

4. A croissant shop has plain croissants, cherry croissants, chocolate croissants, almond croissants, apple croissants, and broccoli croissants. How many ways are there to choose?
   a) a dozen croissants?

      $.\frac{(12+6-1)!}{12!(6-1)!} = 6188$
   b) two dozen croissants with at least two of each kind?

      $C(17,12) = \frac{17!}{12!(12-12)!}$

      =6188
   c) two dozen croissants with at least five chocolate croissants and at least three almond croissants?

      $C(21,16) = \frac{21!}{16!(21-16)!}$

      $= 20\ 349$

5. This procedure is used to break ties in games in the championship round of the World Cup soccer tournament. Each team selects five players in a prescribed order. Each of these players takes a penalty kick, with a player from the first team followed by a player from the second team and so on, following the order of players specified. If the score is still tied at the end of the 10 penalty kicks, this procedure is repeated. If the score is still tied after 20 penalty kicks, a sudden-death shootout occurs, with the first team scoring an unanswered goal victorious.
   a) How many different scoring scenarios are possible if the game is settled in the first round of 10 penalty kicks, where the round ends once it is impossible for a team to equal the number of goals scored by the other team?

      2 wins from 4 games: $C\ (4,2) = \frac{4!}{2!(4-2)!} = 6$

      1 win from 3 games: $C\ (3,1) = \frac{3!}{2!(3-1)!} = 3$

2 wins and 1 tie = 6 * C (3,1) * 2 = 36
1 win and 3 ties = 3 * C (4,3) * 2^3 = 96

Since there are 2 teams:
2 * (36 + 96) = 2 * 132 = 264 possible scoring scenarios

b) How many different scoring scenarios for the first and second groups of penalty kicks are possible if the game is settled in the second round of 10 penalty kicks?

10 penalty kicks: 2^{10} = 1024 (2 possible outcomes per penalty kick)
Game not settled in the first round: 1024 – 264 = 760
Game settled in the second round: 264

Number of scoring scenarios: 760 * 264 = 200,640

c) How many scoring scenarios are possible for the full set of penalty kicks if the game is settled with no more than 10 total additional kicks after the two rounds of five kicks for each team?

Game not settled in the first round: 1024 – 264 = 760
Game not settled in the second round: 1024 – 264 = 760
Sudden - death shoot out: 10 (5 rounds of penalty kicks)

Number of scoring scenarios: 760 * 760 * 10 = 5,776,000

6. A professor gives a multiple-choice quiz that has ten questions, each with four possible responses, (a, b, c, d). What is the minimum number of students that must be in the professor's class to guarantee that at least three answer sheets must be identical? (Assume that no answers are left blank.)

$Number\ of\ possible\ responses\ for\ one\ question\ =\ C(4,1)\ =\ 4$

$Number\ of\ possible\ answer\ sheets\ =\ 4^{10}\ =\ 1{,}048{,}576\ possible\ answer\ sheets$

Now, it may so happen that if the total number of candidates appearing in the exam ≤ 64, each candidate answers differently.

However, if there are 1048576 + 1 = 1048577 candidates appearing for the exam, and there are only a possible set of 1048576 answers to select from, so at least two candidates must answer identically.

To guarantee that at least three answer sheets must be identical,

1048577 + 1048576 = 2097153 answer sheets must be used.

7. In a secondary examination, 75% of the students have passed in history and 65% in Mathematics, while 50% passed both in history and mathematics. If 35 candidates failed in both the subjects, what is the number of candidates sit for the exam?

   $P(H) = 0.75$ $P(M) = 0.65$ $P(H \cap M) = 0.5$
   $P(H' \cap M') = 35$ students
   $P(H \cup M) = P(H) + P(M) - P(H \cap M)$
   $= 0.75 + 0.65 - 0.5$
   $= 0.9$

   $P(H \cup M)' = P(H' \cap M')$

   $0.1 = 35$ students

   $P(H \cup M) = 315$ students

8. An integer from 300 through 780, inclusive is to be chosen at random, find the probability that the number is chosen will have 1 as at least one digit.

   Possible outcomes = 780 – 299 = 481
   1 in 3 digits: 0
   1 in 2 digits: 311, 411, 511, 611, 711 = 5
   1 in 1 digit:
   301, 310, 312, 313, 314, 315, 316, 317, 318, 319, 321, 331, 341, 351, 361, 371, 381, 391 = 18
   401, 410, 412, 413, 414, 415, 416, 417, 418, 419, 421, 431, 441, 451, 461, 471, 481, 491 = 18
   501, 510, 512, 513, 514, 515, 516, 517, 518, 519, 521, 531, 541, 551, 561, 571, 581, 591 = 18
   601, 610, 612, 613, 614, 615, 616, 617, 618, 619, 621, 631, 641, 651, 661, 671, 681, 691 = 18
   701, 710, 712, 713, 714, 715, 716, 717, 718, 719, 721, 731, 741, 751, 761, 771 = 16
   (4 * 18) + 16 + 5 + 0 = 93

   Probability = $\frac{93}{481}$

9. Two blue and four yellow cars are to be parked in a row of 10 parking lots. Assume that cars of the same colour are not distinguishable, and the parking lots are chosen at random.

   a) In how many ways can the cars be parked in the parking lots?

   Let yellow cars be circles, blue cars be triangles, number 1 to 10 parking lots

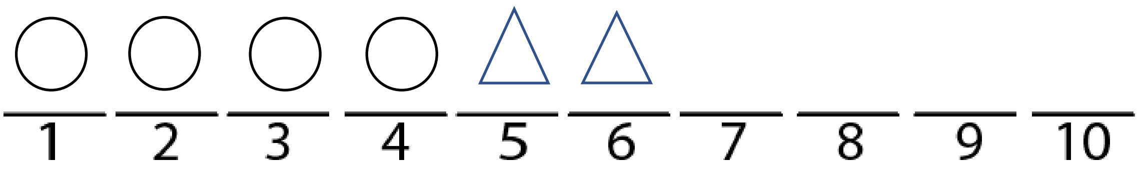


   Figure shows 1 of many ways' cars can be arranged.

   We can specify an arrangement by choosing four locations from the slot locations {1, 2, 3, 4, 5, 6, 7, 8, 9,10}.

   Since there is an order to Parking lots being vacant, the ways of parking spots (objects) being vacant is,

$$10P4 \;=\; \frac{10!}{(10-4)!} = 5040\, ways$$

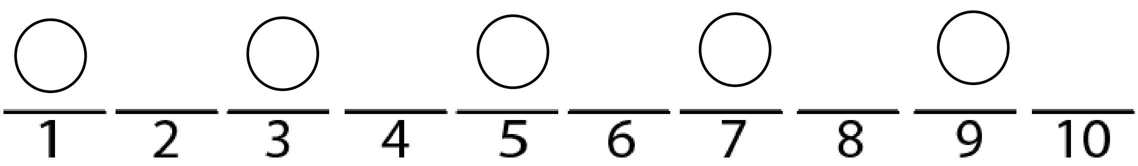
   Since there are 10 parking spots (objects) with 6 types: Yellow cars (4), Blue cars (2). Total number of different arrangements is,

$$= \frac{10P4}{4! \times 2!} = 105$$

   Number of ways can the cars be parked in the parking lots is 105 ways.

   b) In how many ways can the cars be parked so that the empty lots are next to each one another? Find the probability that the empty lots are next to one another?

   Let, the circles represent cars either yellow or blue,



Shown 10 parking lots but given is 6 cars, consisting blue and yellow. Cars must be parked in alternative lots, because empty lots must be next to each one another. For 6 cars to be parked alternatively, there should be at least 11 parking spots. Given only 10 parking lots, therefore probability that empty lots are next to one another is 0.

10. A coach wishes to give a message to a trainee. The probabilities that he uses email, letter and handphone are 0.4,0.1 and 0,5 respectively. He uses only one method. The probabilities of the trainee receive the message if the coach uses email, letter or handphone are 0.6,0.8 and 1 respectively

    a) Find the probability the trainee receives the message

       P(E)= 0.4 P(L) = 0.1 P(H) = 0.5
       P(M|E) = 0.6 P(M|L) = 0.8 P(M|H) = 1
       P(M) = P(M|E) P(E) + P(M|L) P(L) + P(M|H) P(H)
       = 0.6(0.4) + 0.8(0.1) + 0.5(1)
       = 0.82

    b) Given that the trainee receives the messages, find the conditional probabilities that he receives it via email

       $.P(E|M) = \frac{\mathrm{P(M|E)\ P(E)}}{P(M)}$

       $= \frac{0.6(0.4)}{0.82}$

       . $= 0.29$

11. In a recent News, it was reported that light trucks, which include SUV's, pick-up trucks, and minivans, accounted for 40% of all personal vehicles on the road in 2012. Assume the rest are cars. Of every 100,000 cars accidents, 20 involve a fatality; of every 100,000 light trucks accidents, 25 involve a fatality. If a fatal accident is chosen at random, what is the probability the accident involved a light truck?

A – Light trucks (SUV's, pick-up trucks, and minivans)

A' – Cars

B – Fatal accident

B' – Not a fatal accident

Given, $P(\frac{B}{A'}) = \frac{20}{100000}$

$P(\frac{B}{A}) = \frac{25}{100000}$

$P(A) = 0.4$

$\therefore P(A') = 1 - P(A) = 1 - 0.4 = 0.6$

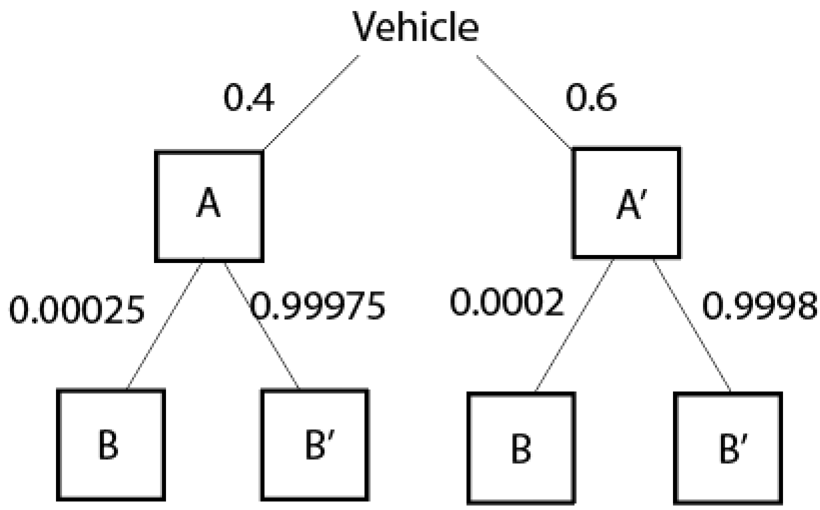
What is the probability accident chosen at random involving a light truck, given it is a fatal accident? $P(\frac{A}{B})$

Using Bayes' Theorem,

$$P(A|B) = \frac{P(B|A)P(A)}{\sum_{i=1}^{n}(B|A_i)P(A_i)}$$

$$\therefore\ P(A|B) = \frac{P(B|A)P(A)}{P(B|A)P(A) + P(B|A')P(A')}$$

Tree Diagram.



Referring to tree diagram,

$$P(A|B) = \frac{P(B|A)P(A)}{P(B|A)P(A) + P(B|A')P(A')}$$

$$P(A|B) = \frac{(0.00025)(0.4)}{(0.00025)(0.4) + (0.0002)(0.6)} = 0.4545$$

Probability accident chosen at random involving a light truck, given it is a fatal accident is 0.4545.

12. There are 9 letters having different colours (red, orange, yellow, green, blue, indigo, velvet) and 4 boxes of different shapes (tetrahedron, cube, polyhedron, dodecahedron). How many ways are there to place these 9 letters into the 4 boxes such that each box contain at least 1 letter?

No. of possible ways, without restriction = 4^9 = 262144
Disallowed ways: 4 * 3^9 = 78732
All letters in 1 box: 4 * 19 = 4 ways
Allowed ways of letters into boxes: 262144 – 78732 + 4 = 183416