ASSIGNMENT 3

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GROUP NUMBER: 7

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QUESTION 1 **[25 marks]**

a) Let $A = \{1, 2, 3, 4, 5, 6, 7, 8\}$, $B = \{2, 5, 9\}$, and $C = \{a, b\}$. Find each of the following: (9 marks)

i. $A - B$

$\boldsymbol{A - B} = \{\mathbf{1, 3, 4, 6, 7, 8}\}$

ii. $(A \cap B) \cup C$

$(\mathbf{A} \cap \mathbf{B}) = \{\mathbf{2, 5}\}$

$(\mathbf{A} \cap \mathbf{B}) \cup \mathbf{C} = \{\mathbf{2{,}5, a, b}\}$

iii. $A \cap B \cap C$

$\mathbf{A} \cap \mathbf{B} = \{\mathbf{2, 5}\}$

$\mathbf{A} \cap \mathbf{B} \cap \mathbf{C} = \{\ \}$

iv. $B \times C$

$\boldsymbol{B} = \{\mathbf{2, 5, 9}\}$ **and** $\boldsymbol{C} = \{\boldsymbol{a, b}\}$

$\boldsymbol{B} \times \boldsymbol{C} = \{\ (\mathbf{2}, \boldsymbol{a}), (\mathbf{2}, \boldsymbol{b}), (\mathbf{5}, \boldsymbol{a}), (\mathbf{5}, \boldsymbol{b}), (\mathbf{9}, \boldsymbol{a}), (\mathbf{9}, \boldsymbol{b})\}$

v. P(C)

$\mathbf{P}(\mathbf{C}) = \{\ \}, \{\mathbf{a}\}, \{\mathbf{b}\}, \{\mathbf{a, b}\}$

b) By referring to the properties of set operations, show that: (4 marks)

$$(P \cap ((P' \cup Q)')) \cup (P \cap Q) = P$$

$= (\boldsymbol{P} \cap ((\boldsymbol{P}' \cup \boldsymbol{Q})')) \cup (\boldsymbol{P} \cap \boldsymbol{Q})$ **[De Morgan's Law]**

$= (\boldsymbol{P} \cap (\boldsymbol{P}'' \cap \boldsymbol{Q}')) \cup (\boldsymbol{P} \cap \boldsymbol{Q})$ **[Double Negation Law]**

$= (\boldsymbol{P} \cap (\boldsymbol{P} \cap \boldsymbol{Q}')) \cup (\boldsymbol{P} \cap \boldsymbol{Q})$ **[Associative Law]**

$= (\boldsymbol{Q}' \cap (\boldsymbol{P} \cap \boldsymbol{P})) \cup (\boldsymbol{P} \cap \boldsymbol{Q})$ **[Idempotent Law]**

$= (\boldsymbol{Q}' \cap \boldsymbol{P}) \cup (\boldsymbol{P} \cap \boldsymbol{Q})$ **[Commutative Law]**

$= (\boldsymbol{P} \cap \boldsymbol{Q}') \cup (\boldsymbol{P} \cap \boldsymbol{Q})$ **[Distributive Law]**

$= \boldsymbol{P} \cap (\boldsymbol{Q}' \cup \boldsymbol{Q})$ **[Negation Law]**

$= \boldsymbol{P} \cap \boldsymbol{T}$ **[Identity Law]**

$= \boldsymbol{P}$

c) Construct the truth table for, **A** = (¬*p* ∨ *q*) ↔ (*q* → *p*). (4 marks)

p	**q**	**-p**	**(¬*p* ∨ *q*)**	**(*q* → *p*)**	**A**
T	T	F	T	T	T
T	F	F	F	T	F
F	T	T	T	F	F
F	F	T	T	T	T

d) Proof the following statement using direct proof

"For all integer *x*, if *x* is odd, then (*x*+2)2 is odd" (4 marks)

$\boldsymbol{P(x) = x}$ **is odd**, $\mathbf{Q(x)}$ $\boldsymbol{(x+2)^2}$ **is odd**

Since, $\forall$ **x (P(x) → Q(x))**

Therefore,
$\boldsymbol{a = 2n + 1}$
$\boldsymbol{a + 2 = 2n + 1 + 2}$
$\boldsymbol{a + 2 = 2n + 3}$
$\boldsymbol{(a+2)^2 = (2n+3)^2}$
$\boldsymbol{(a+2)^2 = 4n^2 + 12n + 9}$
$\boldsymbol{(a+2)^2 = 2(2n^2 + 6n) + 9}$
$\boldsymbol{(a+2)^2 = 2m + 9}$ **(where m is $\boldsymbol{2n^2 + 6n}$)**
$\boldsymbol{(a+2)^2 is\ an\ odd\ integer}$

e) Let *P* (*x*, *y*) be the propositional function *x* ≥ *y*. The domain of discourse for *x* and *y* is the set of all positive integers. Determine the truth value of the following statements. Give the value of *x* and *y* that make the statement TRUE or FALSE. (4 marks)

i. $\exists x\ \exists y\ P(x, y)$

Statement is true because,

For values such as
$$\boldsymbol{P(3,2) then\ 3 \geq 2\ but\ for\ values\ such\ as\ P(1,2) the\ 1 < 2\ so\ there\ exist\ P(x,y)}$$

ii. $\forall x\ \forall y\ P(x, y)$

Statement is False because,
For values such as

$$\boldsymbol{P(4,2)\ then\ 4 \geq 2\ but\ for\ values\ such\ as\ P(1,3)\ then\ 1 < 3\ so\ there\ exist\ P(x,y)}$$

QUESTION 2 **[25 marks]**

a) Suppose that the matrix of relation *R* on {1, 2, 3} is $\begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{bmatrix}$ relative to the ordering 1, 2, 3. (7 marks)

i. Find the domain and the range of *R*.
$\boldsymbol{R}$ = {(1, 1), (1, 2), (2, 2), (3, 1)}
Domain = {1, 2, 3}
Range = {1, 2}

ii. Determine whether the relation is irreflexive and/or antisymmetric. Justify your answer.

- **It is not irreflexive as not all elements, $\boldsymbol{(x\ , x) \notin R}$**
- **It is antisymmetric as $\boldsymbol{(1,2) \in R}$ and $\boldsymbol{(3,1) \in R}$ but $\boldsymbol{(2,1) \notin R}$ and $\boldsymbol{(1,3) \notin R}$, respectively.**

b) Let $S = \{(x, y) \mid x + y \geq 9\}$is a relation on $X = \{2, 3, 4, 5\}$. Find: (6 marks)

i. The elements of the set *S*.
$\boldsymbol{S}$ = {(4, 5), (5, 4), (5, 5)}

ii. Is *S* reflexive, symmetric, transitive, and/or an equivalence relation? Justify your answer.

- **It is not reflexive as $\boldsymbol{\nabla x \in X, (x,x) \notin S}$.**
- **It is symmetric as $\boldsymbol{\nabla x, y \in X, (x,y) \in S\ \rightarrow\ (y,x) \notin S}$.**
- **It is not transitive as $\boldsymbol{M_s \otimes M_s \neq M_s}$.**
- **It is not an equivalence relation as it is not reflexive and not transitive but only symmetric.**

c) Let $X = \{1, 2, 3\}$, $Y = \{1, 2, 3, 4\}$, and $Z = \{1, 2\}$. (6 marks)

i. Define a function f: $X \rightarrow Y$ that is one-to-one but not onto.

$\boldsymbol{f(x) = x}$

It is one-to-one but not onto as not all elements in *Y* is the image of some elements in *X*. (does not take value 4 in co-domain)

ii. Define a function g: $X \rightarrow Z$ that is onto but not one-to-one.

$\boldsymbol{g(1) = 1, g(2) = 2, g(3) = 2}$

It is onto but not one-to-one as value 2 in *Z* take both value 2 and 3 in *X*.

iii. Define a function h: $X \rightarrow X$ that is neither one-to-one nor onto.

$\boldsymbol{h(x) = 2}$

It is not one-to-one as 2 in co-domain have more than 1 arrow point to it and it also not onto as not all elements in co-domain is the image of element in domain.

d) Let m and n be functions from the positive integers to the positive integers defined by the equations:

$$m(x) = 4x + 3, \ n(x) = 2x - 4$$

(6 marks)

i. Find the inverse of m.

$$\boldsymbol{m(x) = 4x + 3}$$
$$\boldsymbol{x = 4y + 3}$$
$$\boldsymbol{x - 3 = 4y}$$
$$\boldsymbol{y = \frac{x - 3}{4}}$$
$$\boldsymbol{m^{-1}(x) = \frac{x - 3}{4}}$$

ii. Find the compositions of n o m.

$$\boldsymbol{n \circ m = n(m(x))}$$
$$\boldsymbol{= 2(4x + 3) - 4}$$
$$\boldsymbol{= 8x + 6 - 4}$$
$$\boldsymbol{= 8x + 2}$$
$$\boldsymbol{= 2(4x + 1)}$$

QUESTION 3 **[15 marks]**

a) Given the recursively defined sequence.

$$a_k = a_{k-1} + 2k, \text{ for all integers } k \geq 2,\ a_1 = 1$$

i) Find the first three terms. (2 marks)

$\boldsymbol{a_1 = 1}$

$\boldsymbol{a_2 = a_1 + 2(2) = 1 + 4 = 5}$

$\boldsymbol{a_3 = a_2 + 2(3) = 5 + 6 = 11}$

ii) Write the recursive algorithm. (5 marks)

Input: k

Output: $\boldsymbol{f(k)}$

```
f(k) {
        if (k = 1)
                return 1
        return f(k − 1) + 2k
    }
```

b) A certain computer algorithm executes twice as many operations when it is run with an input of size k as it is run with an input of size k-1 (where k is an integer that is greater than 1). When the algorithm is run with an input of size 1, it executes seven operations. Let r_k = the number of executes with an input size k. Find a recurrence relation for r_1, r_2, …. r_k. (4 marks)

$\boldsymbol{r_k = 2r_{k-1}}$

Initial condition, $\boldsymbol{r_1 = 7}$

$\therefore\ \boldsymbol{r_k = 2r_{k-1},\ k \geq 2 \text{ with } r_1 = 7}$

c) Given the recursive algorithm:

Input: n

Output: $S(n)$

```
S(n) {
    if (n = 1)
        return 5
    return 5*S(n-1)
    }
```

Trace $S(4)$. (4 marks)

$S(1)$ **n = 1** **because n = 1** **return 5**
$S(2)$ **n = 2** **because n ≠ 1** **return 5 * $S(2)$**
$S(3)$ **n = 3** **because n ≠ 1** **return 5 * $S(2)$**
$S(4)$ **n = 4** **because n ≠ 1** **return 5 * $S(3)$**

return 5
$S(1)$ = 5

return 5 * 5 = 25
$S(2)$ = 25

return 5 * 25 = 125
$S(3)$ = 125

return 5 * 125 = 625
$S(4)$ = 625

<u>QUESTION 4</u> **[25 marks]**

a) Hexadecimal numbers are made using the sixteen digits 0, 1, 2, 3, 4, 5, 6, 7, 8, 9, A, B, C, D, E, F. They are denoted by the subscript 16. How many hexadecimal numbers begin with one of the digits 3 through B, end with one of the digits 5 through F and are 4 digits long?
9 * 16 * 16 * 11 = 25 344

(4 marks)

b) Suppose that in a certain state, all automobile license plates have four letters followed by three digits. How many license plates could begin with A and end in 0?
1 * 26 * 26 * 26 * 9 * 10 * 1 = 1581 840

(4 marks)

c) How many arrangements in a row of no more than three letters can be formed using the letters of word *COMPUTER* (with no repetitions allowed)?
(8 * 7 * 6) + (8 * 7) + 8 = 400

(5 marks)

d) A computer programming team has 13 members. Suppose seven team members are women and six are men. How many groups of seven can be chosen that contain four women and three men?
C (6,3) * C (7,4) = 700

(4 marks)

e) How many distinguishable ways can the letters of the word *PROBABILITY* be arranged?

$$\frac{\mathbf{11}!}{\mathbf{1}!\,\mathbf{1}!\,\mathbf{1}!\,\mathbf{2}!\,\mathbf{1}!\,\mathbf{2}!\,\mathbf{1}!\,\mathbf{1}!\,\mathbf{1}!} = \mathbf{9\,979\,200}$$

(4 marks)

f) A bakery produces six different kinds of pastry. How many different selections of ten pastries are there?

$$\frac{(\mathbf{6}+\mathbf{10}-\mathbf{1})!}{\mathbf{10}!\,(\,\mathbf{6}-\mathbf{1})!} = \mathbf{3003}$$

(4 marks)

QUESTION 5 **[10 marks]**

a) Eighteen persons have first names Ali, Bahar, and Carlie and last names Daud and Elyas. Show that at least three persons have the same first and last names. (4 marks)

Each first name has 2 last name possibilities, therefore each first name has 2 ways to form a name.

$$\textbf{Number of names} = 2 + 2 + 2 = 6 \textbf{ names}$$

$$\textbf{pigeonhole} = 6, \textbf{pigeons} = 18 \textbf{ people.}$$

$$\mathbf{k} = \left\lceil \frac{18}{6} \right\rceil = 3 \qquad (\textbf{Shown})$$

b) How many integers from 1 through 20 must you pick in order to be sure of getting at least one that is odd? (3 marks)

$$\textbf{Odd integers available} = 1, 3, 5, 7, 9, 11, 13, 15, 17, 19$$

$$\textbf{Number of integers that is odd} = 10$$

$$\textbf{Integers must be picked in order to get at least one that is odd} = 10$$

c) How many integers from 1 through 100 must you pick in order to be sure of getting one that is divisible by 5? (3 marks)

$$\boldsymbol{Integers\ divisible\ by}\ 5 = 5, 10, 15, 20, 25, 30, 35, 40, 45, 50, 55, 60, 65, 70, 75, 80, 85, 90, 95, 100$$

$$\boldsymbol{Number\ of\ integers\ that\ is\ divisible\ by}\ 5 = 20$$

$$\boldsymbol{Integers\ must\ be}\ \textbf{picked in order to get at least one that is}\ \boldsymbol{divisible\ by}\ 5 = 20$$