**SECI 1013**

**DISCRETE STRUCTURE**

**SEMESTER 1 2020/2021**

**TUTORIAL 4:**

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No. Markah

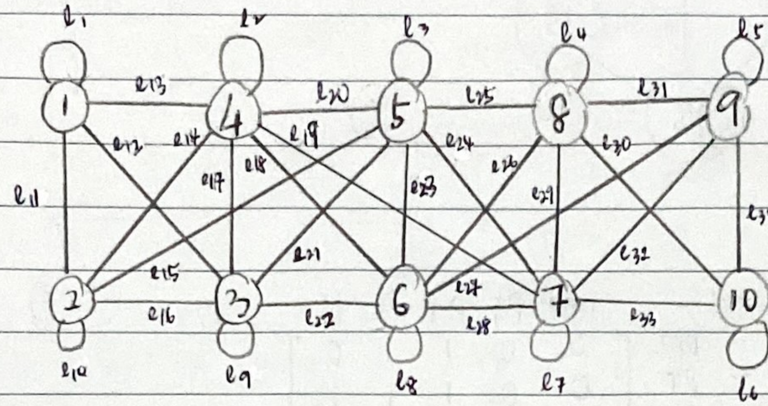
## Tutorial 4

1. $V(G) = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$

$|V - W| \leq 3$

$V = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$

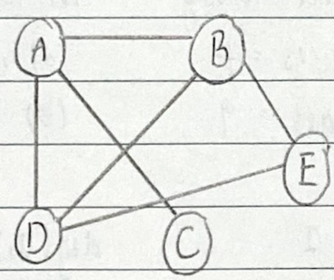
$W = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$




Number of edges, $e(G)$ is 34.

Jum-lah

2. (a) A = Ahmad, B = Bakri, C = Chong, D = David, E = Ehsan




$$\text{adjancency matrix} = \begin{array}{c} \\ A \\ B \\ C \\ D \\ E \end{array} \begin{array}{c} \begin{array}{ccccc} A & B & C & D & E \end{array} \\ \begin{bmatrix} 0 & 1 & 1 & 1 & 0 \\ 1 & 0 & 0 & 1 & 1 \\ 1 & 0 & 0 & 0 & 0 \\ 1 & 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 1 & 0 \end{bmatrix} \end{array}$$

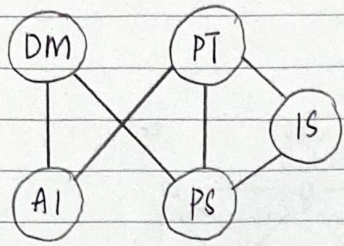
(b) DM = Discrete Mathematics

PT = Programming Technique

AI = Artificial Intelligence

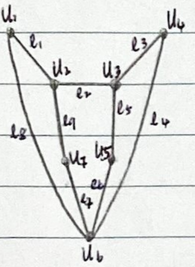
PS = Probability Statistic

IS = Information System




Adjacency matrix =
$$\begin{array}{c|c} & \begin{array}{ccccc} DM & PT & AI & PS & IS \end{array} \\ \begin{array}{c} DM \\ PT \\ AI \\ PS \\ IS \end{array} & \begin{bmatrix} 0 & 0 & 1 & 1 & 0 \\ 0 & 0 & 1 & 1 & 1 \\ 1 & 1 & 0 & 0 & 0 \\ 1 & 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 1 & 0 \end{bmatrix} \end{array}$$

3.




right-hand drawing

(a) vertices = 7

(b) edges = 9

$deg(v_1) = 2$

$deg(v_2) = 3$

$deg(v_3) = 3$

$deg(v_4) = 2$

$deg(v_5) = 2$

$deg(v_6) = 4$

$deg(v_7) = 2$

left-hand drawing

(a) vertices = 7

(b) edges = 9

$deg(u_1) = 2$

$deg(u_2) = 3$

$deg(u_3) = 3$

$deg(u_4) = 2$

$deg(u_5) = 2$

$deg(u_6) = 4$

$deg(u_7) = 2$

- Both drawings have same number of vertices and edges.
- Both drawings have 1 vertex with 4 degree, 4 vertices with 2 degree, 2 vertices with 3 degree.
- $f(v_1) = u_1$ $f(v_2) = u_2$ $f(v_3) = u_3$ $f(v_4) = u_4$ $f(v_5) = u_5$ $f(v_6) = u_6$ $f(v_7) = u_7$
- adjacency matrices

$$\begin{array}{c} \\ v_1 \\ v_2 \\ v_3 \\ v_4 \\ v_5 \\ v_6 \\ v_7 \end{array}\begin{array}{c} \begin{array}{ccccccc} v_1 & v_2 & v_3 & v_4 & v_5 & v_6 & v_7 \end{array} \\ \begin{bmatrix} 0 & 1 & 0 & 0 & 0 & 1 & 0 \\ 1 & 0 & 1 & 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 1 & 1 & 0 & 1 \\ 0 & 1 & 0 & 0 & 0 & 1 & 0 \end{bmatrix} \end{array} \qquad \begin{array}{c} \\ u_1 \\ u_2 \\ u_3 \\ u_4 \\ u_5 \\ u_6 \\ u_7 \end{array}\begin{array}{c} \begin{array}{ccccccc} u_1 & u_2 & u_3 & u_4 & u_5 & u_6 & u_7 \end{array} \\ \begin{bmatrix} 0 & 1 & 0 & 0 & 0 & 1 & 0 \\ 1 & 0 & 1 & 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 1 & 1 & 0 & 1 \\ 0 & 1 & 0 & 0 & 0 & 1 & 0 \end{bmatrix} \end{array}$$



4. adjacency matrix, $A_G$      adjacency matrix, $A_H$

$$\begin{array}{c} \\ v_1 \\ v_2 \\ v_3 \\ v_4 \end{array}\begin{array}{c} \begin{array}{cccc} v_1 & v_2 & v_3 & v_4 \end{array} \\ \begin{bmatrix} 1 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{bmatrix} \end{array} \qquad \begin{array}{c} \\ v_1 \\ v_2 \\ v_3 \\ v_4 \end{array}\begin{array}{c} \begin{array}{cccc} v_1 & v_2 & v_3 & v_4 \end{array} \\ \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 1 & 2 \\ 0 & 1 & 1 & 0 \\ 0 & 2 & 0 & 0 \end{bmatrix} \end{array}$$

incidence matrix, $I_G$      incidence matrix, $I_H$

$$\begin{array}{c} \\ v_1 \\ v_2 \\ v_3 \\ v_4 \end{array}\begin{array}{c} \begin{array}{cccccc} e_1 & e_2 & e_3 & e_4 & e_5 & e_6 \end{array} \\ \begin{bmatrix} 2 & 1 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 1 & 1 & 1 & 1 & 1 \\ 0 & 0 & 0 & 0 & 1 & 1 \end{bmatrix} \end{array} \qquad \begin{array}{c} \\ v_1 \\ v_2 \\ v_3 \\ v_4 \end{array}\begin{array}{c} \begin{array}{cccccc} e_1 & e_2 & e_3 & e_4 & e_5 & e_6 \end{array} \\ \begin{bmatrix} 2 & 0 & 0 & 0 & 0 & 0 \\ 0 & 2 & 1 & 1 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 & 2 \\ 0 & 0 & 1 & 1 & 0 & 0 \end{bmatrix} \end{array}$$

5. $G_1$ : vertices = 5 $G_2$ : vertices = 5

edges = 5 edges = 5

$\deg(v_1) = 2$ $\deg(u_1) = 1$

$\deg(v_2) = 2$ $\deg(u_2) = 3$

$\deg(v_3) = 3$ $\deg(u_3) = 2$

$\deg(v_4) = 2$ $\deg(u_4) = 2$

$\deg(v_5) = 1$ $\deg(u_5) = 2$

adjacency matrices

$$A_{G_1} = \begin{array}{c} \\ v_1 \\ v_2 \\ v_3 \\ v_4 \\ v_5 \end{array} \begin{array}{c} \begin{array}{ccccc} v_1 & v_2 & v_3 & v_4 & v_5 \end{array} \\ \begin{bmatrix} 0 & 1 & 0 & 1 & 0 \\ 1 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 1 & 1 \\ 1 & 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \end{bmatrix} \end{array} \qquad A_{G_2} = \begin{array}{c} \\ u_5 \\ u_3 \\ u_2 \\ u_4 \\ u_1 \end{array} \begin{array}{c} \begin{array}{ccccc} u_5 & u_3 & u_2 & u_4 & u_1 \end{array} \\ \begin{bmatrix} 0 & 1 & 0 & 1 & 0 \\ 1 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 1 & 1 \\ 1 & 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \end{bmatrix} \end{array}$$

Both graphs have same number of vertices and edges.

Both graphs have 1 vertex with 1 degree, 1 vertex with 3 degree and 3 vertices with 2 degree.

$f(v_1) = u_5$ $f(v_2) = u_3$ $f(v_3) = u_2$ $f(v_4) = u_4$ $f(v_5) = u_1$

Therefore, both graphs are isomorphic.

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6.

a. Trail because the walk has no repeated edges

b. Walk because it is a finite alternating sequence of adjacent vertices and edges from $v_4$ to $v_5$

c. Trivial walk because there exist one vertex ($v_2$) and no edges,

d. Circuit because it is a closed walk with no repeated edges.

e. Closed walk because the start and end vertex are the same ($v_2$)

f. Path because it is a trail from $v_3$ to $v_2$ that does not have repeated vertex.

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a. $(v_1, e_1, v_2, e_3, v_3, e_5, v_4)$ There are 3 paths from $v_1$ to $v_4$

$(v_1, e_1, v_2, e_2, v_3, e_5, v_4)$

$(v_1, e_1, v_2, e_4, v_3, e_5, v_4)$

b. $(v_1, e_1, v_2, e_3, v_3, e_5, v_4)$ There are 9 trails from $v_1$ to $v_4$

$(v_1, e_1, v_2, e_2, v_3, e_5, v_4)$

$(v_1, e_1, v_2, e_4, v_3, e_5, v_4)$

$(v_1, e_1, v_2, e_2, v_3, e_3, v_2, e_4, v_3, e_5, v_4)$

$(v_1, e_1, v_2, e_4, v_3, e_3, v_2, e_2, v_3, e_5, v_4)$

$(v_1, e_1, v_2, e_4, v_3, e_2, v_2, e_3, v_3, e_5, v_4)$

$(v_1, e_1, v_2, e_2, v_3, e_4, v_2, e_3, v_3, e_5, v_4)$

$(v_1, e_1, v_2, e_3, v_3, e_2, v_2, e_4, v_3, e_5, v_4)$

$(v_1, e_1, v_2, e_3, v_3, e_4, v_2, e_2, v_3, e_5, v_4)$

c. There are too many walks from $v_1$ to $v_4$ as walk is finite alternating sequence of adjacent vertices and edges from $v_1$ to $v_4$

Thus, there are infinity walks from $v_1$ to $v_4$

8.

a.

| vertex | $v_1$ | $v_2$ | $v_3$ | $v_4$ | $v_5$ |
|---|---|---|---|---|---|
| degree | 2 | 4 | 2 | 4 | 4 |

Graph in (a) has an Euler Circuit because all vertices have even degree

$(v_1, e_1, v_2, e_2, v_5, e_3, v_2, e_4, v_3, e_5, v_4, e_6, v_5, e_7, v_4, e_8, v_1)$

b.

| vertex | $v_0$ | $v_1$ | $v_2$ | $v_3$ | $v_4$ | $v_5$ | $v_6$ | $v_7$ | $v_8$ | $v_9$ |
|---|---|---|---|---|---|---|---|---|---|---|
| degree | 2 | 5 | 2 | 2 | 2 | 4 | 2 | 3 | 3 | 2 |

Grap in (b) does not have Euler Circuit because 3 vertices $(v_1, v_7, v_8)$ has odd degree

9.

a.

| vertex | $u$ | $v_1$ | $v_0$ | $v_7$ | $v_2$ | $v_3$ | $v_4$ | $v_5$ | $v_6$ | $w$ |
|---|---|---|---|---|---|---|---|---|---|---|
| degree | 3 | 2 | 2 | 2 | 4 | 2 | 4 | 2 | 4 | 3 |

$(u, v_1, v_0, v_7, u, v_2, v_3, v_4, v_6, v_5, w)$

Graph in (a) has an Euler path from $u$ to $w$ because it is connected and vertices $u$ and $w$ are the only vertices with odd degree

b.

| vertex | a | b | c | d | e | f | g | h | u | w |
|---|---|---|---|---|---|---|---|---|---|---|
| degree | 2 | 2 | 2 | 2 | 3 | 4 | 2 | 3 | 3 | 3 |

Graph in (b) does not have Euler path from $u$ to $w$ because as there are two other vertices with odd degree (e and h) besides $u$ and $w$.

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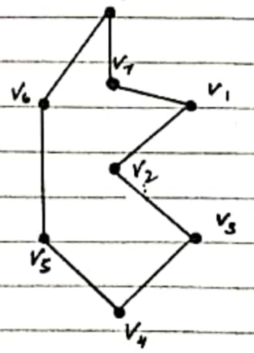
a.




$(v_0, v_7, v_1, v_2, v_3, v_4, v_5, v_6, v_0)$

b. There is no Hamiltonian Circuit in graph (b) because graph (b) cannot form simple circuit which includes all vertices once except for the first and last vertices.

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Question 11

$m = 3, \quad n = 100$

$$l = \frac{(m-1)n+1}{m}$$

$$= \frac{(3-1)100+1}{3}$$

$$= 67$$

Question 12

a) Root = a

b) Internal vertices = a, b, e, g, n, d, h, j

c) Leaves = c, f, k, l, m, r, s, i, o, p, q

d) Children of n = r, s

e) Parent of e = b

f) Siblings of k = l, m

g) Proper ancestors of q = a, d, j

h) Proper descendants of b = e, k, l, m, f, g, n, r, s

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Question 13

Preorder - a, b, e, k, l, m, f, g, n, r, s, c, d, h, o, i, j, p, q

Inorder - k, e, l, m, b, f, r, n, s, g, a, c, o, h, d, i, p, j, q

Postorder - k, l, m, e, f, r, s, n, g, b, c, o, h, i, p, q, j, d, a

Question 14

| | |
|---|---|
| AB | 1 |
| GH | 1 |
| BC | 2 |
| AF | 3 |
| BF | 4 |
| CD | 4 |
| BG | 5 |
| CG | 6 |
| DH | 6 |
| DG | 7 |
| ED | 7 |
| GF | 8 |
| EH | 8 |

AB - 1
GH - 1
BC - 2
AF - 3
CD - 4
BG - 5
ED - 7

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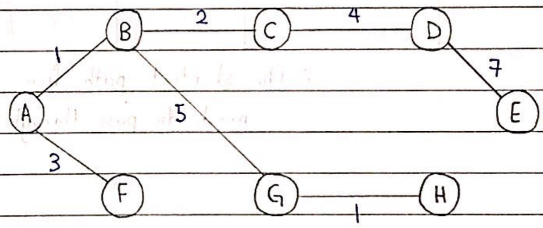

Question 15

| No | S | N | L(M) | L(N) | L(O) | L(P) | L(Q) | L(R) | L(S) | L(T) |
|---|---|---|---|---|---|---|---|---|---|---|
| 1. | { } | {M, N, O, P, Q, R, S, T} | 0 | ∞ | ∞ | ∞ | ∞ | ∞ | ∞ | ∞ |
| 2. | {M} | {N, O, P, Q, R, S, T} | 0 | 4 | ∞ | 2 | ∞ | 5 | ∞ | ∞ |
| 3. | {M, P} | {N, O, Q, R, S, T} | 0 | 4 | ∞ | 2 | 6 | 5 | 5 | ∞ |
| 4. | {M, N, P} | {O, Q, R, S, T} | 0 | 4 | 10 | 2 | 6 | 5 | 5 | ∞ |
| 5. | {M, N, P, S} | {O, Q, R, T} | 0 | 4 | 10 | 2 | 6 | 5 | 5 | ∞ |
| 6. | {M, N, P, R, S} | {O, Q, T} | 0 | 4 | 7 | 2 | 6 | 5 | 5 | 6 |
| 7. | {M, N, P, Q, R, S} | {O, T} | 0 | 4 | 7 | 2 | 6 | 5 | 5 | 6 |
| 8. | {M, N, P, Q, R, S, T} | {O} | 0 | 4 | 7 | 2 | 6 | 5 | 5 | 6 |

∴ The shortest path from M to T is 6 which needs to pass through vertex R.