SECI 1013

DISCRETE STRUCTURE

SEMESTER 1 2020/2021

TUTORIAL 3 :

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Tutorial 3

1\. a) i. $A - B = \{1,2,3,4,5,6,7,8\} - \{2,5,9\}$

$= \{1,3,4,6,7,8\}$

ii. $(A \cap B) \cup C = (\{1,2,3,4,5,6,7,8\} \cap \{2,5,9\}) \cup \{a,b\}$

$= \{2,5\} \cup \{a,b\}$

$= \{2,5,a,b\}$

iii. $A \cap B \cap C = \{1,2,3,4,5,6,7,8\} \cap \{2,5,9\} \cap \{a,b\}$

$= \{2,5\} \cap \{a,b\}$

$= \emptyset$

iv. $B \times C = \{2,5,9\} \times \{a,b\}$

$= \{(2,a),(2,b),(5,a),(5,b),(9,a)(9,b)\}$

v. $P(C) = \{\emptyset, \{a\}, \{b\}, \{a,b\}\}$

b) $(P \cap ((P' \cup Q)')) \cup (P \cap Q)$

$= (P \cap (P'' \cap Q')) \cup (P \cap Q)$ De Morgan's law

$= (P \cap (P \cap Q')) \cup (P \cap Q)$ Double complement law

$= ((P \cap P) \cap Q') \cup (P \cap Q)$ Associative law

$= (P \cap Q') \cup (P \cap Q)$ Idempotent law

$= P \cap (Q' \cup Q)$ Distributive law

$= P \cap U$ Complement law

$= P$ Identity law

c) Truth Table A

p	q	$\neg p$	$\neg p \vee q$	$q \to p$	$(\neg p \vee q) \leftrightarrow (q \to p)$
T	T	F	T	T	T
T	F	F	F	T	F
F	T	T	T	F	F
F	F	T	T	T	T

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d) If x is odd, then $(x+2)^2$ is odd.

$P(x)$ = x is odd

$Q(x)$ = $(x+2)^2$ is odd

Assume x is odd, show $(x+2)^2$ is odd

$x = 2k+1$

$$\begin{aligned}
(x+2)^2 &= ((2k+1)+2)^2 \\
&= (2k+3)^2 \\
&= 4k^2+12k+9 \\
&= -(-4k^2-12k-9) \\
&= -(-4k^2-12k-8-1) \\
&= -(-4k^2-12k-8)+1 \\
&= (4k^2+12k+8)+1 \\
&= 4(k^2+3k+2)+1 \\
&= 4l+1, \text{ where } l = k^2+3k+2
\end{aligned}$$

$\therefore$ since $4l+1$ is odd, thus $(x+2)^2$ is odd integer. (shown)

e) i) $\exists x \exists y\, P(x,y)$

$P(x,y) = x \geq y$

Let $x=2, y=1$

$P(2,1) = 2 \geq 1$

$= \text{TRUE}$

ii) $\forall x \forall y\, P(x,y)$

$P(x,y) = x \geq y$

Let $x=1, y=2$

$P(1,2) = 1 \geq 2$

$= \text{FALSE}$

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2\. a) Relation $R = \{(1,1),(1,2),(2,2),(3,1)\}$

i. Domain of R is $\{1,2,3\}$

Range of R is $\{1,2\}$

ii. The relation is not irreflexive because there are not all 0's in main diagonal of matrix.

iii. The relation is antisymmetric because $(1,2)$ and $(3,1) \in R$ but $(2,1)$ and $(1,3) \notin R$

b) i. set $S = \{(4,5),(5,4),(5,5)\}$

ii. $M_s = \begin{array}{c c} & \begin{array}{cc} 4 & 5 \end{array} \\ \begin{array}{c} 4 \\ 5 \end{array} & \begin{bmatrix} 0 & 1 \\ 1 & 1 \end{bmatrix} \end{array}$ S is not reflexive relation because $(4,4) \notin S$.

$M_s^T = \begin{bmatrix} 0 & 1 \\ 1 & 1 \end{bmatrix}$ S is symmetric relation because $M_s = M_s^T$.

$\begin{bmatrix} 0 & 1 \\ 1 & 1 \end{bmatrix} \otimes \begin{bmatrix} 0 & 1 \\ 1 & 1 \end{bmatrix} \neq \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}$ S is not transitive relation because $M_s \otimes M_s \neq M_s$

Thus, set S is not equivalence relation because it is not reflexive, symmetric and not transitive relation.

c) i.

$$\begin{array}{ccc} X & f & Y \\ 1 & \longrightarrow & 1 \\ 2 & \longrightarrow & 2 \\ 3 & \longrightarrow & 3 \\ & & 4 \end{array}$$

$f(x) = y$

f is one-to-one but not onto function because value 4 has no arrow pointed towards it.

ii.

$$\begin{array}{ccc} X & g & Z \\ 1 & \longrightarrow & 1 \\ 2 & \longrightarrow & 2 \\ 3 & & \end{array}$$

$g(x) = z$

assume $g(1) = 1$, $g(2) = 2$

g is onto function as both 1 and 2 is taken but not one-to-one function because the domain is not X but $\{1,2\}$.

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iii.

$$\begin{array}{ccc} x & h & x \\ 1 & \longrightarrow & 1 \\ 2 & \nearrow & 2 \\ 3 & \nearrow & 3 \end{array}$$

define h by $h(x) = 1$, thus $h(1) = h(2) = h(3)$

h is not one-to-one as $h(1) = h(2) = h(3)$

h is not onto function because value 2 and 3 has no arrow pointed.

d) i. $m(x) = 4x + 3$

$m^{-1}(y) = x$

$y = 4x + 3$

$4x = y - 3$

$x = \dfrac{y-3}{4}$

$m^{-1}(y) = \dfrac{y-3}{4}$

ii. $n \circ m = (n \circ m)(x)$

$= n(m(x))$

$= n(4x + 3)$

$= 2(4x + 3) - 4$

$= 8x + 6 - 4$

$= 8x + 2$

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Question 3

a. $a_k = a_{k-1} + 2k \quad k \geq 2 \;, \quad a_1 = 1$

i. $a_1 = 1$

$a_2 = a_{2-1} + 2(2) = a_1 + 4 = 1 + 4 = 5$

$a_3 = a_{3-1} + 2(3) = a_2 + 6 = 5 + 6 = 11$

ii.
```
a (k)
 {
     if (k=1)
         return 1
     else
         return  a(k-1) + (2*k)
 }
```

b. $r_1 = 7$

$r_2 = 2(7) = 14$

$r_3 = 2(14) = 28$

$r_4 = 2(28) = 56$

$\vdots$

$r_k = 2r_{k-1} \;, \quad k \geq 2 \;, \quad r_1 = 7$

c. $S(4)$

$\downarrow$

$5 * S(3) = 625 \qquad \therefore S(4) = 625$

$\downarrow$

$5 * S(2) = 125$

$\downarrow$

$5 * S(1) = 25$

$\downarrow$

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Question 4

a. ___ ___ ___ ___

1st digit = 9 ways

2nd digit = 16 ways

3rd digit = 16 ways

4th digit = 11 ways

total ways = $9 \times 16 \times 16 \times 11$

= 25344 ways

b. A ___ ___ ___ ___ ___ 0

Letters Digits

Letters = 26

Digits = 10

${}^{1}P_{1} \times 26 \times 26 \times 26 \times 10 \times 10 \times {}^{1}P_{1} = 1757600$ ways

∴ total ways = 1757600 ways

c. ___ ___ ___

1 letter = ${}^{8}P_{1} = \frac{8!}{(8-1)!} = 8$ ways

2 letters = ${}^{8}P_{2} = \frac{8!}{(8-2)!} = 56$ ways

3 letters = ${}^{8}P_{3} = \frac{8!}{(8-3)!} = 336$ ways

total ways = 8 + 56 + 336

= 400 ways

d. ways to choose 4 women out of 7 = ${}^{7}C_{4}$

$= \frac{7!}{4!(7-4)!}$

= 35

total ways = 35×20

= 700 ways

ways to choose 3 men out of 6 = ${}^{6}C_{3}$

$= \frac{6!}{3!(6-3)!}$

= 20

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e.

$$\text{total ways} = \frac{11!}{1! \times 1! \times 1! \times 2! \times 1! \times 2! \times 1! \times 1! \times 1!}$$

$$= 9979200 \text{ ways}$$

f. $n = 6$, $r = 10$

$$C(6+10-1, 10) = \frac{(6+10-1)!}{10!\,(6-1)!}$$

$$= 3003 \text{ ways}$$

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QUESTION 5

a. let pigeons, n = persons
= 18

pigeonholes, m = having same first and last names
= 6 (Ali Daud, Bahar Daud, Carlie Daud, Ali Elyas, Bahar Elyas, Carlie Elyas)

$k = \left\lceil \frac{n}{m} \right\rceil = \left\lceil \frac{18}{6} \right\rceil = 3$ ∴ shown

Thus, there are at least 3 persons have the same first and last names

b. num of intergers = 20
odd integers = |1, 3, 5, 7, 9, 11, 13, 15, 17, 19| = 10 numbers
even integers = 20 - 10
= 10

pigeonholes, k = 10 pigeons, $n > 10$ because $n > k$

Thus, the least numbers to pick in order to be sure of getting at least one that is odd is 11

c. num of integers = 100
set of numbers divisible by 5 = | 5, 10, 15, 20, 25, 30, 35, 40, 45, 50, 55, 60, 65, 70, 75, 80, 85, 90, 95, 100|
= 20 numbers
numbers not divisible by 5 = 100 - 20
= 80

pigeonholes, k = 80 pigeons, $n > 80$

Thus, the number of integers to pick in order to be sure of getting one that is divisible by 5 is 81.