UNIVERSITI TEKNOLOGI MALAYSIA

DISCRETE STRUCTURE ASSIGNMENT 1

COURSE CODE	: SECI1013
COURSE	: DISCRETE STRUCTURE
SCHOOL	: SCHOOL OF COMPUTING
FACULTY	: ENGINEERING
DATE OF SUBMISSION	: 30TH OF NOVEMBER 2020
SECTION NO.	: 03
LECTURER'S NAME	: DR NOR AZIZAH ALI
GROUP	: 11
NAME & MATRIC NO.	:

NO.	NAME	MATRIC NO.
1	ONG HAN WAH	A20EC0129
2	FARAH IRDINA BINTI AHMAD BAHARUDIN	A20EC0035
3	NURARISSA DAYANA BINTI MOHD SUKRI	A20EC0120

DISCRETE STRUCTURE (SECI 1013)

2020/2021 – SEMESTER 1

ASSIGNMENT# 1

1. Let the universal set be the set **R** of all real numbers and let $A=\{x\in\mathbf{R} \mid 0 < x \leq 2\}$, $B=\{x\in\mathbf{R} \mid 1 \leq x < 4\}$ and $C=\{x\in\mathbf{R} \mid 3 \leq x < 9\}$. Find each of the following:

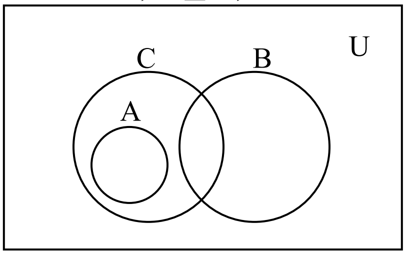
   a) $A \cup C = \{x\in\mathbf{R} \mid 0 < x \leq 2 \text{ or } 3 \leq x < 9\}$

   b) $(A \cup B)' = \{x\in\mathbf{R} \mid x \leq 0 \text{ or } x \geq 4\}$

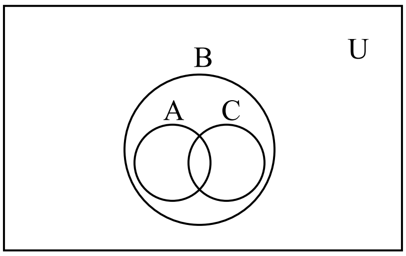
   c) $A' \cup B' = \{x\in\mathbf{R} \mid x < 1 \text{ or } x > 2\}$

2. Draw Venn diagrams to describe sets A, B, and C that satisfy the given conditions.

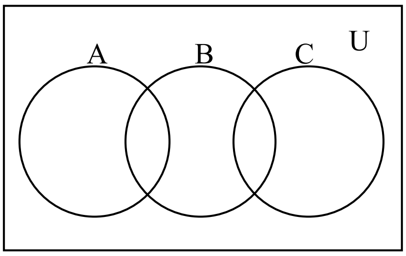
   a) $A \cap B = \emptyset, A \subseteq C, C \cap B \neq \emptyset$



   b) $A \subseteq B, C \subseteq B, A \cap C \neq \emptyset$



   c) $A \cap B \neq \emptyset, B \cap C \neq \emptyset, A \cap C = \emptyset, A \not\subset B, C \not\subset B$



3. Given two relations S and T from A to B,

   $S \cap T = \{(x,y) \in A\times B \mid (x,y) \in S \text{ and } (x,y) \in \mathrm{T}\}$

   $S \cup T = \{(x,y) \in A\times B \mid (x,y) \in S \text{ or } (x,y) \in \mathrm{T}\}$

   Let $A=\{-1, 1, 2, 4\}$ and $B=\{1,2\}$ and defined binary relations S and T from A to B as follows:

   For all $(x,y) \in A\times B$, $\quad x\, S\, y \leftrightarrow |x| = |y|$

   For all $(x,y) \in A\times B$, $\quad x\, T\, y \leftrightarrow x-y$ is even

   State explicitly which ordered pairs are in $A\times B$, S, T, $S \cap T$, and $S \cup T$.

   $A \times B = \{(-1,1), (-1,2), (1,1), (1,2), (2,1), (2,2), (4,1), (4,2)\}$

   $S = \{(-1,1), (1,1), (2,2)\}$

   $T = \{(-1,1), (1,1), (2,2), (4,2)\}$

   $S \cap T = \{(-1,1), (1,1), (2,2)\}$

   $S \cup T = \{(-1,1), (1,1), (2,2), (4,2)\}$

4. Show that $\neg\, ((\neg p\wedge q) \vee (\neg p\wedge\neg q)) \vee (p\wedge q) \equiv p$. State carefully which of the laws are used at each stage.

$\neg\, ((\neg p\wedge q) \vee (\neg p\wedge\neg q)) \vee (p\wedge q)$	$\equiv$	$\neg\, (\neg p\wedge(q\vee\neg q)) \vee (p\wedge q)$	(Distributive law)
	$\equiv$	$\neg\, (\neg p\wedge q) \vee (p\wedge q)$	(Negation law)
	$\equiv$	$\neg\, (\neg p) \vee (p\wedge q)$	(Identity law)
	$\equiv$	$p \vee (p\wedge q)$	(Double negation law)
	$\equiv$	p	(Absorption law)

5. $R_1=\{(x,y) \mid x+y \leq 6\}$; R_1 is from X to Y; $R_2=\{(y,z) \mid y>z\}$; R_2 is from Y to Z; ordering of X, Y, and Z: 1, 2, 3, 4, 5.

$$R_1 = \{(1,1), (1,2), (1,3), (1,4), (1,5), (2,1), (2,2), (2,3), (2,4), (3,1), (3,2), (3,3), (4,1), (4,2), (5,1)\}$$

$$R_2 = \{(2,1), (3,1), (3,2), (4,1), (4,2), (4,3), (5,1), (5,2), (5,3), (5,4)\}$$

Find:

a) The matrix A_1 of the relation R_1 (relative to the given orderings)

$$A_1 = \begin{array}{c} \\ 1 \\ 2 \\ 3 \\ 4 \\ 5 \end{array} \begin{array}{c} \begin{array}{ccccc} 1 & 2 & 3 & 4 & 5 \end{array} \\ \begin{bmatrix} 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 0 \\ 1 & 1 & 1 & 0 & 0 \\ 1 & 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 \end{bmatrix} \end{array}$$

b) The matrix A_2 of the relation R_2 (relative to the given orderings)

$$A_2 = \begin{array}{c} \\ 1 \\ 2 \\ 3 \\ 4 \\ 5 \end{array} \begin{array}{c} \begin{array}{ccccc} 1 & 2 & 3 & 4 & 5 \end{array} \\ \begin{bmatrix} 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 \\ 1 & 1 & 0 & 0 & 0 \\ 1 & 1 & 1 & 0 & 0 \\ 1 & 1 & 1 & 1 & 0 \end{bmatrix} \end{array}$$

c) Is R_1 reflexive, symmetric, transitive, and/or an equivalence relation?

R_1 is not reflexive, symmetric and not transitive.
Thus, R_1 is not an equivalence relation.

d) Is R_2 reflexive, antisymmetric, transitive, and/or a partial order relation?

R_2 is irreflexive, antisymmetric but not transitive.
Thus, R_2 is not a partial order relation.

6. Suppose that the matrix of relation R_1 on $\{1, 2, 3\}$ is

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 1 \\ 1 & 0 & 1 \end{bmatrix}$$

relative to the ordering 1, 2, 3, and that the matrix of relation R_2 on $\{1, 2, 3\}$ is

$$\begin{bmatrix} 0 & 1 & 0 \\ 0 & 1 & 0 \\ 1 & 0 & 1 \end{bmatrix}$$

relative to the ordering 1, 2, 3.

$R_1 = \{(1,1), (2,2), (2,3), (3,1), (3,3)\}$
$R_2 = \{(1,2), (2,2), (3,1), (3,3)\}$

Find:

a) The matrix of relation $R_1 \cup R_2$

$R_1 \cup R_2 = \{(1,1), (1,2), (2,2), (2,3), (3,1), (3,3)\}$

$$R_1 \cup R_2 = \begin{array}{c} \\ 1 \\ 2 \\ 3 \end{array} \begin{array}{c} \begin{array}{ccc} 1 & 2 & 3 \end{array} \\ \begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & 1 \\ 1 & 0 & 1 \end{bmatrix} \end{array}$$

b) The matrix of relation $R_1 \cap R_2$

$R_1 \cap R_2 = \{(2,2), (3,1), (3,3)\}$

$$R_1 \cap R_2 = \begin{array}{c} \\ 1 \\ 2 \\ 3 \end{array} \begin{array}{c} \begin{array}{ccc} 1 & 2 & 3 \end{array} \\ \begin{bmatrix} 0 & 0 & 0 \\ 0 & 1 & 0 \\ 1 & 0 & 1 \end{bmatrix} \end{array}$$

7. If $f:\mathbf{R} \to \mathbf{R}$ and $g:\mathbf{R} \to \mathbf{R}$ are both one-to-one, is $f + g$ also one-to-one? Justify your answer.

Let x_n be real numbers with n is a Natural Number. Suppose that,

$(f + g)(x_1) = (f + g)(x_2)$
$f(x_1) + g(x_1) = f(x_2) + g(x_2)$

Since f is one to one, we have,
$f(x_1) = f(x_2) \to x_1 = x_2$

Similarly,
$g(x_1) = g(x_2) \to x_1 = x_2$

Therefore, f + g is one-to-one.

8. With each step you take when climbing a staircase, you can move up either one stair or two stairs. As a result, you can climb the entire staircase taking one stair at a time, taking two at a time, or taking a combination of one- or two-stair increments. For each integer $n \geq 1$, if the staircase consists of n stairs, let c_n be the number of different ways to climb the staircase. Find a recurrence relation for $c_1, c_2, \ldots, c_n$.

Let, c_n = number of different ways to climb the staircase

When $n = 1$, there is only one way to climb the stair. Therefore, $c_1 = 1$

When $n = 2$, there is two ways to climb the stair which is either one or two steps. Therefore, $c_2 = 2$

If $n \geq 3$, there is a combination of either one or two steps as the last step taken. Thus, the total number of ways to climb the stair is the sum of number of ways for one step and two steps climbing.

$$c_n = c_{n-1} + c_{n-2}\text{, when } n \geq 3$$

9. The Tribonacci sequence (t_n) is defined by the equations,

$$t_0 = 0,\ t_1 = t_2 = 1,\ t_n = t_{n-1} + t_{n-2} + t_{n-3} \text{ for all } n \geq 3.$$

a) Find t_7.

$t_3 = t_2 + t_1 + t_0 = 1+1+0 = 2$

$t_4 = t_3 + t_2 + t_1 = 2+1+1 = 4$

$t_5 = t_4 + t_3 + t_2 = 4+2+1 = 7$

$t_6 = t_5 + t_4 + t_3 = 7+4+2 = 13$

$t_7 = t_6 + t_5 + t_4 = 13+7+4 = 24$

b) Write a recursive algorithm to compute t_n, $n \geq 3$.

```
t(n) {
        if (n==0)
                return 0
        else if (n==1 or n==2)
                return 1
else
        return t(n-1) + t(n-2) + t(n-3)
}
```