SECI1013-06

DISCRETE STRUCTURE

Assignment 4

Group 13

SEMESTER I, SESI 2020/2021

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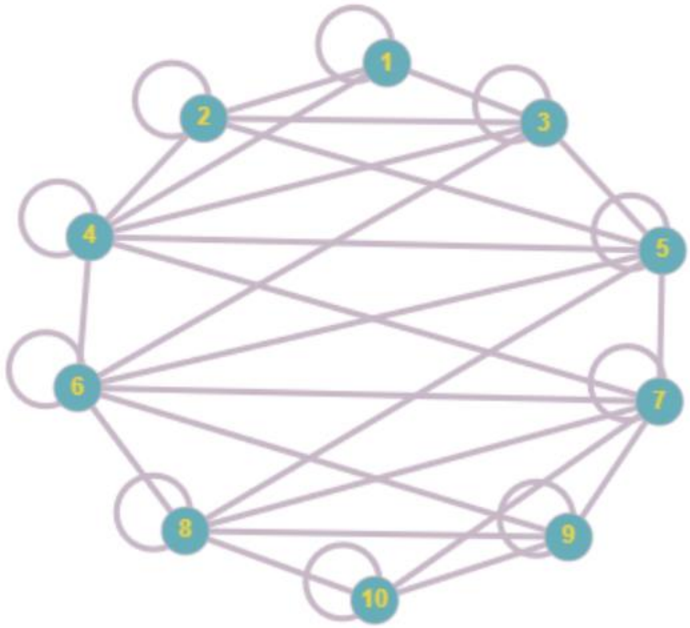
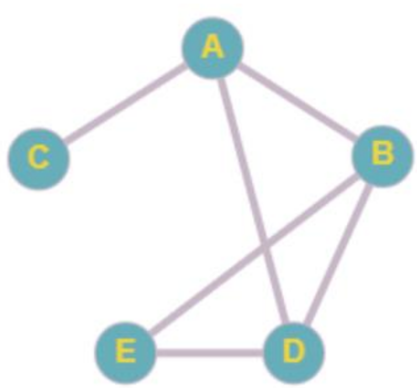
1. Let G be a graph with (G) = {1, 2, …. , 10}, such that two numbers 'v' and 'w' in $V(G)$ are adjacent if and only if | v – w| ≤ 3. Draw the graph G and determine the numbers of edges, (G).



Number of edges, (G): 34

2. Model the following situation as graphs, draw each graphs and gives the corresponding adjacency matrix.

(a) Ahmad and Bakri are friends. Ahmad is also friends with David and Chong. David, Bakri and Ehsan all friends. (Note that you may use the representation of A= Ahmad; B = Bakri; C = Chong; D = David; E= Ehsan)



$$A_G = \begin{array}{c} \\ A \\ B \\ C \\ D \\ E \end{array} \begin{array}{c} \begin{array}{ccccc} \text{A} & \text{B} & \text{C} & \text{D} & \text{E} \end{array} \\ \begin{bmatrix} 0 & 1 & 1 & 1 & 0 \\ 1 & 0 & 0 & 1 & 1 \\ 1 & 0 & 0 & 0 & 0 \\ 1 & 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 1 & 0 \end{bmatrix} \end{array}$$

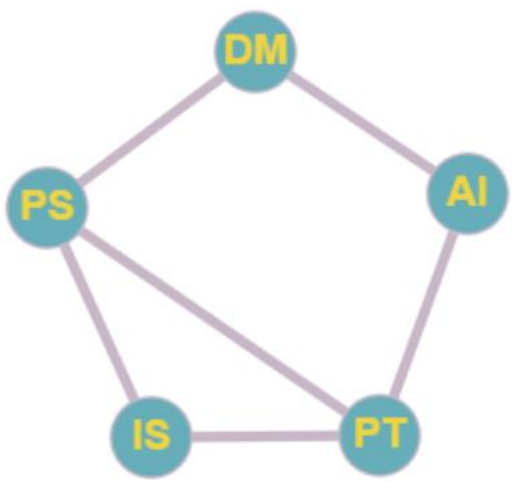
(b) There are 5 subjects to be scheduled in the exam week: Discrete Mathematics (DM), Programming Technique (PT), Artificial Intelligence (AI), Probability Statistic (PS) and Information System (IS). The following subjects cannot be scheduled in the same time slot:-

i. DM and IS

ii. DM and PT

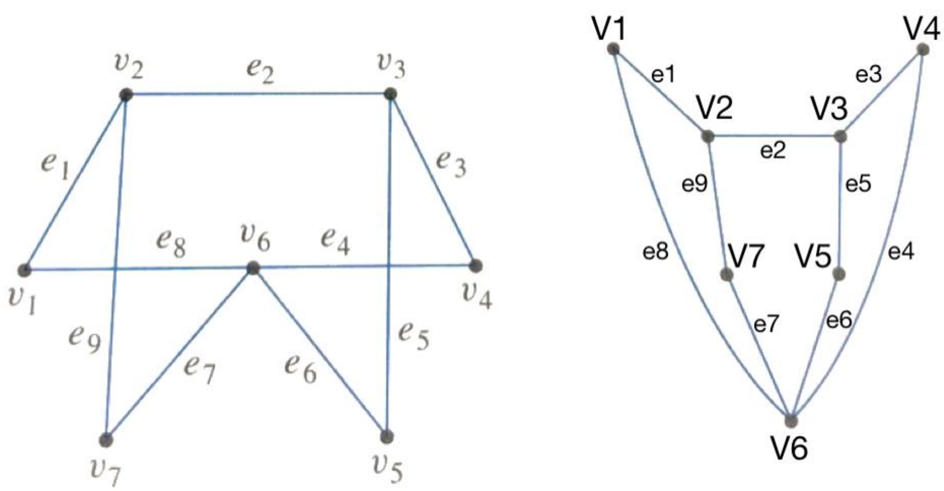
iii. AI and PS

iv. IS and AI

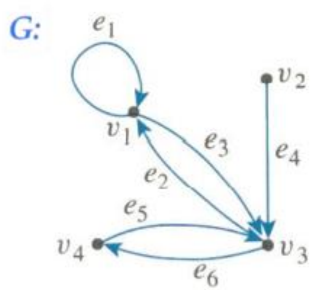


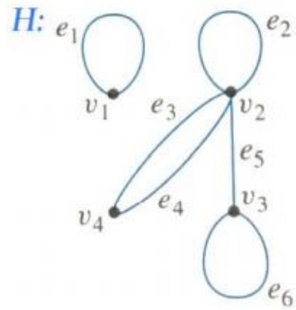
$$A_G = \begin{array}{c} \\ DM \\ PT \\ AI \\ PS \\ IS \end{array} \begin{array}{c} \begin{array}{ccccc} \text{DM} & \text{PT} & \text{AI} & \text{PS} & \text{IS} \end{array} \\ \begin{bmatrix} 0 & 0 & 1 & 1 & 0 \\ 0 & 0 & 1 & 1 & 1 \\ 1 & 1 & 0 & 0 & 0 \\ 1 & 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 1 & 0 \end{bmatrix} \end{array}$$

3. Show that the two drawing represent the same graph by labeling the vertices and edges of the right-hand drawing to correspond to left-hand drawing.



4. Find the adjacency and incidence matrices for the following graphs.





$A_G =$

	V_1	V_2	V_3	V_4
V_1	1	0	1	0
V_2	0	0	1	0
V_3	1	0	0	1
V_4	0	0	1	0

$I_G =$

	e_1	e_2	e_3	e_4	e_5	e_6
V_1	2	1	1	0	0	0
V_2	0	0	0	1	0	0
V_3	0	1	1	1	1	1
V_4	0	0	0	0	1	1

$A_H =$

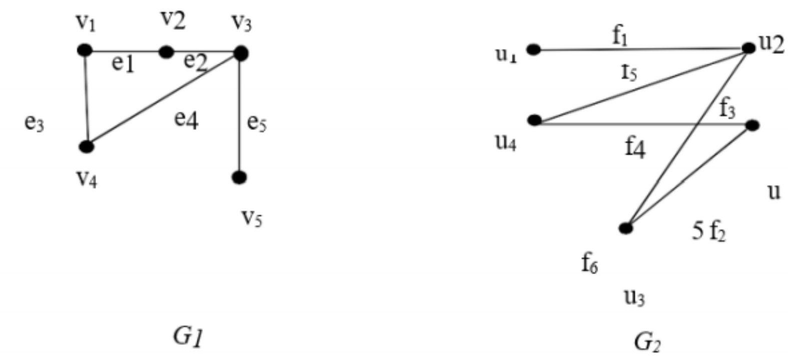
	V_1	V_2	V_3	V_4
V_1	1	0	0	0
V_2	0	1	1	2
V_3	0	1	1	0
V_4	0	2	0	0

$I_H =$

	e_1	e_2	e_3	e_4	e_5	e_6
V_1	2	0	0	0	0	0
V_2	0	2	1	1	1	0
V_3	0	0	0	0	1	2
V_4	0	0	1	1	0	0

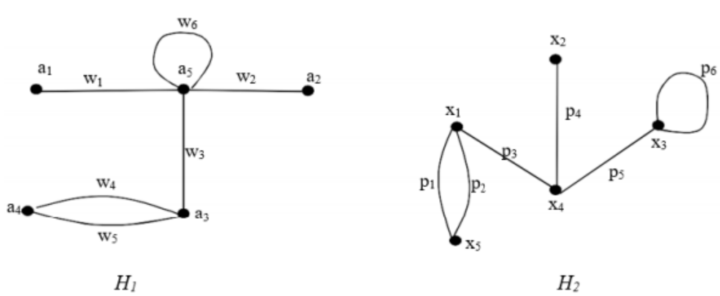
5. Determine whether the following graphs are isomorphic.

a)



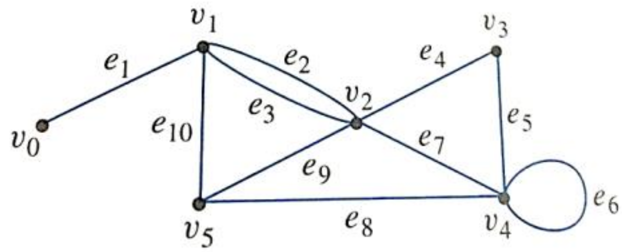
- Both have 5 vertices and 5 edges.
- Both are connected and simple graph.
- Both have 1 vertex with 3 degree, 3 vertices with 2 degree and 1 vertex with 1 degree.
- $f(v_1) = u_5$ $f(v_2) = u_4$
  $f(v_3) = u_2$ $f(v_4) = u_3$
  $f(v_5) = u_1$
- G_1 and G_2 are isomorphic.

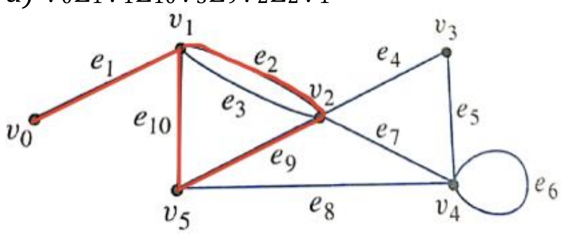
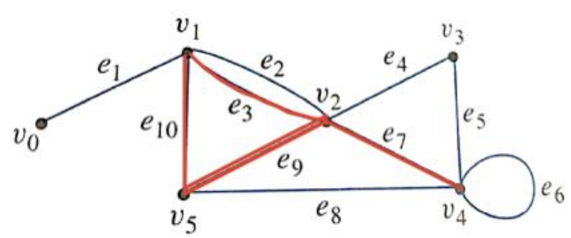
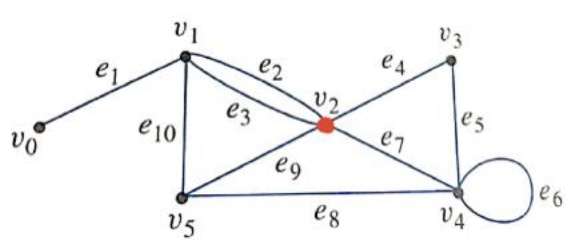
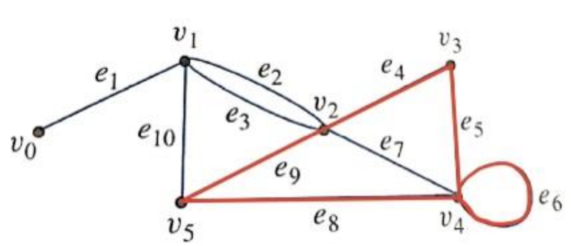
b)

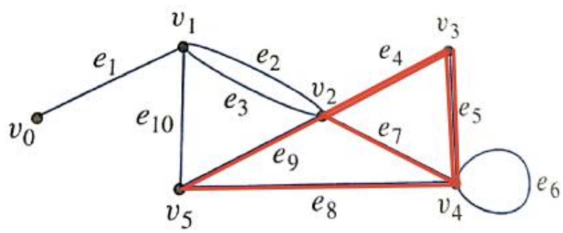
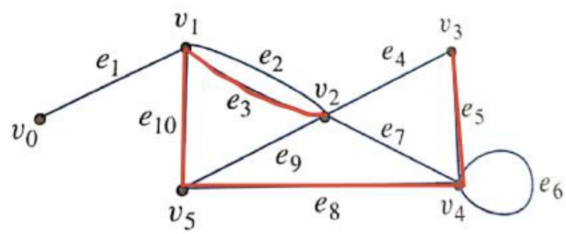


- Both have 5 vertices and 6 edges.
- Both are connected and simple graph.
- In graph H_1, it have 1 vertex with 5 degree, 1 vertex with 3 degree, 1 vertex with 2 degree and 2 vertex with 1 degree, while in graph H_2, , it have 3 vertex with 3 degree, 1 vertex with 2 degree and 1 vertex with 1 degree.
- Therefore f: $H_1 \longrightarrow H_2$ cannot be defined.
- H_1 and H_2 are not isomorphic.

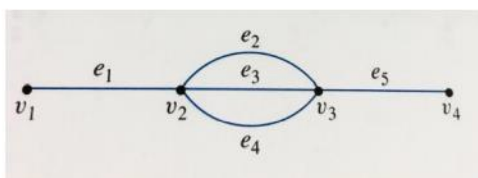
6. In the graph below, determine whether the following walks are trails, paths, closed walks, circuits/cycles, simple circuits or just walks.



a) $V_0E_1V_1E_{10}V_5E_9V_2E_2V_1$ 	It is a **trail** because it does not contain repeated edge.
b) $V_4E_7V_2E_9V_5E_{10}V_1E_3V_2E_9V_5$ 	It is a **walk**. It has repeated edges and vertices
c) V_2 	It is a **trivial walk** because it consists of the single vertex V_2. The walk contains zero edges. It has length of zero.
d) $V_5E_9V_2E_4V_3E_5V_4E_6V_4E_8V_5$ 	It is a **closed circuit**. The first and last vertex are the same. It has repeated vertices but no repeated edges.

e) $V_2E_4V_3E_5V_4E_8V_5E_9V_2E_7V_4E_5V_3E_4V_2$ 	It is **closed walk.** The first and last vertex is the same. There is repeated vertices and edges
f) $V_3E_5V_4E_8V_5E_{10}V_1E_3V_2$ 	It is a **path**. It has no repeated vertices.

7. Consider the following graph.



a) How many paths are there from $v1$ to $v4$?

Answer: **3**

V1 > E1 > V2 > E2 > V3 > E5 > V4
V1 > E1 > V2 > E3 > V3 > E5 > V4
V1 > E1 > V2 > E4 > V3 > E5 > V4

b) How many trails are there from $v1$ to $v4$?

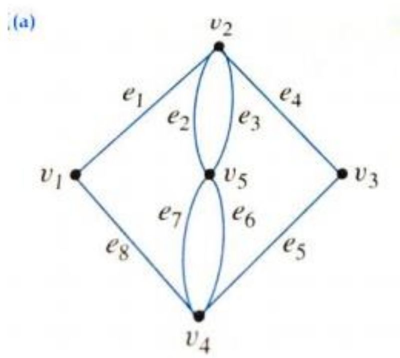
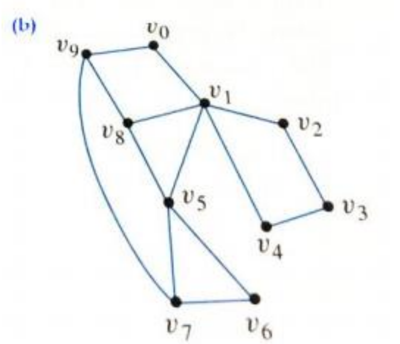
Answer: **9**

V1 > E1 > V2 > E2 > V3 > E5 > V4
V1 > E1 > V2 > E3 > V3 > E5 > V4
V1 > E1 > V2 > E4 > V3 > E5 > V4
V1 > E1 > V2 > E2 > V3 > E3 > V2 > E4 > V3 > E5 > V4
V1 > E1 > V2 > E2 > V3 > E4 > V2 > E3 > V3 > E5 > V4
V1 > E1 > V2 > E3 > V3 > E2 > V2 > E4 > V3 > E5 > V4
V1 > E1 > V2 > E3 > V3 > E4 > V2 > E2 > V3 > E5 > V4
V1 > E1 > V2 > E4 > V3 > E2 > V2 > E3 > V3 > E5 > V4
V1 > E1 > V2 > E4 > V3 > E3 > V2 > E2 > V3 > E5 > V4

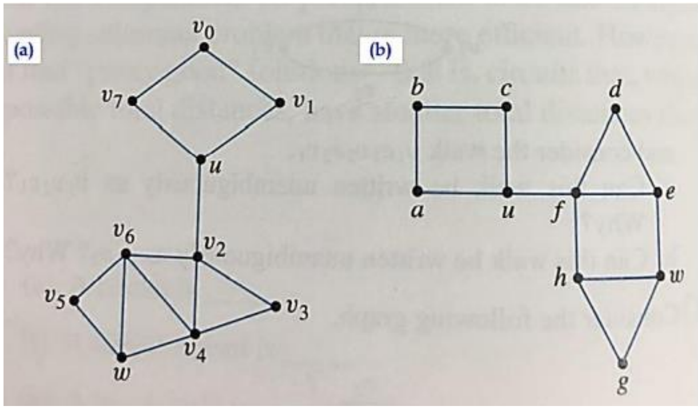
c) How many walks are there from $v1$ to $v4$?

From $v1$ to $v4$ **has infinity number of walks**. This is because a walk is a finite alternating sequence of adjacent vertices and edges of G. Since repeated vertices and edges are allowed, the length is also no restricted, therefore it can be started at $v1$, keep repeated visit any vertices and edges in any pattern, any number of times, but lastly end with $v4$. Through all these, they all consider as a valid walk.

8. Determine which of the graphs in (a) – (b) have Euler circuits. If the graph does not have a Euler circuit, explain why not. If it does have a Euler circuit, describe one.

	It **has Euler circuit.** Every vertex has even degree. For v1 and v3, they have 2 degree. For v2 and v4, they have 4 degree. V2 > E2 > V5 > E7 > V4 > E8 > V1 > E1 > V2 > E3 > V5 > E6 > V4 > E5 > V3 > E4 > V2
	It **does not have Euler circuit.** To have Euler circuit, every vertex must have even degree. However, in this circuit V7, V8 and V9 has odd degree of 3.

9. For each of graph in (a) – (b), determine whether there is an Euler path from u to w. If there is, find such a path.

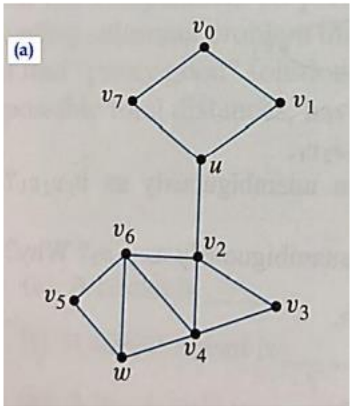


a) (u, v_7, v_0, v_1, u, v_2, v_3, v_4, v_2, v_6, v_4, w, v_6, v_5, w) is a Euler path from u to w. This is because every vertex except u and w which is the starting point and ending pint vertex, therefore there exists a Euler path.

b) No Euler path. This is because Euler path should have every vertex having even degree except the starting and ending vextex. Although u and w have odd degree, but since vertex h and e also have add degree. Therefore a Euler path cannot be formed.

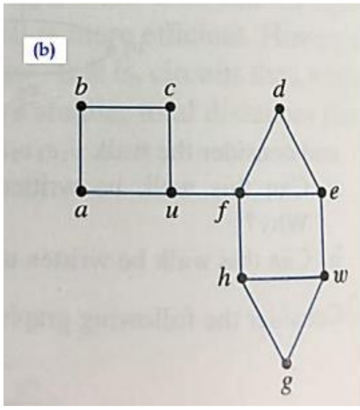
10. For each of graph in (a) – (b), determine whether there is Hamiltonian circuit. If there is, exhibit one.

a)



In Hamiltonian circuit, every vertex appears exactly once, expect for the first and the last, which are the same. However, it does not include every edge.
This graph **does not consist of Hamiltonian circuit** because u and v2 only have 1 edge connecting them. Once go from u to v2, it is impossible to get back from v2 to u to form a circuit.

b)



In Hamiltonian circuit, every vertex appears exactly once, expect for the first and the last, which are the same. However, it does not include every edge.
This graph **does not consist of Hamiltonian circuit** because u and f only have 1 edge connecting them. Once go from u to f, it is impossible to get back from f to u to form a circuit.

11. How many leaves does a full 3-ary tree with 100 vertices have?

In 3-ary tree with 100 vertices, the m = 3, n = 100, leaves, l = ?.

Using the formula,

$$l = n - \left\lceil \frac{(n-1)}{m} \right\rceil$$

$$l = 100 - \left\lceil \frac{(100-1)}{3} \right\rceil$$

$$l = 100 - \left\lceil \frac{99}{3} \right\rceil$$

$$l = 100 - 33$$

$$l = 67$$

12. Find the following vertex/vertices in the rooted tree illustrated below.

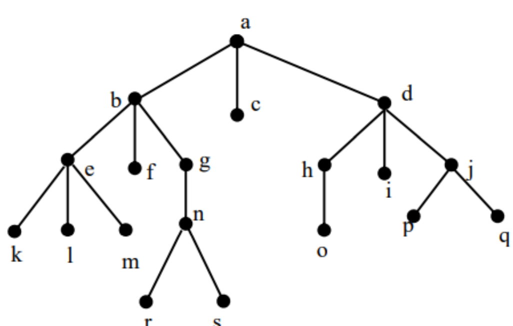
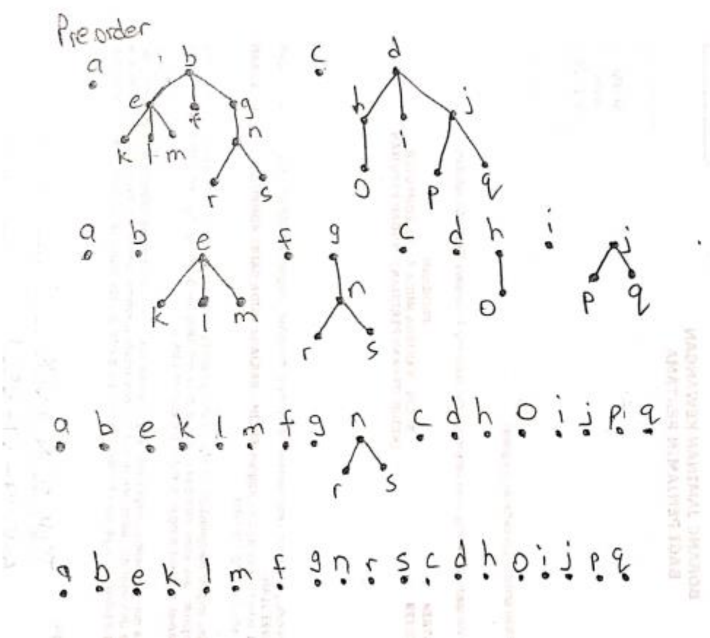


Figure 1

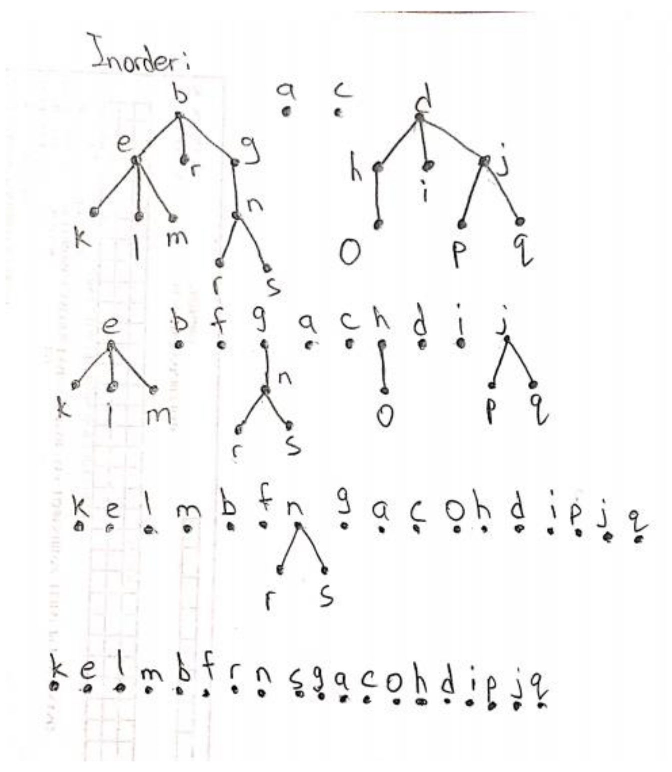
a) **Root** = a
b) **Internal vertices** = a, b, d, e, g, h, j, n
c) **Leaves** = c, f, k, l, m, r, s, o, i, p, q
d) **Children of n** = r, s
e) **Parent of e** = b
f) **Siblings of k** = l, m
g) **Proper ancestors of q** = a, d, j
h) **Proper descendants of b** = e, f, g, k, l, m, n, r, s

13. In which order are the vertices of ordered rooted tree in Figure 1 is visited using preorder, inorder and postorder.

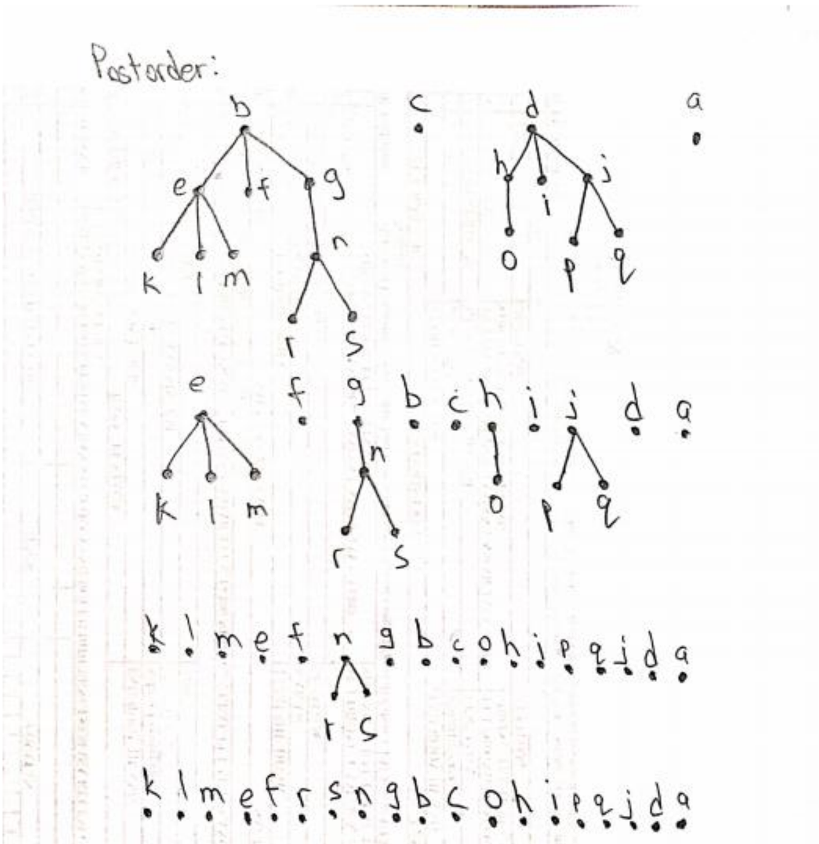
Preorder: a, b, e, k, l, m, f, g, n, r, s, c, d, h, o, i, j, p, q



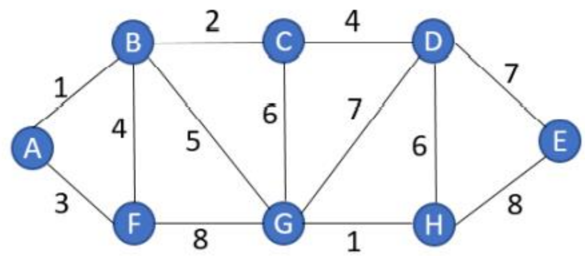
Inorder: k, e, l, m, b, f, r, n, s, g, a, c, o, h, d, i, p, j, q



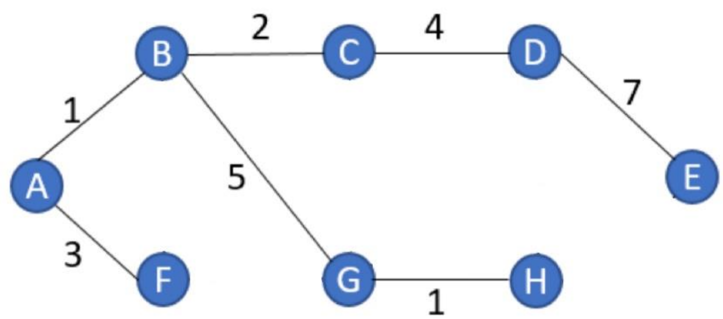
Postorder: k, l, m, e, f, r, s, n, g, b, c, o, h, i, p, q, j, d, a



14. Find the minimum spanning tree for the following graph using Kruskal's algorithm.



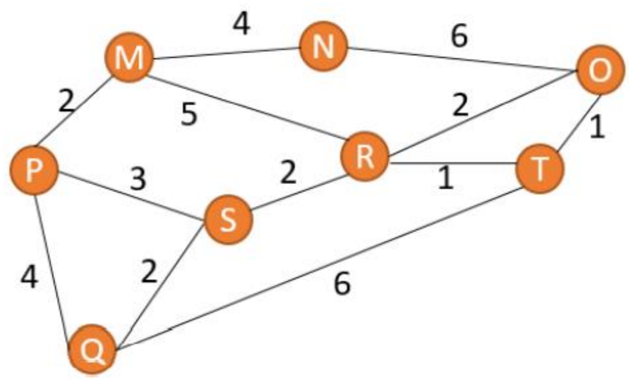
Edges	Oder of size
AB	1
GH	1
BC	2
AF	3
BF	4
CD	4
BG	5
CG	6
DH	6
DG	7
DE	7
FG	8
EH	8



$\text{Total weight} = 1 + 1 + 2 + 3 + 4 + 5 + 7$

$= 23$

15. Use Dijsktra's algorithm to find the shortest path from M to T for the following graph.



No	S	N	L(M)	L(N)	L(O)	L(P)	L(Q)	L(R)	L(S)	L(T)
1	{}	{M,N,P,O,R,S,Q,T}	0	∞	∞	∞	∞	∞	∞	∞
2	{M}	{N,P,O,R,S,Q,T}	0	4	∞	2	∞	5	∞	∞
3	{M,P}	{N,O,R,S,Q,T}	0	4	∞	2	6	5	5	∞
4	{M,P,N}	{O,R,S,Q,T}	0	4	10	2	6	5	5	∞
5	{M,P,N,R}	{O,S,Q,T}	0	4	7	2	6	5	5	6
6	{M,P,N,R,S}	{O,Q,T}	0	4	7	2	6	5	5	6
7	{M,P,N,R,S,T}	{O,Q}	0	4	7	2	6	5	5	6

Shortest path : M – R – T

Shortest length $= 5 + 1$
$= 6$