UTM UNIVERSITI TEKNOLOGI MALAYSIA

SCHOOL OF COMPUTING
Faculty of Engineering

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DISCRETE STRUCTURE (SECI1013-03)

ASSIGNMENT 2

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ASSIGNMENT 2

1. Consider 3-digit number whose digit are either 2,3, 4, 5, 6 or 7

   a. How many numbers are there?

   Case 1: choose the first digit = 6 ways
   Case 2: choose the second digit = 6 ways
   Case 3: choose the third digit = 6 ways
   ∴ Total ways numbers can be formed = $6 \times 6 \times 6 = 216$ *ways*

   b. How many numbers are there if the digits are instinct?

   Case 1: choose the first digit = 6 ways
   Case 2: choose the second digit = 5 ways
   Case 3: choose the third digit = 4 ways
   ∴ Total ways numbers can be formed = $6 \times 5 \times 4 = 120$ *ways*

   c. How many numbers between 300 to 700 is only odd digits allow?

   Case 1: choose the first digit (3,4,5,6) = 4 ways
   Case 2: choose the second digit = 6 ways
   Case 3: choose the third digit (3,5,7) = 3 ways
   ∴ Total ways numbers can be formed = $4 \times 6 \times 3 = 72$ *ways*

2. Anita invited 10 of her friends, a couple, four men and four women to her anniversary dinner, how many ways Anita can arrange them around a round dinner table

   a. Men insist to sit next to each other

   Arrange men = 5! ways
   Arrange women = 5! ways
   (6-1)! x 5! = 14,400
   ∴ Total = 14,400 ways

   b. The couple insisted to sit next to each other

   Let the couple be a single unit,
   Ways to arrange people = 8!

Ways to arrange couple = 2!
(9-1)! x 2! = 80,640
∴ Total ways to arrange = 80,640

c. Men and women sit in alternate seat

Ways to arrange men = (5-1)!
Ways to arrange women to sit between men= 5!
∴ Total ways = 2,880

d. Before her friend left, Anita want to arrange a photoshoot. How many ways can her photographer arrange in a row, if Anita and her husband stand next to each other
Assuming anita and her husband as a single unit, there will be 10 of Anita's friend and 1 couple.
Ways to arrange = 11! x 2!
∴ Total ways = 79,833,600

3. In a school sport day, five sprinters are competing in a 100 meter race. How many ways are there for the sprinter to finish

a. If no ties

Case 1: winner = 5 ways
Case 2: runner-up = 4 ways
Case 3: third place = 3 ways
Case 4: fourth place = 2 ways
Case 5: last place = 1 way
∴ Total ways for sprinters to finish = 5! = 120 *ways*

b. Two sprinters tie

If there is a tie, the sprinters can finish in 1^{st}, 2^{nd}, 3^{rd}, 4^{th} place = 4! ways
Choosing two sprinters forming a tie = 5C2 = 10
∴ Total = 240 ways

c. Two group of two sprinters tie

If there are 2 ties, the sprinters can finish in 1^{st}, 2^{nd}, 3^{rd} place = 3! ways
Choosing two sprinters forming a tie = 5C2 = 10
Choose two groups of tied sprinters for 3 position = 3C2 = 3

$\therefore$ Total = 180 ways

4. A croissant shop has plain croissants, cherry croissants, chocolate croissants, almond croissants, apple croissants, and broccoli croissants. How many ways are there to choose
   a. a dozen croissants?

$n = 6$
$r = 12$
$C(n + r - 1, r) = \frac{(n+r-1)!}{r!(n-1)!} = \frac{17!}{12!5!} = 6,188$

   b. two dozen croissants with at least two of each kind?

$n = 6$
$r = 12$
$C(n + r - 1, r) = \frac{(n+r-1)!}{r!(n-1)!} = \frac{17!}{12!5!} = 6,188$

   c. two dozen croissants with at least five chocolate croissants and at least three almond croissants?

Remaining croissants after choosing 5 chocolate croissants and 3 almond croissants,
$24 - 5 - 3 = 16$
$n = 6$
$r = 16$
$C(n + r - 1, r) = \frac{(n+r-1)!}{r!(n-1)!} = \frac{21!}{16!5!} = 20,349$

5. This procedure is used to break ties in games in the championship round of the World Cup soccer tournament. Each team selects five players in a prescribed order. Each of these players takes a penalty kick, with a player from the first team followed by a player from the second team and so on, following the order of players specified. If the score is still tied at the end of the 10 penalty kicks, this procedure is repeated. If the score is still tied after 20 penalty kicks, a sudden-death shootout occurs, with the first team scoring an unanswered goal victorious.

   a. How many different scoring scenarios are possible if the game is settled in the first round of 10 penalty kicks, where the round ends once it is impossible for a team to equal the number of goals scored by the other team?

   Two scenario that will ended with the first 10 penalty kicks

   First scenario, if one team wins 2 games with additional 1 win or 1 tie
   Second scenario, if one team wins 1 game with additional 3 wins or ties

   2 wins and 1 ties/wins: x C(3.1) x 2 = 36
   1 win and 3 ties/wins: C(3,1) x C(4,3) x = 96
   Number of Scenarios = 2 x (36+96)

= 264

b. How many different scoring scenarios for the first and second groups of penalty kicks are possible if the game is settled in the second round of 10 penalty kicks?

If the game is settled in the second round of 10 penalty kicks
The number of possible outcomes is

1024-264 = 760 the scenarios result in the game not being settled in the first round of 10 penalty kicks.

First round: 760 scenarios
Second round: 264 scenarios

Number of scenarios = 760 x 264 = 200,640

c. How many scoring scenarios are possible for the full set of penalty kicks if the game is settled with no more than 10 total additional kicks after the two rounds of five kicks for each team?

If first and second round not finish

First round: 760 scenarios
Second round: 760 scenarios

Sudden death: 10 since we have 5 kick for each team (2x5 =10)

Number of scenarios: 760 x 760 x 10 = 5,776,000

6. A professor gives a multiple-choice quiz that has ten questions, each with four possible responses, a,b,c,d. What is the minimum number of students that must be in the professor's class in order to guarantee that at least three answer sheets must be identical?

(Assume that no answers are left blank.)

Possible distinct answers = 4^{10}

To ensure at least 3 answer sheets are identical, there must be minimum of,

$= 4^{10} \times 2 + 1$

=2,097,153 students

7. In a secondary examination, 75% of the students have passed in history and 65% in Mathematics, while 50% passed both in history and mathematics. If 35 candidates failed in both the subjects, what is the number of candidates sit for the exam?

Let x be number of students,

Students passed in History = $0.75x$

Students passed in Mathematics = $0.65x$

Passed in both = $0.5x$

Students passed only in History

$= (0.75x) - (0.5x)$

$= 0.25x$

Students passed only in Mathematics

$= (0.65x) - (0.5x)$

$= 0.15x$

Total students passed = $0.5x + 0.25x + 0.15x = 0.9x$

Total students failed = $x - 0.9x$

$35 = 0.1x$

$x = 350$ *students*

8. An integer from 300 through 780, inclusive is to be chosen at random, find the probability that the number is chosen will have 1 as at least one digit.

Total number of possible outcomes = 780 -299

= 481

(_ 1 _) = $\frac{5 \times 1 \times 9}{481}$ = 0.094

(_ _ 1) = $\frac{4 \times 9 \times 1}{481} + \frac{1 \times 7 \times 1}{481}$ = 0.089

(_ 1 1) = $\frac{5 \times 1 \times 1}{481}$ = 0.010

Probability = 0.094 + 0.089 + 0.01 = 0.193

9. Two blue and four yellow cars are to be parked in a row of 10 parking lots. Assume that cars of the same color are not distinguishable, and the parking lots are chosen at random.

   a. In how many ways can the cars be parked in the parking lots?

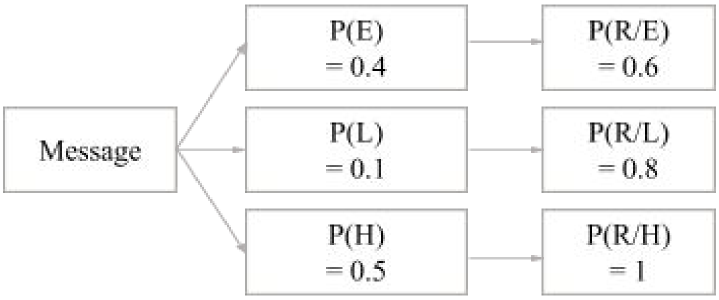
$\frac{10!}{2!\,4!} = 75{,}600$

b. In how many ways can the cars be parked so that the empty lots are next to each one another? Find the probability that the empty lots are next to one another?

$\frac{6!}{2!\,4!}$ x 7 = 105

10. A coach wishes to give a message to a trainee. The probabilities that he uses email, letter and handphone are 0.4,0.1 and 0,5 respectively. He uses only one method. The probabilities of the trainee receive the message if the coach uses email, letter or handphone are 0.6,0.8 and 1 respectively

E = Email
L = Letter
H = Handphone



a. Find the probability the trainee receives the message

T = Trainee receives the message

$P\,(T) = \left(P\,(E) \times P\,\left(\frac{R}{E}\right)\right) + \left(P\,(L) \times P\,\left(\frac{R}{L}\right)\right) + \left(P\,(H) \times P\,\left(\frac{R}{H}\right)\right)$

= (0.4×0.6) + (0.1×0.8) + (0.5×1)
= 0.82

b. Given that the trainee receives the messages, find the conditional probabilities that he receives it via email

$P\,\left(\frac{E}{R}\right) = \frac{P(E) \times P(\frac{R}{E})}{P(T)}$

$= \frac{(0.4)(0.6)}{0.82}$

$= 0.293$

11. In recent News, it was reported that light trucks, which include SUV's, pick-up trucks and minivans, accounted for 40% of all personal vehicles on the road in 2012. Assume the rest are cars. Of every 100,000 cars accidents, 20 involve a fatality; of every 100,000 light trucks accidents, 25 involve a fatality. If a fatal accident is chosen at random, what is the probability the accident involved a light truck?

L = light truck
L' = cars
F = fatal accident
F' = not fatal accident

$P\ (L) = 0.4$
$P\ (L') = 0.6$
$P\ \left(\frac{F}{L}\right) = \frac{25}{100000} = 0.00025$
$P\ \left(\frac{F}{L'}\right) = \frac{20}{100000} = 0.0002$
$P\ \left(\frac{L}{F}\right) = \frac{P(\frac{F}{L})P(L)}{P(\frac{F}{L})P(L) + P\left(\frac{F}{L'}\right)P(L')}$
$= \frac{(0.00025)(0.4)}{(0.00025)(0.4) + (0.0002)(0.6)}$
$= 0.455$

12. There are 9 letters having different colors (red, orange, yellow, green, blue, indigo, velvet) and 4 boxes of different shapes (tetrahedron, cube, polyhedron, dodecahedron). How many ways are there to place these 9 letters into the 4 boxes such that each box contains at least 1 letter?

Total letters = 9
Total boxes = 4
Possible ways to put letters in boxes = $4^9 = 262144$

Based on inclusion-exclusion, there is 4 choices for which boxes get chosen and 2 choices per letter if the letters are put into two boxes,
$= 4 \times 3^9 = 78732$

Ways to put all letters into two box = $4 \times 2^9 = 2048$

Ways to put all letters into one box = $4 \times 1^9 = 4$ ways

Therefore, ways are there to place these 9 letters into the 4 boxes such that each box contain at least 1 letter is

262144 – 78732 + 2048+ 4 = 183416 ways