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Assignment 3

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Tutorial 3

1. $A = \{1, 2, 3, 4, 5, 6, 7, 8\}$
$B = \{2, 5, 9\}$
$C = \{a, b\}$

(a) i) $A - B = \{1, 3, 4, 6, 7, 8\}$
ii) $(A \cap B) \cup C = \{2, 5, a, b\}$
iii) $A \cap B \cap C = \{\emptyset\}$
iv) $B \times C = \{(2,a), (2,b), (5,a), (5,b), (9,a), (9,b)\}$
v) $P(C) = \{\{a\}, \{b\}, \{a,b\}, \emptyset\}$

(b) $(P \cap ((P' \cup Q)')) \cup (P \cap Q)$
$= (P \cap (P'' \cap Q')) \cup (P \cap Q)$ - De morgan's law
$= (P \cap (P \cap Q')) \cup (P \cap Q)$ - Double complement law
$= ((P \cap P) \cap Q') \cup (P \cap Q)$ - Associative law
$= (P \cap Q') \cup (P \cap Q)$ - Idempotent law
$= P \cap (Q' \cup Q)$ - Distributive law
$= P \cap U$ - Complement law
$= P$ (shown) - Properties of universal set

(c) $A = (\neg p \vee q) \leftrightarrow (q \rightarrow p)$

p	q	$\neg p \vee q$	$q \rightarrow p$	A
T	T	T	T	T
T	F	F	T	F
F	T	T	F	F
F	F	T	T	T

(d) "For all integer x, if x is odd, then $(x+2)^2$ is odd"

Direct proof: if x is odd, $x = 2k+1$

$$
\begin{aligned}
(x+2)^2 &= x^2 + 4x + 4 \\
&= (2k+1)^2 + 4(2k+1) + 4 \\
&= 4k^2 + 4k + 1 + 8k + 4 + 4 \\
&= 4k^2 + 12k + 8 + 1 \\
&= 2(2k^2 + 6k + 4) + 1 \\
&= 2m + 1 \text{ (odd)}
\end{aligned}
$$

(e) $P(x, y)$: $x \geq y$

i) $\exists x \exists y\, P(x,y)$ = True

Let $x = 3$, $y = 2$

$x \geq y$

$\therefore 3 \geq 2$

ii) $\forall x \forall y\, P(x, y)$ = False

Let $x = 2$, $y = 3$

$x \geq y$

$\therefore 2 < 3$

2.

$$M_R = \begin{array}{c c} & \begin{array}{ccc} 1 & 2 & 3 \end{array} \\ \begin{array}{c} 1 \\ 2 \\ 3 \end{array} & \begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{bmatrix} \end{array}$$

$R = \{ (1,1), (1,2), (2,2), (3,1) \}$

(a) i) Domain $= \{1, 2, 3\}$

Range $= \{1, 2\}$

ii) R is antisymmetric.

R is not irreflexive because the diagonal of matrices R is not all '0'.

R is antisymmetric because $(x, y) \in R$ and $(y, x) \notin R$ when $x \neq y$. For example, $(1,2)$ exist but $(2,1)$ does not exist.

(b) $S = \{ (x,y) \mid x + y \geq 9 \}$; $X = \{2, 3, 4, 5\}$

i) $S = \{ (4,5), (5,4), (5,5) \}$

ii) S is not reflexive because there exist at least one $(x, x) \notin R$ such as $(4,4)$

S is symmetric because $(4,5)$ and $(5,4)$ both exist.

S is not transitive because Matrics of S, $M_S \otimes M_S \neq M_S$

S is not equivalance because S is not all reflexive symmetric and transitive.

$$M_S = \begin{array}{c c} & \begin{array}{cccc} 2 & 3 & 4 & 5 \end{array} \\ \begin{array}{c} 2 \\ 3 \\ 4 \\ 5 \end{array} & \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 1 \end{bmatrix} \end{array} \otimes \begin{array}{c c} & \begin{array}{cccc} 2 & 3 & 4 & 5 \end{array} \\ \begin{array}{c} 2 \\ 3 \\ 4 \\ 5 \end{array} & \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 1 \end{bmatrix} \end{array} = \begin{array}{c c} & \begin{array}{cccc} 2 & 3 & 4 & 5 \end{array} \\ \begin{array}{c} 2 \\ 3 \\ 4 \\ 5 \end{array} & \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 1 & 1 \end{bmatrix} \end{array}$$

(c) $X = \{1,2,3\}$, $Y = \{1,2,3,4\}$, $Z = \{1,2\}$

(i) $f(x) = x$

$f = \{(1,1), (2,2), (3,3)\}$

(ii) $g(1) = 1$, $g(2) = 2$, $g(3) = 1$

$g = \{(1,1), (2,2), (3,1)\}$

(iii) $h(x) = 1$

$h = \{(1,1), (2,1), (3,1)\}$

(d) $m(x) = 4x + 3$

$n(x) = 2x - 4$

i) Let $y = m(x)$

$$y = 4x + 3$$

$$4x = y - 3$$

$$x = \frac{y-3}{4}$$

$$m(y) = \frac{y-3}{4}$$

$$\therefore m^{-1}(x) = \frac{x-3}{4}$$

ii) $n \circ m(x) = n(4x + 3)$

$$= 2(4x + 3) - 4$$

$$= 8x + 6 - 4$$

$$= 8x + 2$$

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Question 3

a) $a_k = a_{k-1} + 2k$

i) $a_2 = a_1 + 2(2)$
$= 1 + 4$
$= 5$

$a_3 = a_2 + 2(3)$
$= 5 + 6$
$= 11$

$a_4 = a_3 + 2(4)$
$= 11 + 8$
$= 19$

ii) Input : n , integer ≥ 2

```
f(n)
{
    if (n=1)
        return 1
    return f(n-1)+2n
}
```

b) $r_1 = 7$
$r_2 = 14$
$r_3 = 28$
$r_k = 2r_{k-1}$, $k > 1$

c) S(1) = return 5
S(2) = return 5×5
= 25
S(3) = return 5×25
= 125
S(4) = return 5×125
= 625

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Question 4

a) $\underline{\ }\ \underline{\ }\ \underline{\ }\ \underline{\ }$
$9 \times 16 \times 16 \times 11$

Number of hexadecimal numbers $= 9 \times 16 \times 16 \times 11$
$= 25344$

b) $\underline{A}\ \underline{\ }\ \underline{\ }\ \underline{\ }\ \underline{\ }\ \underline{\ }\ \underline{0}$
$1 \times 26 \times 26 \times 26 \times 10 \times 10 \times 1$

Number of license plates $= 1 \times 26 \times 26 \times 26 \times 10 \times 10 \times 1$
$= 1757600$

c) $\underline{\ }\ \underline{\ }\ \underline{\ }$
$8 \times 7 \times 6$

Pick 3 letters $= {}^{8}C_{3}$
$= 56$

Pick 2 letters $= {}^{8}C_{2}$
$= 28$

Arrange 3 letters $= {}^{3}P_{3}$
$= 6$

Arrange 2 letters $= {}^{2}P_{2}$
$= 2$

Total arrangements $= (56 \times 6) + (28 \times 2) + 1$
$= 393$

d) Choose four women $= {}^{7}C_{4}$
$= 35$

Choose three men $= {}^{6}C_{3}$
$= 20$

Number of possible groups $= 35 \times 20$
$= 700$

e) Arrange 11 letters $= {}^{11}P_{11}$
$= 39916800$

Similar letters $(2B, 2I) = 2! \times 2!$
$= 4$

Number of distinguishable arrangements $= \frac{39916800}{4}$
$= 9979200$

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Question 4 (continued)

f) $r=10$

$n=6$

$C(n+r-1, r) = \frac{(n+r-1)!}{r!(n-1)!}$

$= \frac{15!}{10!(5)!}$

$= 3003$ selections

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Question 5

a) A = Ali D = Daud
B = Bahar E = Elyas
C = Carlie

{A, D}, {A, E}, {B, D}, {B, E}, {C, D}, {C, E}

$\lceil \frac{18}{6} \rceil = 3$

b) $1 \rightarrow 20 = 20$ integers

10 odd, 10 even

Guaranteed odd = 10 even + 1 odd
= 11

∴ Pick 11 integers at least one odd

c) $1 \rightarrow 100 = 100$ integers

5, 10, 15, 20, 25, 30, 35, 40, 45, 50
55, 60, 65, 70, 75, 80, 85, 90, 95, 100

Integers divisible by 5 = 20

Integers not divisible by 5 = 100 - 20
= 80

Guaranteed divisible by 5 = 80 + 1
= 81

∴ Pick 81 integers be sure of getting one that is divirible by 5