SECI Assignment 1

Date. No.

1. Let $A = \{x \in R \mid 0 < x \leq 2\} = \{1, 2\}$

$B = \{x \in R \mid 1 \leq x < 4\} = \{1, 2, 3\}$

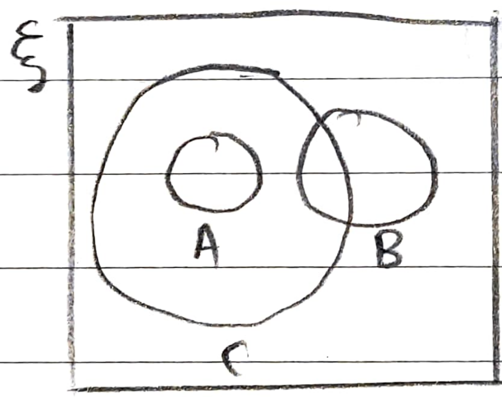
$C = \{x \in R \mid 3 \leq x < 9\} = \{3, \ldots, 8\}$

(a) $A \cup C = \{1, 2, 3, 4, 5, 6, 7, 8\}$

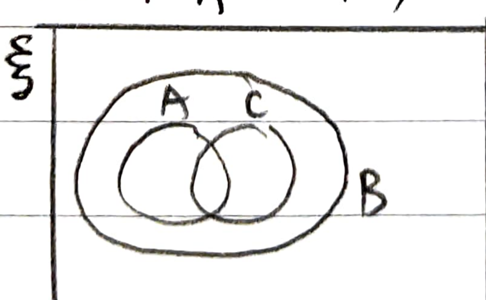
(b) $(A \cup B)' = \{4, 5, 6, 7, 8\}$

(c) $A' \cup B' = \{3, 4, 5, 6, 7, 8\}$

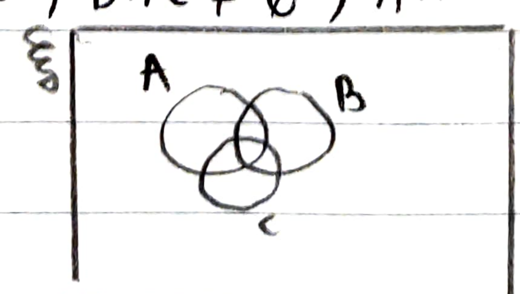
2. (a) $A \cap B = \emptyset$, $A \subseteq C$, $C \cap B \neq \emptyset$



(b) $A \subseteq B$, $C \subseteq B$, $A \cap C \neq \emptyset$



(c) $A \cap B \neq \emptyset$, $B \cap C \neq \emptyset$, $A \cap C = \emptyset$, $A \not\subset B$, $C \not\subset B$



3. $S \cap T = \{(x,y) \in A \times B \mid (x,y) \in S \text{ and } (x,y) \in T\}$

$S \cup T = \{(x,y) \in A \times B \mid (x,y) \in S \text{ OR } (x,y) \in T\}$

Let $A = \{-1, 1, 2, 4\}$, $B = \{1, 2\}$

For all $(x,y) \in A \times B$, $x\,S\,y \leftrightarrow |x| = |y|$

For all $(x,y) \in A \times B$, $x\,T\,y \leftrightarrow x - y$ is even

$A \times B = \{(-1,1), (-1,2), (1,1), (1,2), (2,1), (2,2),$
$(4,1), (4,2)\}$

$S = \{(-1,1), (1,1), (2,2)\}$

$T = \{(-1,1), (1,1), (2,2), (4,2)\}$

$S \cap T = \{(-1,1), (1,1), (2,2)\}$

$S \cup T = \{(-1,1), (1,1), (2,2), (4,2)\}$

4. Show that $\neg((\neg p \wedge q) \vee (\neg p \wedge \neg q)) \vee (p \wedge q) \equiv p$

$\neg((\neg p \wedge q) \vee (\neg p \wedge \neg q)) \vee (p \wedge q)$

$= \neg(\neg p \wedge q) \wedge \neg(\neg p \wedge \neg q) \vee (p \wedge q)$ } De morgan's Law

$= (\neg(\neg p) \vee \neg q) \wedge (\neg(\neg p) \vee \neg(\neg q)) \vee (p \wedge q)$

$= (p \vee \neg q) \wedge (p \vee q) \vee (p \wedge q)$ } Double Complement law

$= p \vee (\neg q \wedge q) \vee (p \wedge q)$ } Distributive Law

$= p \vee (p \wedge q)$ } Complement Law

$= p$

5. $R_1 = \{(x,y) \mid x+y \leq 6\}$ is from X to Y

$R_2 = \{(y,z) \mid y > z\}$ is from Y to Z

Ordering of X, Y, Z : 1, 2, 3, 4, 5

(a)

$$\text{Matrix } A_1 = x \begin{array}{c} \\ 1 \\ 2 \\ 3 \\ 4 \\ 5 \end{array} \begin{array}{c} \begin{array}{ccccc} & & y & & \\ 1 & 2 & 3 & 4 & 5 \end{array} \\ \begin{bmatrix} 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 0 \\ 1 & 1 & 1 & 0 & 0 \\ 1 & 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 \end{bmatrix} \end{array}$$

(b)

$$\text{Matrix } A_2 = y \begin{array}{c} \\ 1 \\ 2 \\ 3 \\ 4 \\ 5 \end{array} \begin{array}{c} \begin{array}{ccccc} & & z & & \\ 1 & 2 & 3 & 4 & 5 \end{array} \\ \begin{bmatrix} 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 \\ 1 & 1 & 0 & 0 & 0 \\ 1 & 1 & 1 & 0 & 0 \\ 1 & 1 & 1 & 1 & 0 \end{bmatrix} \end{array}$$

(c) R_1 : Not reflexive because we do not have (4,4) and (5,5)

Symmetric because $M_{A_1} = M_{A_1}^T$

Not transitive because $M_{A_1} \otimes M_{A_1} \neq M_{A_1}$

$\therefore$ R_1 is not an ~~equivalenfce~~ equivalence relation

(d) R_2 : Irreflexive because the main diagonal are all '0'

Asymmetric because it is irreflexive and antisymmetric

Not transitive because $M_{A_2} \otimes M_{A_2} \neq M_{A_2}$

$\therefore$ R_2 is not a partial order relation.

6.

$$M_{R_1} = \begin{array}{cc} & \begin{matrix} 1 & 2 & 3 \end{matrix} \\ \begin{matrix} 1 \\ 2 \\ 3 \end{matrix} & \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 1 \\ 1 & 0 & 1 \end{bmatrix} \end{array} \qquad M_{R_2} = \begin{array}{cc} & \begin{matrix} 1 & 2 & 3 \end{matrix} \\ \begin{matrix} 1 \\ 2 \\ 3 \end{matrix} & \begin{bmatrix} 0 & 1 & 0 \\ 0 & 1 & 0 \\ 1 & 0 & 1 \end{bmatrix} \end{array}$$

$R_1 = \{(1,1), (2,2), (2,3), (3,1), (3,3)\}$

$R_2 = \{(1,2), (2,2), (3,1), (3,3)\}$

(a) $R_1 \cup R_2 = \{(1,1), (1,2), (2,2), (2,3), (3,1), (3,3)\}$

$$M_{R_1 \cup R_2} = \begin{array}{cc} & \begin{matrix} 1 & 2 & 3 \end{matrix} \\ \begin{matrix} 1 \\ 2 \\ 3 \end{matrix} & \begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & 1 \\ 1 & 0 & 1 \end{bmatrix} \end{array}$$

(b) $R_1 \cap R_2 = \{(2,2), (3,1), (3,3)\}$

$$M_{R_1 \cap R_2} = \begin{array}{cc} & \begin{matrix} 1 & 2 & 3 \end{matrix} \\ \begin{matrix} 1 \\ 2 \\ 3 \end{matrix} & \begin{bmatrix} 0 & 0 & 0 \\ 0 & 1 & 0 \\ 1 & 0 & 1 \end{bmatrix} \end{array}$$

7. If $f:R \to R$ and $g:R \to R$ are both one-to-one,

and $f:R \to R$ and $g:R \to R$ are functions,

$(f+g):R \to R \Rightarrow (f+g)(x) = f(x) + g(x)$

Let $f(x_1) = f(x_2)$, $\quad (f+g)(x_1) = f(x_1) + g(x_1)$

$g(x_1) = g(x_2)$ $\quad = f(x_2) + g(x_2)$

$= (f+g)(x_2)$

$\therefore$ $f+g$ is also one-to-one. #

8. n = number of stairs

C_n = total number of ways to climb

when $n=1$, $C_1 = 1$

$n=2$, $C_2 = 2$ (go by 2 one-stair or 1 two-stair)

The last step of stair taken is either single stair or two.

- The number of ways to climb the staircase and have the final step be a single stair is C_{n-1}
- " " two stairs is C_{n-2}

$\therefore$ $C_n = C_{n-1} + C_{n-2}$, $n \geq 3$

providing $C_1 = 1$, $C_2 = 2$

9. $t_1 = t_2 = t_3 = 1$

Tribonacci sequence, $t_n = t_{n-1} + t_{n-2} + t_{n-3}, n \geq 4$

(a) $t_7 = t_6 + t_5 + t_4$

$= (t_5 + t_4 + t_3) + (t_4 + t_3 + t_2) + (t_3 + t_2 + t_1)$

$= ((t_4 + t_3 + t_2) + t_4 + t_3) + (t_4 + t_3 + t_2) + (t_3 + t_2 + t_1)$

$= ((3+1+1) + 3 + 1) + (3+1+1) + (1+1+1)$

$= 17$ *

OR

$t_4 = t_3 + t_2 + t_1 = 3$

$t_5 = t_4 + t_3 + t_2 = 5$

$t_6 = t_5 + t_4 + t_3 = 9$

$t_7 = t_6 + t_5 + t_4 = 9 + 5 + 3$

$= 17$

(b) $t_n = t_{n-1} + t_{n-2} + t_{n-3}, \quad n \geq 1$

Input : n Output : f(n)

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f(n)
{   if (n=1 or n=2 or n=3)
        return 1
    return f(n-1) + f(n-2) + f(n-3)
}
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