

Bubble Sort [10 7 18 16 5 3 11 15 9 23 28 33]

[11]

33	33	33	33	33	33	33	33	33	33	33	33
28	28	28	28	28	28	28	28	28	28	28	28
23	23	23	23	23	23	23	23	23	23	23	23
9	9	9	9	9	9	9	9	9	9	9	9
15	15	15	15	15	15	15	15	15	15	15	15
11	11	11	11	11	11	11	11	11	11	11	11
3	3	3	3	3	3	3	3	3	3	3	3
5	5	5	5	5	5	5	5	5	5	5	5
16	16	16	16	16	16	16	16	16	16	16	16
18	18	18	18	18	18	18	18	18	18	18	18
7	7	7	7	7	7	7	7	7	7	7	7
10	10	10	10	10	10	10	10	10	10	10	10

[0]

Pass 1 : comparison $\Rightarrow 12-1=11$

[11]

33	33	33	33	33	33	33	33	33	33	33	33
28	28	28	28	28	28	28	28	28	28	28	28
23	23	23	23	23	23	23	23	23	23	23	23
18	18	18	18	18	18	18	18	18	18	18	18
9	9	9	9	9	9	9	9	9	9	9	9
15	15	15	15	15	15	15	15	15	15	15	15
11	11	11	11	11	11	11	11	11	11	11	11
3	3	3	3	3	3	3	3	3	3	3	3
5	5	5	5	5	5	5	5	5	5	5	5
16	16	16	16	16	16	16	16	16	16	16	16
10	10	10	10	10	10	10	10	10	10	10	10
7	7	7	7	7	7	7	7	7	7	7	7

[0]

Pass 2 : comparison $\Rightarrow 12-2=10$

[11]	33	33	33	33	33	33	33	33	33	33
	28	28	28	28	28	28	28	28	28	28
	23	23	23	23	23	23	23	23	23	23
	18	18	18	18	18	18	18	18	18	18
	16	16	16	16	16	16	16	16	16	16
	9	9	9	9	9	9	15	15	15	15
	15	15	15	15	15	15	9	9	9	9
	11	11	11	11	11	11	11	11	11	11
	3	3	3	3	3	3	10	10	10	10
	5	5	5	5	5	5	3	3	3	3
	10	10	10	10	10	10	5	5	5	5
[9]	7	7	7	7	7	7	7	7	7	7

Pass 3: comparison $\Rightarrow 12-3 = 9$

[11]	33	33	33	33	33	33	33	33	33
	28	28	28	28	28	28	28	28	28
	23	23	23	23	23	23	23	23	23
	18	18	18	18	18	18	18	18	18
	16	16	16	16	16	16	16	16	16
	15	15	15	15	15	15	15	15	15
	9	9	9	9	9	11	11	11	11
	11	11	11	11	11	9	9	9	9
	10	10	10	10	10	10	10	10	10
	3	3	3	3	3	7	7	7	7
	5	5	5	5	5	5	5	5	5
[9]	7	7	7	7	7	7	7	7	7

Pass 4: comparison $\Rightarrow 12-4 = 8$

[11]	33	33	33	33	33	33	33	33
	28	28	28	28	28	28	28	28
	23	23	23	23	23	23	23	23
	18	18	18	18	18	18	18	18
	16	16	16	16	16	16	16	16
	15	15	15	15	15	15	15	15
	11	11	11	11	11	11	11	11
	9	9	9	9	10	10	10	10
	10	10	10	10	9	9	9	9
	7	7	7	7	7	7	7	7
	3	3	3	3	3	3	3	3
[9]	5	5	5	5	5	5	5	5

Pass 5: comparison $\Rightarrow 12-5 = 7$

33	33	33	33	33	33	33
28	28	28	28	28	28	28
23	23	23	23	23	23	23
18	18	18	18	18	18	18
16	16	16	16	16	16	16
15	15	15	15	15	15	15
11	11	11	11	11	11	11
10	10	10	10	10	10	10
9	9	9	9	9	9	9
7	7	7	7	7	7	7
5	5	5	5	5	5	5
3	3	3	3	3	3	3

Pass 6 = comparison $\Rightarrow 12-6=6$

33	33	33	33	33	33
28	28	28	28	28	28
23	23	23	23	23	23
18	18	18	18	18	18
16	16	16	16	16	16
15	15	15	15	15	15
11	11	11	11	11	11
10	10	10	10	10	10
9	9	9	9	9	9
7	7	7	7	7	7
5	5	5	5	5	5
3	3	3	3	3	3

Pass 7: comparison $\Rightarrow 12-7=5$

33	33	33	33	33
28	28	28	28	28
23	23	23	23	23
18	18	18	18	18
16	16	16	16	16
15	15	15	15	15
11	11	11	11	11
10	10	10	10	10
9	9	9	9	9
7	7	7	7	7
5	5	5	5	5
3	3	3	3	3

Pass 8: comparison $\Rightarrow 12-8=4$

33	33	33	33
28	28	28	28
23	23	23	23
18	18	18	18
16	16	16	16
15	15	15	15
11	11	11	11
10	10	10	10
9	9	9	9
7	7	7	7
5	5	5	5
3	3	3	3

Pass 9 = comparison
 $= 12-9=3$

33	33	33	33
28	28	28	28
23	23	23	23
18	18	18	18
16	16	16	16
15	15	15	15
11	11	11	11
10	10	10	10
9	9	9	9
7	7	7	7
5	5	5	5
3	3	3	3

Pass 10: comparison
 $= 12-10=2$

33	33
28	28
23	23
18	18
16	16
15	15
11	11
10	10
9	9
7	7
5	5
3	3

Pass 11: comparison $\Rightarrow 12-11=1$

Index	11	10	9	8	7	6	5	4	3	2	1
[11]	33	33	33	33	33	33	33	33	33	33	33
[10]	28	28	28	28	28	28	28	28	28	28	28
[9]	23	23	23	23	23	23	23	23	23	23	23
[8]	9	9	9	9	18	18	18	18	18	18	18
[7]	15	15	15	15	15	16	16	16	16	16	16
[6]	11	11	11	11	11	11	15	15	15	15	15
[5]	3	3	3	3	3	3	3	11	11	11	11
[4]	5	5	5	5	5	5	5	5	10	10	10
[3]	16	16	16	16	16	15	11	3	3	9	9
[2]	18	18	18	18	9	9	9	9	3	3	7
[1]	7	7	7	7	7	7	7	7	7	3	5
[0]	10	10	10	10	10	10	10	10	5	5	3
largest	11	10	9	2	3	3	3	0	2	1	0

largest:
index

Insertion Sort

[11]	33	33	33
[10]	28	28	28
[9]	23	23	23
[8]	9	9	9
[7]	15	15	15
[6]	11	11	11
[5]	3	3	3
[4]	5	5	5
[3]	16	16	16
[2]	18	18	18
[1]	7	10	10
[0]	10	18	7

Pass 1
insert
index (1)
item 7

[11]	33	33
[10]	28	28
[9]	23	23
[8]	9	9
[7]	15	15
[6]	11	11
[5]	3	3
[4]	5	5
[3]	16	16
[2]	18	18
[1]	10	10
[0]	7	7

Pass 2
insert
index (2)
item 18

[11]	33	33
[10]	28	28
[9]	23	23
[8]	9	9
[7]	15	15
[6]	11	11
[5]	3	3
[4]	5	5
[3]	16	18
[2]	18	18
[1]	10	10
[0]	7	7

Pass 3
insert
index (3)
item 16

[11]	33	33	33	33
[10]	28	28	28	28
[9]	23	23	23	23
[8]	9	9	9	9
[7]	15	15	15	15
[6]	11	11	11	11
[5]	3	3	3	3
[4]	5	18	18	18
[3]	18	18	16	16
[2]	16	16	16	10
[1]	10	10	10	7
[0]	7	7	7	7

Pass 4
insert
index (4)
item 5

Insertion Sort (continued)

[11]	33	33	33	33	33	33	33
[10]	28	28	28	28	28	28	28
[9]	23	23	23	23	23	23	23
[8]	9	9	9	9	9	9	9
[7]	15	15	15	15	15	15	15
[6]	11	11	11	11	11	11	11
[5]	3	18	18	18	18	18	18
[4]	18	18	16	16	16	16	16
[3]	16	16	16	10	10	10	10
[2]	10	10	10	10	7	7	7
[1]	7	7	7	7	7	5	5
[0]	5	5	5	5	5	5	3

Pass 5

insert

index (5)

item 3

33	33	33	33
28	28	28	28
23	23	23	23
9	9	9	9
15	15	15	15
11	18	18	18
18	18	16	16
16	16	16	11
10	10	10	10
7	7	7	7
5	5	5	5
3	3	3	3

Pass 6

insert

index (6)

item 11

33	33	33	33
28	28	28	28
23	23	23	23
9	9	9	9
15	18	18	18
18	18	16	16
16	16	16	15
11	11	11	11
10	10	10	10
7	7	7	7
5	5	5	5
3	3	3	3

Pass 7

insert

index (7)

item 15

Insertion sort (Continued)

[11]	33	33	33	33	33	33	33
[10]	28	28	28	28	28	28	28
[9]	23	23	23	23	23	23	23
[8]	9	18	18	18	18	18	18
[7]	18	18	16	16	16	16	16
[6]	16	16	16	15	15	15	15
[5]	15	15	15	15	11	11	11
[4]	11	11	11	11	11	10	10
[3]	10	10	10	10	10	10	9
[2]	7	7	7	7	7	7	7
[1]	5	5	5	5	5	5	5
[0]	3	3	3	3	3	3	3

Pass 8

insert
index (8)

item 9

[11]	33	33
[10]	28	28
[9]	23	23
[8]	18	18
[7]	16	16
[6]	15	15
[5]	11	11
[4]	10	10
[3]	9	9
[2]	7	7
[1]	5	5
[0]	3	3

Pass 9

insert
index (9)
item 23

[11]	33	33
[10]	28	28
[9]	23	23
[8]	18	18
[7]	16	16
[6]	15	15
[5]	11	11
[4]	10	10
[3]	9	9
[2]	7	7
[1]	5	5
[0]	3	3

Pass 10

insert
index (10)
item 28

[11]	33	33
[10]	28	28
[9]	23	23
[8]	18	18
[7]	16	16
[6]	15	15
[5]	11	11
[4]	10	10
[3]	9	9
[2]	7	7
[1]	5	5
[0]	3	3

Pass 11

insert
index (11)
item 33

Quick Sort.

[0]	[1]	[2]	[3]	[4]	[5]	[6]	[7]	[8]	[9]	[10]	[11]
10	7	18	16	5	3	11	15	9	23	28	33

Quicksort($T, 0, 11$)

PIVOT

Partition($T, 0, 11$) = 4

[0]	[1]	[2]	[3]	[4]	[5]	[6]	[7]	[8]	[9]	[10]	[11]
7	5	3	9	10	18	16	11	15	23	28	33

Quicksort($T, 0, 4$)

(< pivot)

(> pivot)

Quicksort($T, 5, 11$)

[0]	[1]	[2]	[3]	[4]
7	5	3	9	10

PIVOT

[5]	[6]	[7]	[8]	[9]	[10]	[11]
18	16	11	15	23	28	33

PIVOT

Partition($T, 5, 11$) = 8

Quicksort($T, 0, 2$)

[0]	[1]	[2]	[3]	[4]
5	3	7	9	10

Quicksort($T, 3, 4$)

[5]	[6]	[7]	[8]	[9]	[10]	[11]
16	11	15	18	23	28	33

[5]	[6]	[7]	[8]
16	11	15	18

PIVOT

Partition($T, 5, 8$) = 7

[9]	[10]	[11]
23	28	33

PIVOT

[9]	[10]	[11]
23	28	33

PIVOT

Partition($T, 0, 2$) = 1

PIVOT

Partition($T, 3, 4$) = 3

Quicksort($T, 0, 1$)

[0]	[1]	[2]
3	5	7

[3]	[4]
9	10

Quicksort($T, 1, 3$)

Quicksort($T, 4, 4$)

[0]	[1]
3	5

Quicksort($T, 2, 2$)

Quicksort($T, 3, 3$)

Quicksort($T, 4, 4$)

[5]	[6]	[7]
11	15	16

PIVOT

[5]	[6]	[7]
11	15	16

[5]

[6]

PIVOT

[5]	[6]	[7]
15	16	

[5]

[6]

[6]

[7]

[5]

[6]

[9]

[10]

[9]

[10]

[9]

[10]

[9]

[10]

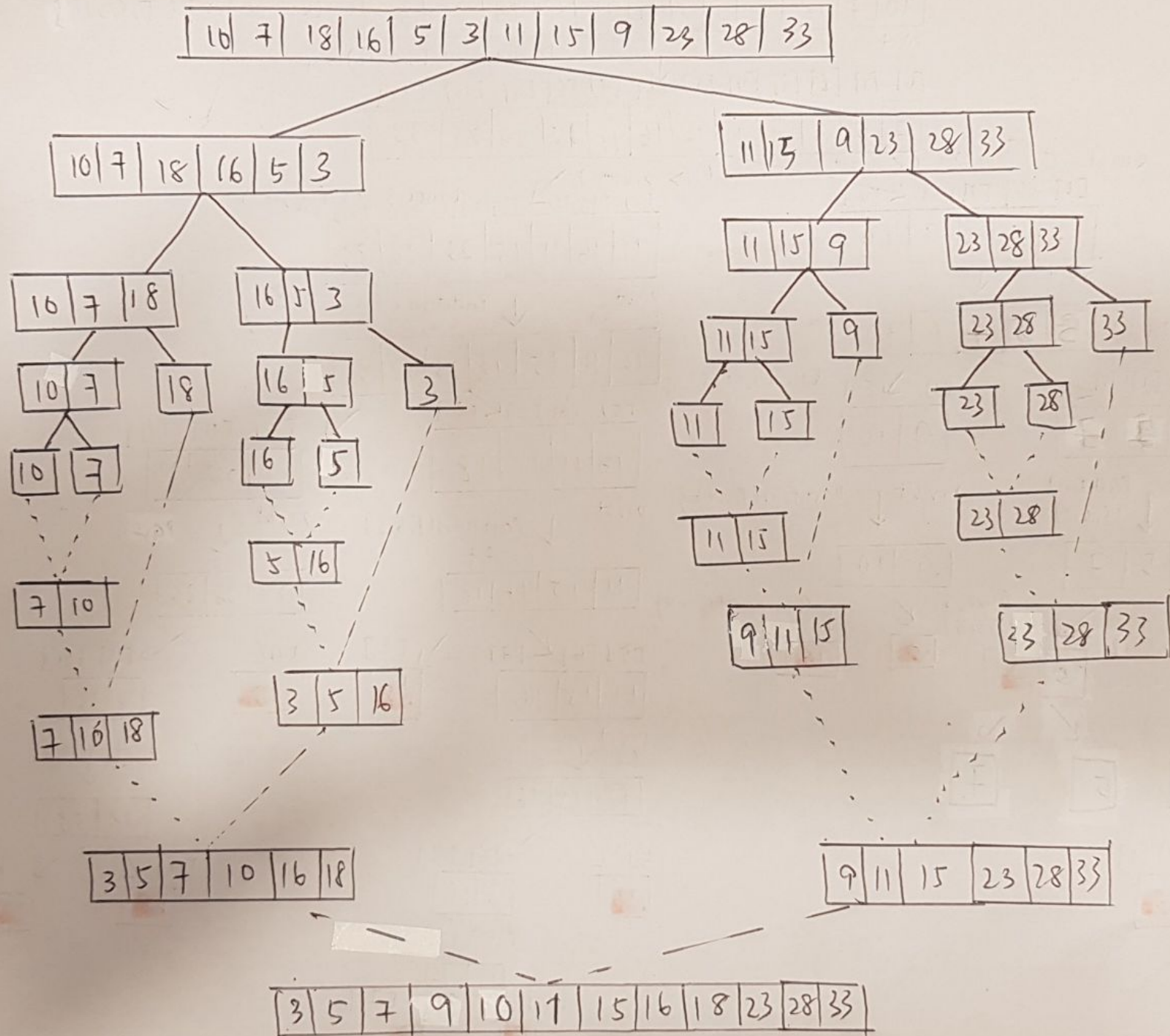
[9]

[10]

[9]

[10]

Merge sort



Efficiency of Sorting Technique.

1. Bubble Sort.

- To determine the efficiency of bubble sort algorithm, we can use the number of comparison & number of exchange.
- The number of comparison is same.
- Base on the question, the number of comparison such that $(12-1) + (12-2) + (12-3) + \dots + 2+1 = 11+10+9+\dots+1 = \frac{n(n-1)}{2}$
 $= O(n^2)$
 $= 66$
- Where the number of swaps = 66

2. Selection Sort. and Insertion Sort

- The number of comparison in selection sort such that $(12-1) + (12-2) + (12-3) + \dots + 2+1 = \frac{n(n-1)}{2} = O(n^2) = 66$

3. Quick Sort.

- The efficiency of Quick sort depends on the pivot value, where the first element choiced to be the pivot value.
- This question provide a worse case quick sort as it is imbalance partition for the left and right partition.
- Therefore, the number of comparison = $O(n * \log n)$ where it is significantly faster than $O(n^2)$ algorithm.

4. Merge Sort

- The segment for left and right for this question is equal in size.
- The number of iteration is approximately $n \log n$
- The same number of repetition is needed to sort and merge this question's data
- This type of data is an example of average case merge sort where the number of comparison of this data is $O(n \log n)$