



UTM
UNIVERSITI TEKNOLOGI MALAYSIA

SCHOOL OF COMPUTING
Faculty of Engineering

Discrete Structure

Section - 7

Assignment 4

Done by:

Group = 9

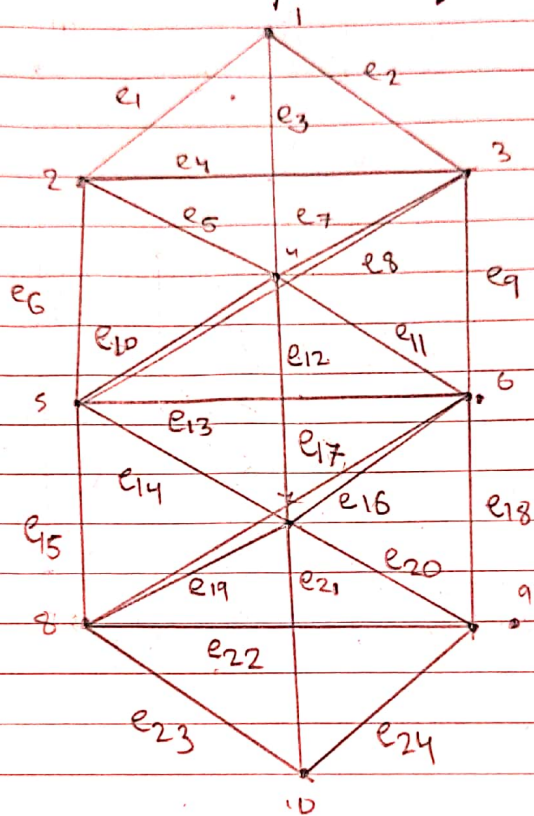
Members:

<u>No.</u>	<u>Name</u>	<u>Matric No.</u>	<u>Questions</u>
1	Muhammad Faiz Aiman Bin Fakhrurrazi	A20EC0209	3,6,9,12,15
2	Savero Fajri Sutino	A19EC3016	2,5,8,11,14
3	Islam Mohammed Ruzhan	A20EC4028	1,4,7,10,13

1) The set of vertices, $V(G) = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$

∴ The graph contains 10 vertices.

The vertices will be adjacent if $|v - w| \leq 3$.



From, the graph, we can say that there are 24 edges.

$$\therefore E(G) = 24$$

Question 2

(a)

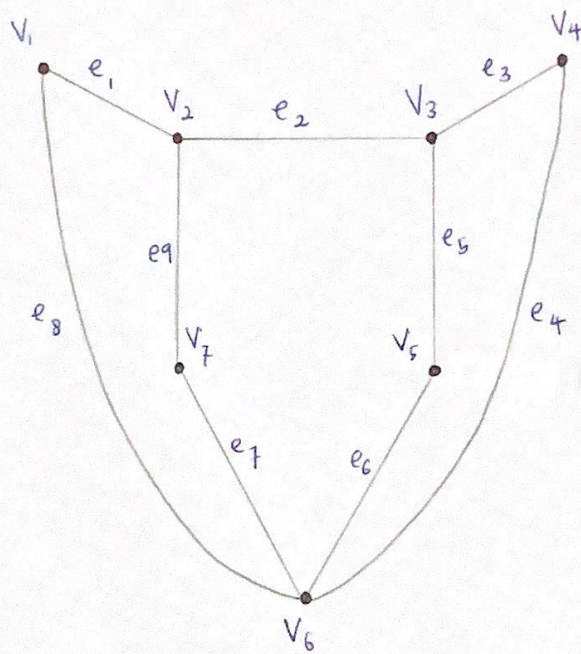
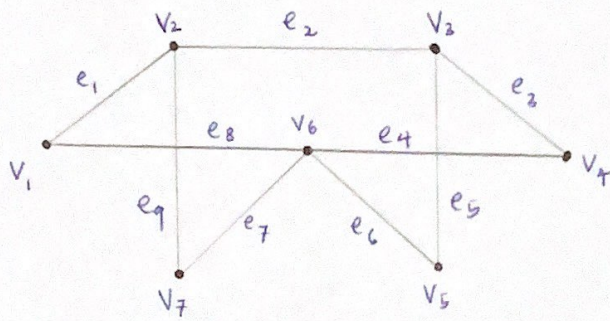
	A	B	C	D	E
A	0	1	1	1	1
B	0	0	0	1	1
C	1	0	0	0	0
D	0	1	1	0	1
E	0	1	1	1	0

all friends of each other

(B)

$$((DM \cup PT)' (A \cup PS)) \cup ((DM \cup A) \cap (PT \cup PS) \cap (A \cup PS)) \\ \cup \\ ((DM \cup PS) \cap (PT \cup PS)) \cap ((A \cup DM) \cap (A \cup PS))$$

Question 3



4) For the graph G ,

$$\text{Vertices} = \{v_1, v_2, v_3, v_4\}$$

$$\text{Edges} = \{e_1, e_2, e_3, e_4, e_5, e_6\}$$

∴ The adjacency matrix =

$$\begin{bmatrix} 1 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{bmatrix}$$

Incidence matrix ∴

$$\begin{bmatrix} 0 & -1 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 1 & 0 & 0 & 0 \\ 0 & 1 & 1 & -1 & -1 & -1 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & -1 \end{bmatrix}$$

For graph H ,

$$\text{Vertices} = \{v_1, v_2, v_3, v_4\}$$

$$\text{Edges} = \{e_1, e_2, e_3, e_4, e_5, e_6\}$$

Adjacency matrix :-

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 1 & 2 \\ 0 & 1 & 0 & 0 \\ 0 & 2 & 0 & 0 \end{bmatrix}$$

Incidence matrix :-

$$\begin{bmatrix} 2 & 0 & 0 & 0 & 0 & 0 \\ 0 & 2 & 1 & 1 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 & 2 \\ 0 & 0 & 1 & 1 & 0 & 0 \end{bmatrix}$$

Question 5.

a) both graph in (a) are isomorphic because they includes the following :

connected and a simple graph

have 5 vertices and edges

have 3 vertices with 2 degree

have 1 vertices with 3 degree

have 1 vertices with 1 degree

b) both graph is not isomorphic because

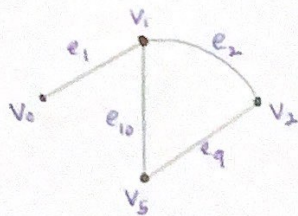
they have the same loop and parallel

have 5 vertices and 6 edges

but they didn't have the same number of degrees and vertices

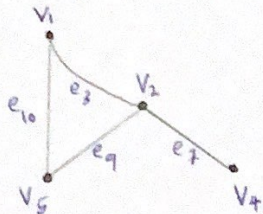
Question 6

a) $V_0 e_1 V_1 e_{10} V_5 e_9 V_2 e_2 V_1$



trail

b) $V_4 e_7 V_2 e_9 V_5 e_{10} V_1 e_3 V_2 e_9 V_5$



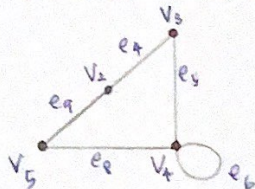
walk

c) V_2



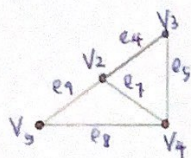
trivial walk

d) $V_5 e_9 V_2 e_4 V_3 e_5 V_4 e_6 V_4 e_8 V_5$



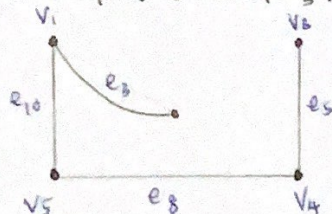
circuit

e) $V_2 e_4 V_3 e_5 V_4 e_8 V_5 e_9 V_2 e_7 V_4 e_5 V_3 e_4 V_2$



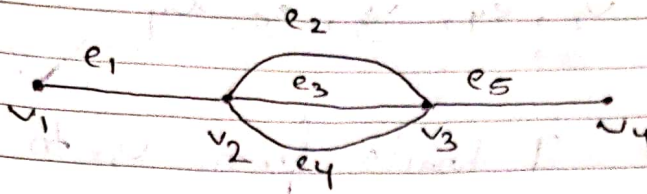
closed walk

f) $V_2 e_5 V_4 e_8 V_5 e_{10} V_1 e_3 V_2$



path

7)



a) The paths from v_1 to v_4 are:

~~v_1, e_1, v_2, e_2~~ i) $v_1, e_1, v_2, e_2, v_3, e_5, v_4$

ii) $v_1, e_1, v_2, e_3, v_3, e_5, v_4$

iii) $v_1, e_1, v_2, e_4, v_3, e_5, v_4$

∴ There are 3 paths from v_1 to v_4

b) The trails from v_1 to v_4 are:

i) $v_1, e_1, v_2, e_4, v_3, e_2, v_2, e_3, v_3, e_5, v_4$

ii) $v_1, e_1, v_2, e_3, v_3, e_2, v_2, e_4, v_3, e_5, v_4$

iii) $v_1, e_1, v_2, e_4, v_3, e_3, v_2, e_2, v_3, e_5, v_4$

iv) $v_1, e_1, v_2, e_3, v_3, e_4, v_2, e_2, v_3, e_5, v_4$

v) $v_1, e_1, v_2, e_2, v_3, e_3, v_2, e_4, v_3, e_5, v_4$

vi) $v_1, e_1, v_2, e_2, v_3, e_4, v_2, e_3, v_3, e_5, v_4$

vii) $v_1, e_1, v_2, e_2, v_3, e_5, v_4$

viii) $v_1, e_1, v_2, e_3, v_3, e_5, v_4$

ix) $v_1, e_1, v_2, e_4, v_3, e_5, v_4$

∴ There are 9 trails from v_1 to v_4 .

c) For walk from v_1 to v_4 , it can be done in many ways including the paths and trails because here, each vertex and edge can be repeated as many times as possible.

Question 8

- Graph (A) is an Euler circuit

because the path travels every edge only once
and it ends in the same vertex

- Graph (B) is not an Euler circuit

because it's not end on the same vertex
without crossing at least an edge more than twice

Question 9

a) possible

$U V_7 V_6 V_1 U V_2 V_3 V_4 V_2 V_6 V_4 W V_6 V_5 W$

b) Not possible

This is because there will be repeated edges.
Based on theorem, every edges should be used exactly once.

10) For graph (a), there is no Hamiltonian Circuit. }

Because there is only one way for going to 'u' to 'v₂' or 'v₂' to 'u.' So the we can never reach to the vertex from where we started.

For graph (b) there is no Hamiltonian Circuit because there is only one way for going from 'u' to 'f' or 'f' to 'u'. So if we start from a vertex, we can never reach the vertex again without using one vertex more than once.

Question 11

3-ary tree = 9

100 vertices = 4

$$\frac{(9-1)n + 1}{4} = \frac{(3-1)100 + 1}{3} = \underline{\underline{67 \text{ leaves}}}$$

Question 12

a) Root = a

b) Internal vertices = a, b, d, e, g, h, j, n

c) Leaves = k, l, m, r, s, f, c, o, i, p, q

d) Children of n = r, s

e) Parent of e = b

f) Siblings of k = l, m

g) Proper ancestors of q = a, d, j

h) Proper descendants of b = e, f, g, k, l, m, n, r, s

13) For Figure 1,

i) Pre order = a, b, e, k, l, m, f, g, n, p, s, c, d, h, o, i, y, p, q.

ii) Post order = k, l, m, e, f, p, s, n, g, b, c, o, h, i, p, q, j, d, a.

iii) In-order = k, e, l, m, b, f, g, p, n, s, a, c, o, h, d, i, p, j, q.

Question 14

$$A-B = 1$$

$$G-H = 1$$

$$B-C = 2$$

$$A-F = 3$$

$$C-D = 4$$

$$B-G = 5$$

$$\underline{C-G = 6}$$

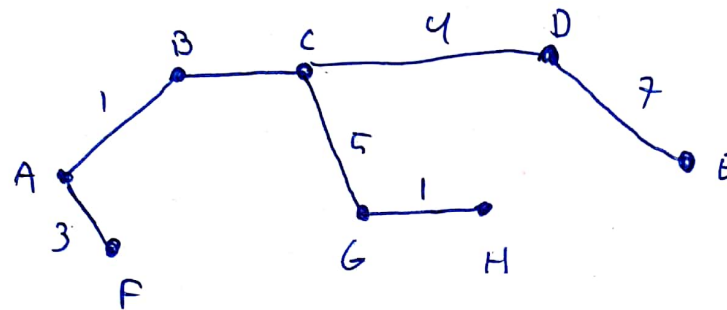
$$\underline{D-H = 6}$$

$$\underline{D-G = 7}$$

$$D-E = 7$$

$$\underline{H-E = 8}$$

f



$$\text{total weight} = \del{24} 23$$

Question 14

$$A-B = 1$$

$$G-H = 1$$

$$B-C = 2$$

$$A-F = 3$$

$$C-D = 4$$

$$B-G = 5$$

$$\underline{C-G = 6}$$

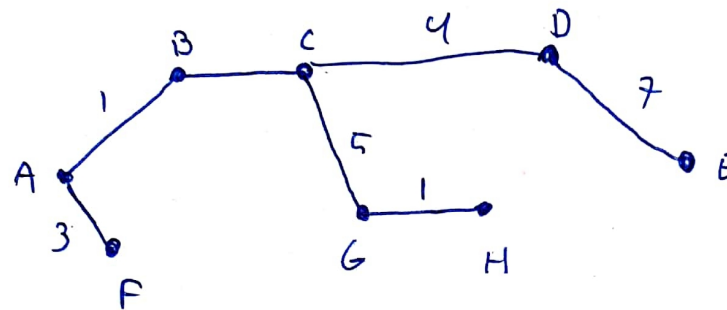
$$\underline{D-H = 6}$$

$$\underline{D-G = 7}$$

$$D-E = 7$$

$$\underline{H-E = 8}$$

f



$$\text{total weight} = 23$$

Question 15

V	M	N	O	P	Q	R	S	T
M	0 _M	4 _M	∞	2 _M	∞	5 _M	∞	∞
P	0 _M	4 _M	∞	2 _M	6 _P	5 _M	5 _P	∞
N	0 _M	4 _M	10 _N	2 _M	6 _P	5 _M	5 _P	∞
R	0 _M	4 _M	7 _R	2 _M	6 _P	5 _M	5 _P	6 _R
S	0 _M	4 _M	7 _R	2 _M	6 _P	5 _M	5 _P	6 _R
Q	0 _M	4 _M	7 _R	2 _M	6 _P	5 _M	5 _P	6 _R
T	0 _M	4 _M	7 _R	2 _M	6 _P	5 _M	5 _P	6 _R
O	0 _M	4 _M	7 _R	2 _M	6 _P	5 _M	5 _P	6 _R

shortest path from M to T is

M to R to T by total weight of 6.