



UTM
UNIVERSITI TEKNOLOGI MALAYSIA

SCHOOL OF COMPUTING

SESSION 2020/2021 SEMESTER 1

SECI 1013 -07

DISCRETE STRUCTURE

ASSIGNMENT 3

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QUESTION 1

a) Given,

$$A = \{1,2,3,4,5,6,7,8\}$$

$$B = \{2,5,9\}$$

$$C = \{a,b\}$$

- i. $A - B = \{1,3,4,6,7,8\}$
- ii. $(A \cap B) = \{2,5\}$
So, $(A \cap B) \cup C = \{2,5\} \cup \{a,b\}$
 $= \{2,5,a,b\}$
- iii. $A \cap B \cap C = \{ \}$
 $A \cap B \cap C$ is empty set.
- iv. $B \times C = \{2,5,9\} \times \{a,b\}$
 $= \{(2,a),(2,b),(5,a),(5,b),(9,a),(9,b)\}$
- v. $P(C) = \text{Power set of } C$
 $= \text{set of all subset of } C$
 $= \{\{a\},\{b\},\{2,b\},\{ \}\}$
 $P(C) = 2^4 - 4 = 4$

b) $(P \cap ((P' \cup Q)')) \cup (P \cap Q) = P$
 $= (P \cap (P' \cup Q')) \cup (P \cap Q)$ [De Morgan's law]
 $= P \cap (P \cup Q') \cup (P \cap Q)$ [Distributive law]
 $= (P \cap P) \cup (P \cap Q') \cup (P \cap Q)$ [Idempotent law]
 $= (P \cap Q') \cup (P \cap Q)$ [Absorption law]
 $= P \cap (Q \cup Q')$ [Complement law]
 $= P$ [Properties of Universal set]
 [Proved]

c) $A = (\neg p \vee q) \leftrightarrow (q \rightarrow p)$

p	q	$\neg p$	$(\neg p \vee q)$	$(q \rightarrow p)$	A
T	T	F	T	T	T
T	F	F	F	T	F
F	T	T	T	F	T
F	F	T	T	T	T

- d)** $P(x) = x$ is an odd integer
 $Q(x) = (x+2)$ is an odd integer
 $\forall x (P(x) \rightarrow Q(x))$
 let a is an odd integers

$$a = 2n + 1$$

$$\Rightarrow a^2 = (2n + 1)^2$$

$$\Rightarrow a^2 = 4n^2 + 4n + 1$$

$$\Rightarrow a^2 = 2(2n^2 + 2n) + 1$$

$$\text{let } m = 2n^2 + 2n$$

$$a^2 = 2m + 1$$

so, a^2 is an odd integers

That's why, for all integer x , if x odd then x^2 is odd

- e)** Ans:
- i. $\exists x \exists y P(x, y)$
 It is true, if any $x \geq y$ or $y \leq x$
 - ii. $\forall x \forall y P(x, y)$
 It is false, if $x < y$ or $y > x$

QUESTION 2

a) $\begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{bmatrix}$

i. Domain = $\{1,2,3\}$ Range= $\{1,2\}$

ii. Answer :

– Not irreflexive because $(2,2) \in R$ or $(1,1) \in R$

– Antisymmetric because $(1,2) \in R$ but $(2,1) \notin R$

b) Let $S = \{(x,y) | x+y \geq 9\}$ is a relation on $X = \{2, 3, 4, 5\}$

i. $S = \{(4,5), (5,4), (5,5)\}$

ii. Answer:

– **Not reflexive** since not all $(4,4) \notin S$

– **Symmetric** since $(4,5) \in S$ and $(5,4) \in S$ or $M_S = M_S^T$

– **Not transitive** because $M_S \otimes M_S \neq M_S$

$$- \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 1 \end{bmatrix} \otimes \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 1 \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 1 & 1 \end{bmatrix}$$

– Not reflexive, symmetric and not transitive, hence not **equivalence relation**

c) Let $X = \{1, 2, 3\}$, $Y = \{1, 2, 3, 4\}$, and $Z = \{1, 2\}$.

i. Define a function $f: X \rightarrow Y$ that is one-to-one but not onto.

Ans: Define f by $f(x) = x$

ii. Define a function $g: X \rightarrow Z$ that is onto but not one-to-one

Ans: Define g by $g(1) = 1, g(2) = 1, g(3) = 2$

iii. Define a function $h: X \rightarrow X$ that is neither one-to-one nor onto.

Ans: Define h by $h(x) = 1$

d) Ans :

i. $m(x) = 4x + 3$

$$y = 4x + 3$$

$$4x = y - 3$$

$$x = \frac{y - 3}{4}$$

$$f^{-1}(x) = \frac{x-3}{4}$$

ii. $m(x) = 4x + 3 \quad n(x) = 2x - 4$

$$(n \circ m)(x) = n(m(x))$$

$$n(m(x)) = n(4x + 3)$$

$$n(m(x)) = 2(4x + 3) - 4$$

$$n(m(x)) = 8x + 6 - 4 = 8x + 2$$

QUESTION 3

a) Ans:

i. Given,

$$a_k = a_{k-1} + 2k, k \geq 2.$$

Also,

$$a_1 = 1,$$

Thus,

$$a_2 = a_1 + 2 \times 2 = (1 + 4) = 5$$

$$a_3 = a_2 + 2 \times 3 = (5 + 6) = 11$$

$$a_4 = a_3 + 2 \times 4 = (11 + 8) = 19$$

Ans : 5, 11, 19

ii. **ALGORITHM**

Input : k

Output : A(k)

A(k){

 if (k=1)

 return 1

 return A(k-1)+2k

}

b) Ans:

r_k = number of executes with an input size k , $r_1=7$

$$r_k = 2r_{k-1}, \quad k > 1, \quad r_1 = 7$$

$$r_2 = 2r_1 = 2^1 r_1$$

$$r_3 = 2r_2 = 2^2 r_1$$

$$r_4 = 2r_3 = 2^3 r_1 \dots$$

$$r_k = 2r_{k-1} = 2^{(k-1)} r_1$$

The recurrence relation for $r_1, r_2, \dots, r_k$ is

$$r_k = 2^{(k-1)} r_1, \text{ when } k > 1, \text{ with } r_1 = 7$$

c) Ans

S(4)

n=4

because $n \neq 1$

return $5 * S(3)$



S(3)

n=3

because $n \neq 1$

return $5 * S(2)$



S(2)

n=2

because $n \neq 1$

return $5 * S(1)$



S(1)

n=1

because $n=1$

return 5

S(4)=**625**

return $5 * 125$

S(3) = 125

return $5 * 25$

S(2)=25

return $5 * 5$

S(1)=5

return 5

ANS: S(4)=**625**

QUESTION 4

a) Ans :

Let the number be WXYZ where W, X, Y, Z are the digits from 0 to F

We can take 9 possible values from 3 to B

Each of X and Y can take all possible 16 values, since no restriction is mentioned.

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Hence, there are $9 \times 16 \times 16 \times 5 = \mathbf{11520}$ such numbers

b) Since first letter and last digit are fixed, hence we only need to fix the 3 letters and 2 digits.

Each of the 3 letters can be fixed in 6 ways

Each of the 3 letters can be fixed in 6 ways

Hence there are $6 \times 6 \times 6 \times 10 \times 10 = \mathbf{21600}$ such number plates

c) This is a permutation problem.

The word COMPUTER has 8 distinct letters. We can have single letter, two letter or three letter words

Single letter words = 8

two letter words = $P(8,2) = 8 \times 7 = 56$

Three letter words = $P(8,3) = 8 \times 7 \times 6 = 336$

Total such words = $8 + 56 + 336 = \mathbf{396}$

d) No of ways to select 4 out of 7 women = $C(7,4) = (7 \times 6 \times 5) / (3!) = 35$

No of ways to select 3 out of 6 men = $C(6,3) = (6 \times 5 \times 4) / (3!) = 20$

To select the teams, we need to select the women and the men. Hence, we multiply these ways.

Total such teams = $35 \times 20 = \mathbf{700}$

e) We identify the repetition of letters in the given word. Out of the total 11 letters, we have

P, R, O, A, L, T, Y - 7 letters that occur once

B, I - 2 letters that occur twice

If there was no repetition, there would have been $11!$ arrangements. But the $2!$ arrangements of the 2 B's and 2 I's does not create any new word. Hence, total possible arrangements are

$$11! / (2! \times 2!) = \mathbf{9979200}$$

- f) Let there be x_1 pastries of first kind, x_2 pastries of second kind, and so on till x_6 pastries of the sixth kind. The number of selections are non-negative integer solutions to the equation

$$x_1 + x_2 + x_3 + x_4 + x_5 + x_6 = 10$$

We know that no of solutions to

$$x_1 + x_2 + \dots + x_k = n$$

are $C(n + k - 1, k - 1)$

Hence, it's no of solutions are $C(10 + 6 - 1, 6 - 1)$

$$= C(15, 5) = \mathbf{3003}$$

QUESTION 5

- a) We are given that 18 persons have first names Ali, Bahar and Carlie and last names Daud and Elyas.

We are asked to prove that at least 3 people have same first & last name.

Here, this question is about persons and their belonging names, we have to use the pigeonhole principle (generalized).

This principle says - When n pigeons are placed into k pigeonholes then there exists a pigeonhole with at least n/k pigeons.

To use this in given example, we have to find k .

K = Number of combinations of first names and last names combinations

Hence, $K = 2 \times 3 = 6$ (by multiplicity principle)

Thus, there are 6 different (first name + second name) combinations.

But there are $n = 18$ persons having any one of these combinations.

Thus, by pigeonhole principle, there are at least n/k persons having same first & last name.

Hence there are at least $18/6 = 3$ people with same first & last name.

- b) We see that as there are equal number of odd and even integers from 1 to 20, the probability of picking an odd integer is $1/2$ in every pick.

Hence even if you pick 10 integers randomly, there is a chance that all of them are even.

Thus, you have pick 11 numbers to be sure that at least 1 of them is odd.

- c) Again, just like in solution b, here there are **80** integers in the range of 1 -100 which are **not divisible by 5**.

So, you have to pick **81** integers to be sure that at least 1 of them is divisible by 5.