UTM UNIVERSITI TEKNOLOGI MALAYSIA

SCHOOL OF COMPUTING
Faculty of Engineering

SUBJECT:
DISCRETE STRUCTURE (SECI1013-03)

TOPIC :
ASSIGNMENT 1

NO	STUDENT'S NAME	MATRIX NO
1	NURFARRAHIN BINTI CHE ALIAS	A20EC0121
2	GOO YE JUI	A20EC0191
3	QAISARA BINTI ROHZAN	A20EC0133

LECTURER'S NAME:
DR. NOR AZIZAH ALI

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ASSIGNMENT# 1

1. Let the universal set be the set **R** of all real numbers and let $A=\{x\in\mathbf{R} \mid 0 < x \leq 2\}$, $B=\{x\in\mathbf{R} \mid 1 \leq x < 4\}$ and $C=\{x\in\mathbf{R} \mid 3 \leq x < 9\}$. Find each of the following:

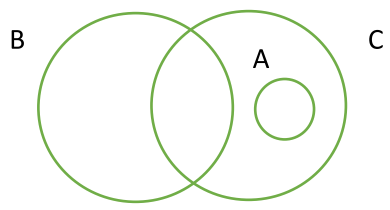
   $A = \{ 1, 2 \}, B = \{ 1, 2, 3 \}, C = \{ 3, 4, 5, 6, 7, 8 \}$

   a) $A \cup C$
   $= \{ 1, 2 \} \cup \{ 3, 4, 5, 6, 7, 8 \}$
   $= \{ 1, 2, 3, 4, 5, 6, 7, 8 \}$
   $= \{ x \in \boldsymbol{R} \mid 1 \leq x < 9 \}$

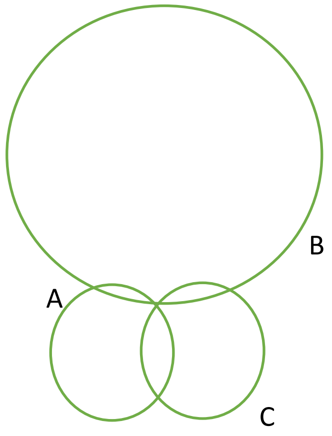
   b) $(A \cup B)'$
   $= \{ x \in \mathbf{R} \mid x <1 \text{ and } x >3\}$

   c) $A' \cup B'$
   $= \{ x \in \mathbf{R} \mid x < 1 \text{ and } x \geq 3\}$

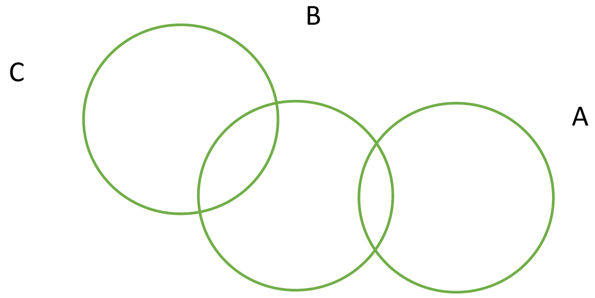
2. Draw Venn diagrams to describe sets A, B, and C that satisfy the given conditions.
   a) $A \cap B = \varnothing, A \subseteq C, C \cap B \neq \varnothing$



   b) $A \subseteq B, C \subseteq B, A \cap C \neq \varnothing$



c) $A \cap B \neq \varnothing, B \cap C \neq \varnothing, A \cap C = \varnothing, A \not\subset B, C \not\subset B$



3. Given two relations S and T from A to B,

   $S \cap T = \{(x,y) \in A \times B \mid (x,y) \in S \text{ and } (x,y) \in T\}$

   $S \cup T = \{(x,y) \in A \times B \mid (x,y) \in S \text{ or } (x,y) \in T\}$

   Let $A=\{-1, 1, 2, 4\}$ and $B=\{1,2\}$ and defined binary relations S and T from A to B as follows:

   For all $(x,y) \in A \times B$, $\quad x\, S\, y \leftrightarrow |x| = |y|$

   For all $(x,y) \in A \times B$, $\quad x\, T\, y \leftrightarrow x - y$ is even

   State explicitly which ordered pairs are in $A \times B$, S, T, $S \cap T$, and $S \cup T$.

   A x B = {(-1,1), (-1,2), (1,1), (1,2), (2,1), (2,2), (4,1), (4,2)}

   S = {(-1,1), (1,1), (2,2)}

   T = {(1,1), (2,2), (4,2)}

   S ∩ T = {(1,1), (2,2)}

   S ∪ T = {(-1,1), (1,1), (2,2), (4,2)}

4. Show that $\neg\, ((\neg p \wedge q) \vee (\neg p \wedge \neg q)) \vee (p \wedge q) \equiv p$. State carefully which of the laws are used at each stage.

$\neg\, ((\neg p \wedge q) \vee (\neg p \wedge \neg q)) \vee (p \wedge q)$

$= (\neg(\neg p \wedge q) \;\wedge\; \neg(\neg p \wedge \neg q)\,) \vee (p \wedge q)$, De Morgan's Law

$= (\neg(\neg p) \vee \neg q) \;\wedge\; (\neg(\neg p) \vee \neg(\neg q)) \vee (p \wedge q)$, De Morgan's Law

$= (\,(\, p \vee \neg q) \wedge (\, p \vee q)\,) \vee (p \wedge q)$, Double Negation Law

$= (p \wedge q) \vee (\,(\, p \vee \neg q) \wedge (\, p \vee q)\,)$, Commutative Law

$= ((p \wedge q) \vee (\, p \vee \neg q)) \wedge ((p \wedge q) \vee (\, p \vee q))$, Distributive Law

$= (((p \wedge q) \vee p) \vee \neg q) \wedge (((p \wedge q) \vee p) \vee q))$, Associative Law

$= (p \vee \neg q) \wedge (p \vee q)$, Absorption Law

$= p \vee (q \wedge \neg q)$, Distributive Law

$= p \vee c$, Negation Law

$= p$,Universal Bound Law

5. $R_1=\{(x,y) \mid x+y \leq 6\}$; R_1 is from X to Y; $R_2=\{(y,z) \mid y>z\}$; R_2 is from Y to Z; ordering of X, Y,
and Z: 1, 2, 3, 4, 5.

Find:

$X, Y, Z = \{1, 2, 3, 4, 5\}$,
$R_1 = \{(1,1), (1,2), (1,3), (1,4), (1,5), (2,1), (2,2), (2,3), (2,4), (3,1), (3,2), (3,3), (4,1), (4,2), (5, 1) \}$
$R_2 = \{ (2,1), (3,1), (3,2), (4,1), (4,2), (4,3), (5,1), (5,2), (5,3), (5,4) \}$

a) The matrix A_1 of the relation R_1 (relative to the given orderings)

$$M_{A1} = \begin{bmatrix} 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 0 \\ 1 & 1 & 1 & 0 & 0 \\ 1 & 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 \end{bmatrix}$$

b) The matrix A_2 of the relation R_2 (relative to the given orderings)

$$M_{A2} = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 \\ 1 & 1 & 0 & 0 & 0 \\ 1 & 1 & 1 & 0 & 0 \\ 1 & 1 & 1 & 1 & 0 \end{bmatrix}$$

c) Is R_1 reflexive, symmetric, transitive, and/or an equivalence relation?

Reflexive: Since for each $x \in A$, $(x,x) \notin R$, thus R is not a reflexive relation.

Symmetric: Since for all $a, b \in A$, $(a, b) \in R$ but $(b, a) \notin R$, thus R is not symmetric.

Transitive: Mr x Mr is not equal to Mr , thus R is not transitive.

So, R is not an equivalence relation.

d) Is R_2 reflexive, antisymmetric, transitive, and/or a partial order relation?

Reflexive: Since for each x ∈ A, (x,x) ∉ R, thus R is not a reflexive relation.

Symmetric: Since for all a, b∈ A, (a, b) ∈R but (b, a) ∉ R, thus R is not symmetric.

Transitive: Mr x Mr is not equal to Mr , thus R is not transitive.

So, R is not partial order relation.

6. Suppose that the matrix of relation R_1 on {1, 2, 3} is

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 1 \\ 1 & 0 & 1 \end{bmatrix}$$

relative to the ordering 1, 2, 3, and that the matrix of relation R_2 on {1, 2, 3} is

$$\begin{bmatrix} 0 & 1 & 0 \\ 0 & 1 & 0 \\ 1 & 0 & 1 \end{bmatrix}$$

relative to the ordering 1, 2, 3. Find:

R_1 = {(1,1), (2,2) , (2,3) , (3,1) , (3,3)}
R_2 = { (1,2) , (2,2) , (3,1) , (3,3) }

a) The matrix of relation $R_1 \cup R_2$

$$\begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & 1 \\ 1 & 0 & 1 \end{bmatrix}$$

b) The matrix of relation $R_1 \cap R_2$

$$\begin{bmatrix} 0 & 0 & 0 \\ 0 & 1 & 0 \\ 1 & 0 & 1 \end{bmatrix}$$

7. If f :**R**→ **R** and g:**R**→ **R** are both one-to-one, is $f + g$ also one-to-one? Justify your answer.

If we let f(x) = x , and g(x) = -x, both will be one - to - one,

But for f + g which is f(x) + g(x) = x + (-x) = 0, all the domain will be mapped to the same point of codomain which is not one - to - one .

8. With each step you take when climbing a staircase, you can move up either one stair or two stairs. As a result, you can climb the entire staircase taking one stair at a time, taking two at a time, or taking a combination of one- or two-stair increments. For each integer $n \geq 1$, if the staircase consists of n stairs, let c_n be the number of different ways to climb the staircase. Find a recurrence relation for $c_1, c_2, \ldots, c_n$.

Let n = 1, c1 = 1

Let n = 2, c2 = 2

For n = 3, if first steps = 1 , c=2 1,1,1, 1,2

if first steps = 2, c=1 2,1

c3 = 3

For n=4, if first step = 1, c=3 1,1,1,1, 1,2,1 1,1,2

if first step = 2, c=2 2,1,1 2,2

c4 = 5

For n=5, if first step = 1, c=5 (1,1,1,2 1,2,2,) from n=3 (1,1,1,1,1 1,2,1,1 1,1,2,1)from n=4

if first step = 2, c=3 (2,1,1,1)from n=3 (2,2,1 2, 1, 2)from n=4

c5 = 8

The number of ways is depends on the last two steps which is (n-1) and (n-2), using Fibonacci sequence , c1 = 1, c2=2 .For n≥3, $c_n = c_{n-1} + c_{n-2}$

9) The Tribonacci sequence (t_n) is defined by the equations,

$$t_0 = 0,\ t_1 = t_2 = 1,\ t_n = t_{n-1} + t_{n-2} + t_{n-3} \text{ for all } n \geq 3.$$

a) Find t_7.

$t_7 = t_6 + t_5 + t_4$

b) Write a recursive algorithm to compute t_n, $n \geq 3$.

```
Input : n
Output : f(n)
f (n)
 {
      return f (n-1) + f (n-2)
```

```
}
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