1.Let the universal set be the set R of all real numbers and let A={x∈R | $0 < x \leq 2$}, B={x∈R | $1 \leq x < 4$} and C={x∈R | $3 \leq x < 9$}. Find each of the following: a) A ∪ C b) (A ∪ B)′ c) A′ ∪ B′

Solution:

a.A ∪ C = { x∈R | $0 < x \leq 2$ or $3 \leq x < 9$ }

b. (A ∪ B) = { x∈R | $0 < x \leq 2$ or $1 \leq x < 4$}

={ x∈R | $0 < x < 4$}
(A ∪ B)′ = R - (A ∪ B)
= { x∈R |$0 \geq x$ or $x \geq 4$ }

c. A′ ∪ B′

A′ ={x∈R | $0 \geq x > 2$}

B′ ={x∈R | $1 > x \geq 4$}

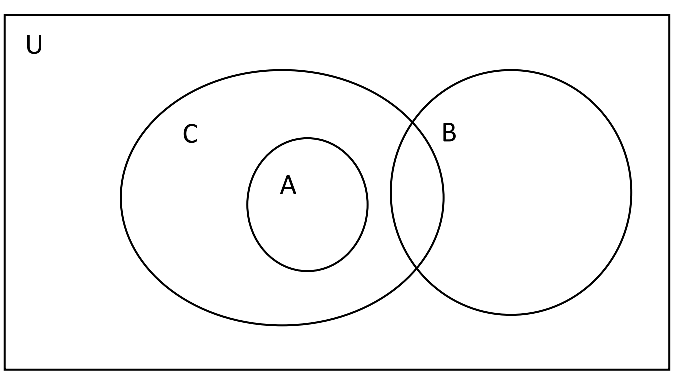
A′ ∪ B′ ={x∈R | $0 \geq x$ or $x > 2$}

2. Draw Venn diagrams to describe sets A, B, and C that satisfy the given conditions. a) $A \cap B = \emptyset$, $A \subseteq C$, $C \cap B \neq \emptyset$ b) $A \subseteq B$, $C \subseteq B$, $A \cap C \neq \emptyset$ c) $A \cap B \neq \emptyset$, $B \cap C \neq \emptyset$, $A \cap C = \emptyset$, $A \not\subseteq B$, $C \not\subseteq B$

Solution:

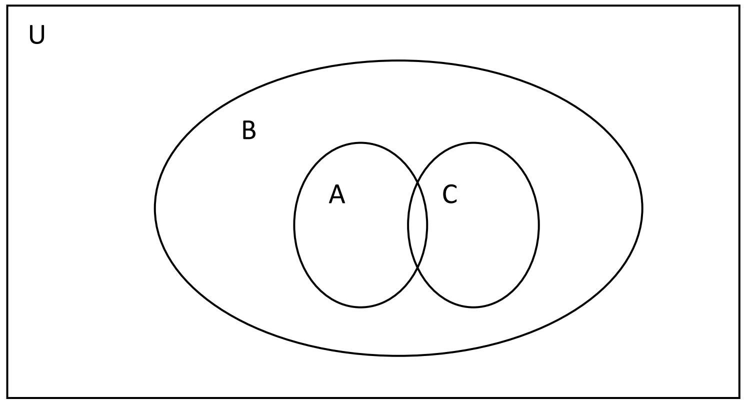
U = universal set

a)

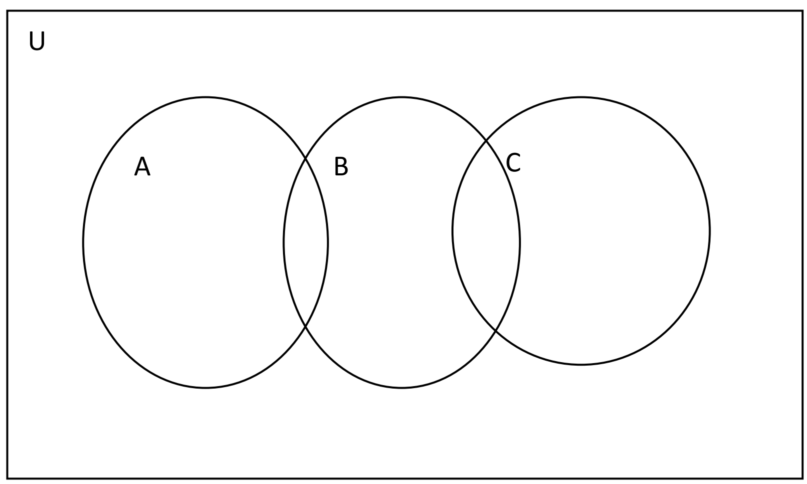


Venn Diagram

b)



c)



3. Given two relations S and T from A to B, S ∩ T = {(x,y) ∈A×B | (x,y) ∈ S and (x,y) ∈ T} S ∪ T = {(x,y) ∈A×B | (x,y) ∈ S or (x,y) ∈ T} Let A={−1, 1, 2, 4} and B={1,2} and defined binary relations S and T from A to B as follows: For all (x,y) ∈A×B, x S y ⟷ |x| = |y| For all (x,y) ∈A×B, x T y ⟷ x− y is even State explicitly which ordered pairs are in A×B, S, T, S ∩ T, and S ∪ T.

Solution:

$A \times B = \{ (-1 , 1) , (-1, 2) , (1,1) , (1,2) ,(2,1) , (2,2) , (4,1) , (4,2) \}$

$S = \{ (-1 , 1), (1,1) , (2,2) \}$

$T = \{ (-1 , 1) , (1,1) , (2,2) , (4,2) \}$

$S \cap T = \{ (-1 , 1), (1,1) , (2,2)\}$

$S \cup T = \{ (-1 , 1) , (1,1) , (2,2) , (4,2) \}$

4. Show that $\neg ((\neg p \wedge q) \vee (\neg p \wedge \neg q)) \vee (p \wedge q) \equiv p$. State carefully which of the laws are used at each stage.

Solution:

$\neg ((\neg p \wedge q) \vee (\neg p \wedge \neg q)) \vee (p \wedge q)$

$= (\neg\neg p \vee \neg q) \wedge (\neg\neg p \vee \neg\neg q) \vee (p \wedge q)$ (De Morgan's law)

$= (p \vee \neg q) \wedge (p \vee q) \vee (p \wedge q)$ (double negation law)

$= (p \vee (\neg q \wedge q)) \vee (p \wedge q)$ (distributive law)

$=(p \vee \mathbf{F}) \vee (p \wedge q)$ (negation law)

$=p \vee (p \wedge q)$

$= p$ (absorption law)

Therefore , Shown $\neg ((\neg p \wedge q) \vee (\neg p \wedge \neg q)) \vee (p \wedge q) \equiv p$.

5. $R1=\{(x,y)\mid x+y \le 6\}$; R1 is from X to Y; $R2=\{(y,z)\mid y>z\}$; R2 is from Y to Z; ordering of X, Y, and Z: 1, 2, 3, 4, 5. 1,2 ,3 ,4 ,5

Find:

a) The matrix A1 of the relation R1 (relative to the given orderings)

b) The matrix A2 of the relation R2 (relative to the given orderings)

c) Is R1 reflexive, symmetric, transitive, and/or an equivalence relation?

d) Is R2 reflexive, antisymmetric, transitive, and/or a partial order relation?

Solution:

a) R1 = {(1,1) , (1,2), (1,3) (1,4) , (1,5), (2,1), (2,2),(2,3),(2,4), (3,1),(3,2),(3,3),(4,1),(4,2),(5,1)}

$$A1= \begin{matrix} 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 0 \\ 1 & 1 & 1 & 0 & 0 \\ 1 & 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 \end{matrix}$$

b) R2 = {(2,1), (3,1), (3,2) , (4,1) , (4,2), (4,3) , (5,1), (5,2), (5,3),(5,4)}

$$A2= \begin{matrix} 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 \\ 1 & 1 & 0 & 0 & 0 \\ 1 & 1 & 1 & 0 & 0 \\ 1 & 1 & 1 & 1 & 0 \end{matrix}$$

c) not reflexive

symmetric

not transitive

so, since equivalence relation must be symmetric , reflexive and transitive

R1 is not an equivalence relation

d) not symmetric

not transitive

not reflexive

so , since partial order must be antisymmetric , reflexive and transitive

R2 is not a partial order relation.

6. Suppose that the matrix of relation R1 on {1, 2, 3} is

$$\begin{matrix} 1 & 0 & 0 \\ 0 & 1 & 1 \\ 1 & 0 & 1 \end{matrix}$$

relative to the ordering 1, 2, 3, and that the matrix of relation R2 on {1, 2, 3} is

$$\begin{matrix} 0 & 1 & 0 \\ 0 & 1 & 0 \\ 1 & 0 & 1 \end{matrix}$$

relative to the ordering 1, 2, 3. Find:

a) The matrix of relation R1∪ R2

b) The matrix of relation R1∩ R2

Solution:

R1 = { (1,1) , (2,2) , (2,3) , (3,1) , (3,3) }

R2 = { (1,2) , (2,2) , (3,1) , (3,3)}

a) R1∪ R2 = { (1,1) , (1,2) , (2,2), (2,3) , (3,1) , (3,3)}

b) R1∩ R2 = { (2,2), (3,1), (3,3)}

7. If f :R→ R and g:R→ R are both one-to-one, is f + g also one-to-one? Justify your answer.

Solution: let ,

f(x) = x and g(x) = -x

so f+g (x) = x-x,

when x=1 ,

f+g (x) = 1 -1 = 0

when x=2

f+g = 2-2 =0

so , as we can see f+g (x) has same co-domain for 2 different domains

and so f+g is not a one to one function.

8. With each step you take when climbing a staircase, you can move up either one stair or two stairs. As a result, you can climb the entire staircase taking one stair at a time, taking two at a time, or taking a combination of one- or two-stair increments. For each integer $n \geq 1$, if the staircase consists of n stairs, let cn be the number of different ways to climb the staircase. Find a recurrence relation for c1, c2, …., cn.

Solution:

Given ,

c1 = 1 , c2 = 2

so,

for n>2 we have combination of more than 2 stairs

so we take c1 as = c_{n-1}

and c2 as = c_{n-2}

Recurrence relation is $c_n = c_{n-1} + c_{n-2}$ for $n \geq 3$.

9. The Tribonacci sequence (tn) is defined by the equations, t0 = 0, t1 = t2 =1, tn = tn-1 + tn-2 + tn-3 for all $n \geq 3$. a) Find t7. b) Write a recursive algorithm to compute tn, $n \geq 3$.

Solution:

$t_0 = 0$, $t_1 = t_2 = 1$,

$t_n = t_{n-1} + t_{n-2} + t_{n-3}$ for all $n \geq 3$

a) $t_3 = t_{3-1} + t_{3-2} + t_{3-3}$

$= t_2 + t_1 + t_0$

$= 1+1+0$

$= 2$

$t_4 = t_{4-1} + t_{4-2} + t_{4-3}$

$= t_3 + t_2 + t_1$

$= 2+1+1$

$= 4$

$t_5 = t_{5-1} + t_{5-2} + t_{5-3}$

$= t_4 + t_3 + t_2$

$= 4+2+1$

$= 7$

$t_6 = t_{6-1} + t_{6-2} + t_{6-3}$

$= t_5 + t_4 + t_3$

$= 7+4+2$

$= 13$

$t_7 = t_{7-1} + t_{7-2} + t_{7-3}$

$= t_6 + t_5 + t_4$

$= 13+7+4$

= 24

b)

Input: n

Output: t(n)

```
t(n) {
If(n=1 or n=2)
        return 1
else if(n=0)
        return 0
return t(n-1) + t(n-2) + t(n-3)
}
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