DESCRETE STRUCTURE ASSIGNMENT NO.2

GROUP 2

Shah Sajid (A20EC4050)

ToyaLazmin Khan (A20EC0284)

Nafis Ahmed (A20EC4044)

Question No 1. Consider 3-digit number whose digit are either 2,3, 4, 5, 6 or 7

a. How many numbers are there?

b. How many numbers are there if the digits are instinct?

c. How many numbers between 300 to 700 is only odd digits allow?

Solution:

(a) Total digits = 6

Therefore, Total numbers = 63 =216

(b)If the digits are instinct, the 3 digits numbers that can be made from 6 digits are: P (6,3) = 120

(c) First place can be filled in= 4 ways

Second place can be filled in= 6 ways

Last Place can be filled in= 3 ways

Therefore, Total ways= 4x6x3 = 72 ways

Question No 2. Anita invited 10 of her friends, a couple, four men and four women to her anniversary dinner,how many ways Anita can arrange them around a round dinner table

a. Men insist to sit next to each other

b. The couple insisted to sit next to each other

c. Men and women sit in alternate seat

d. Before her friend left, Anita want to arrange a photo-shoot. How many ways can her photographer arrange in a row, if Anita and her husband stand next to each other.

Solution:

(a) Men insist to sit next to each other.

Considering all men together and with the remaining 5 women we have total = 6

So, arrangements= (6-1)!

Again, Men can be arranged among themselves in= 5! Ways

Therefore, Total ways= (6-1)! X 5! = 14400 ways

(b) The couple insisted to sit next to each other,

Couple can be arranged among themselves in= 2! Ways

Total arrangements= (9-1)! X 2! Ways = 80640 ways

(c) Men= 5

Women= 5

Women can sit in (5-1)! Ways = 24 ways

Men can sit in between women in= 5! Ways = 120 ways

Total Arrangements= 24 x 120 = 2880

(d) Friends= 10

Total people now can be considered as= 11

Anita and her husband can be arranged in= 2! Ways

Therefore, total ways= 11! X 2! = 79833600

Question No.3 In a school sport day, five sprinters are competing in a 100 meter race. How many ways are there for the sprinter to finish

a. If no ties

b. Two sprinters tie

c. Two group of two sprinters tie

Solution:

(a) 5 sprinters

No ties allowed so all of them get different positions in the race

First position can be obtained by: 5 ways

Second position can be obtained by: 4 ways

Third position can be obtained by: 3 ways

Fourth position can be obtained by: 2 ways

Fifth position can be obtained by: 1 way

Therefore, Total ways= 5x4x3x2x1 = 120 ways

(b) If 2 sprinters tie, then the positioning will be done considering 4 people.

So, Number of ways= 4! = 24 ways

(c) If two group of sprinter ties, we will consider now 3 people

So, number of ways = 3! = 6

Question No 4.A croissant shop has plain croissants, cherry croissants, chocolate croissants, almond croissants, apple croissants, and broccoli croissants. How many ways are there to choose

a. a dozen croissants?

b. two dozen croissants with at least two of each kind?

c. two dozen croissants with at least five chocolate croissants and at least three almond croissants?

Solution:

(a) Types of croissants=6

Here,

n= 6

r= 12

To choose 1 dozen we need,

C (n+r-1, r)

= C (6+12-1, 12)

= C (17,12)

= 6188

(b) Let us first select 2 o feachkind, which are 12 croissants in total. Then, we still need to select the remaining 12 croissants.

n= 6

r=12

Repetition of the croissants is allowed

C (6+12-1,12) = C (17,12) = 6188

(c) From 6 types we have to select 2 dozen that is 24.

Here 5 Chocolate croissants and 3 almond croissants are selected. Then, we still need to select the remaining 24-5-3 = 16 Croissants

n= 6

r= 16

Repetition of croissants is allowed

C (n+r-1, r) = C (21,6) = 20,349

Question No: 5. This procedure is used to break ties in games in the championship round of the World Cup soccer tournament. Each team selects five players in a prescribed order. Each of these players takes a penalty kick, with a player from the first team followed by a player from the second team and so on, following the order of players specified. If the score is still tied at the end of the 10 penalty kicks, this procedure is repeated. If the score is still tied after 20 penalty kicks, a sudden-death shootout occurs, with the first team scoring an unanswered goal victorious. a. How many different scoring scenarios are possible if the game is settled in the first round of 10 penalty kicks, where the round ends once it is impossible for a team to equal the number of goals scored by the other team? b. How many different scoring scenarios for the first and second groups of penalty kicks are possible if the game is settled in the second round of 10 penalty kicks? c. How many scoring scenarios are possible for the full set of penalty kicks if the game is settled with no more than 10 total additional kicks after the two rounds of five kicks for each team?

Solution:

a.) if a team wins 2 games among 4 games we have

C(4,2) = 4C2 = 6

If a team wins 1 game among 3 games we have

C(3,1) = 3C1 = 3

2 wins and 1 ties or wins = C (4,2) X C(3,1) X 2 = 36

1 win and 3 ties or wins = C (3,1) X C(4,3) X 2 X 2 X2 = 96

Since there are 2 teams we multiply it by 2 ;

Scenarios = 2 X (36+96) = 264

b) If 10 penalty kicks are executed we get = $2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2$ = 1024

Unsettled games = 1024 – 264 = 760

So 1st game = 760 ans 2nd game = 264

So total scenarios = 760×264 = 200640

c) for sudden death shootout we have 3 options

A wins , B win or a tie

So, unsettled games scenarios are

1st round = 760

2nd round = 760

For shootout the game was settled so the scenarios are = 2×5 = 10

So, final scenario = 10 $\times 760 \times 760$= 5776000

Question No 6.A professor gives a multiple-choice quiz that has ten questions, each with four possible responses, a,b,c,d. What is the minimum number of students that must be in the professor's class in order to guarantee that at least three answer sheets must be identical? (Assume that no answers are left blank.)

Solution:

selection for 1 question with selection of 4 C(4,1) = 4

So for 10 questions = $4\times 4 \times 4 \times 4 \times 4 \times 4 \times 4 \times 4 \times 4 \times 4$ = 1048576

Let N = number of students (pigeons)

10 question selection = (pigeon hole)

Using ceiling function from pigeon hole principle = $\lceil N/1048576 \rceil$ = 3

So, N = 2097153

Question No: 7In a secondary examination, 75% of the students have passed in history and 65% in Mathematics, while 50% passed both in history and mathematics. If 35 candidates failed in both the subjects, what is the number of candidates sit for the exam?

Solution:

Let total students = 100%

Students who only passed history = 75-50 = 25%

Students who only passed Maths = 65-50 = 15%

Students who only passed math or only history or both = (15+ 25 + 50)% = 90%

Students who failed both the subjects = 100-90 = 10%

So,

(10/100) $\times$ $number\ of\ students$ = 35

So , number of students = 350

Question No:8An integer from 300 through 780, inclusive is to be chosen at random, find the probability that the number is chosen will have 1 as at least one digit.

Solution: Possibility of a number which does not have any ones into it

For 1st digit , we can take 3 to 6 so = P(4,1) = 4

For 2nd and 3rd we take any from 0 to 9 except 1 = $9\times 9 = 81$

So, $81 \times 4 = 324$

If we take 1st digit as 7 except for the 3 digit 780 , arrangements = 1P1 $\times 7P1 \times 9P1 = 63$

For 780 = 1 way

So, arrangements of numbers without ones = 324+ 63+1 = 388 ways

Total numbers = 481

So , arrangements with atleast one digit as 1 = 481 – 388 = 93 ways

Probability = $\frac{93}{481}$

Question No.9 Two blue and four yellow cars are to be parked in a row of 10 parking lots. Assume that cars ofthe same color are not distinguishable, and the parking lots are chosen at random.
a) In how many ways can the cars be parked in the parking lots?
b) In how many ways can the cars be parked so that the empty lots are next to each oneanother? Find the probability that the empty lots are next to one another?

Solution:

BLUE CARS= TWO

YELLOW CARS=FOUR

A) Among 10 cars 6 cars are parked
So arrangement= $10P_6$
Since some color cannot be distinguish, the arrangement is
$10P_6$ / 4!2!=3150

B) W →4 empty lots

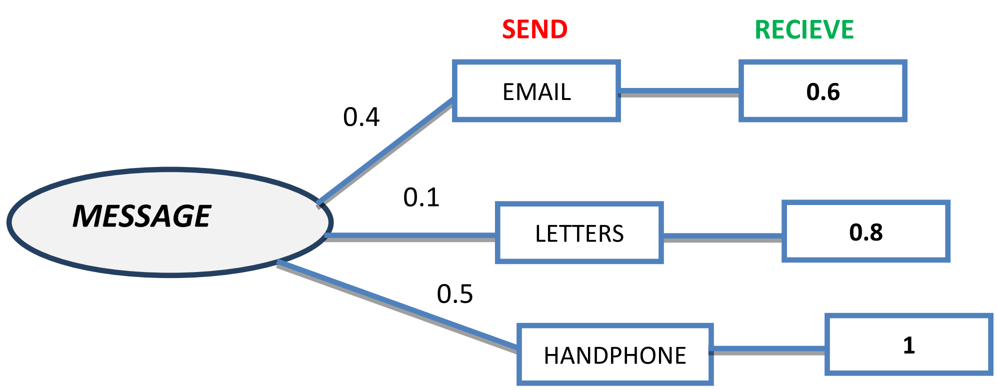
C= cars

WCWCW C WCW C WC W = places where we can put the empty lots
So, arrangements
= $7P_1$*6! / 4! *2!
=105
Probability= 105/3150 = 1/30

Question No.10 **A coach wishes to give a message to a trainee. The probabilities that he uses email, letter andhand phone are 0.4,0.1 and 0,5 respectively. He uses only one method. The probabilities of thetrainee receive the message if the coach uses email, letter or hand phone are 0.6,0.8 and 1respectively**

a) Find the probability the trainee receives the message
b) Given that the trainee receives the messages, find the conditional probabilities that he receivesit via email

Solution:



a) RECEIVING PROABILITY

= 0.4*06+01*0.8+0.5*1.0

= 0.24+0.08+0.5=0.82

b)

Email-B, Receive=A

$$P(B/A)= \frac{P(B \wedge A)}{P(A)}$$

$$\therefore \frac{0.4*0.6}{0.82}=\frac{0.24}{0.82}=\frac{12}{41}$$

Question No11. **In a recent News, it was reported that light trucks, which include SUV's, pick-up trucks and minivans, accounted for 40% of all personal vehicles on the road in 2012. Assume the rest are cars. Of every 100,000 cars accidents, 20 involve a fatality; of every 100,000 light trucks accidents, 25 involve a fatality. If a fatal accident is chosen at random, what is the probability the accident involved a light truck?**

Solution:

Lets take the event

Light truck- L

-Cars- C

Fatal Accident- F

Not a Fatal Accident- A

Given:

P(F/C)= $\frac{20}{100000}$

P(F/L)= $\frac{25}{100000}$

P(L)=0.4

As we know that A and A' are complementary events

P(C)= 1-P(L)= 1-04= 0.6

We have to compute the additional probability of light truck accident given that it is fatal P(L/F)

Consider P(L/F) conditional probability of light truck involved accident given that it is a fatal.

Using Bayes theorem :

P(L/F)=P(F/L)P(L)/P(F/L)P(L)+P(F/C)P(C)

$$P(F/L)=\frac{(.00025)(0.4)}{(.00025)(0.4)+(.00020)(0.6)}$$

P(F/L)=0.4545 OR P(F/L)= 45.45%

So the probability the accident involved a light truck is 45.45%

Question No.12 There are 9 letters having different colors (red, orange, yellow, green, blue, indigo, velvet) and 4boxes of different shapes (tetrahedron, cube, polyhedron, dodecahedron). How many ways are there to place these 9 letters into the 4 boxes such that each box contain at least 1 letter?

Solution:

Total number of letters = 9

Total number of boxes = 4

As given in the question all 9 letters having different colors and we have 4 choices when to put them. Therefore the possible number of ways without restriction is

4^9 =262144

If put in 3 boxes , arrangements;

$4*3^9$ = 78732

If put in 2 boxes , arrangements;

$4*2^9$ = 2048

If put in one box

$4*1^9$ = 4 ways

Finally the numbers of allow assignments of letters to boxes is = 262144-78732 +2048- 4 =185456