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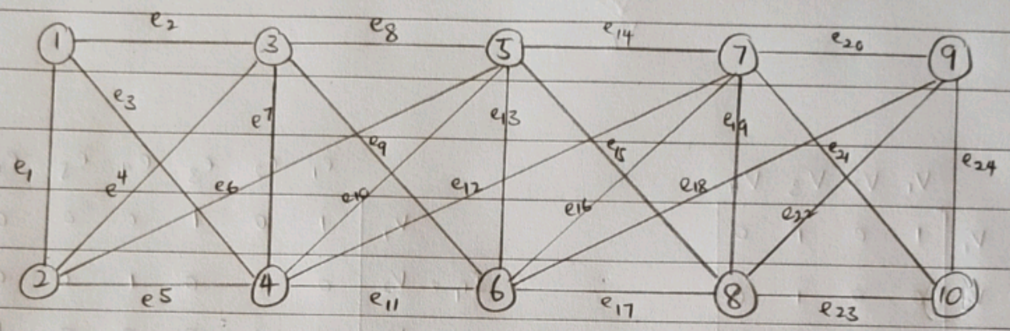
Assignment 4

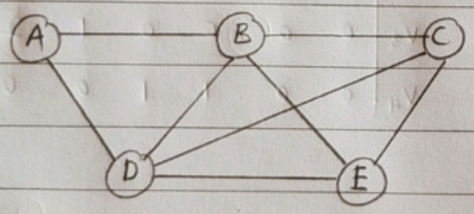
1. $V(G) = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$

set of $|v-w| \leq 3 = \{ \cancel{(1,1)}, (1,2), (1,3), (1,4), \cancel{(2,1)}, \cancel{(2,2)}, (2,3), (2,4),$
$(2,5), \cancel{(3,1)}, \cancel{(3,2)}, \cancel{(3,3)}, (3,4), (3,5), (3,6), \cancel{(4,1)}, \cancel{(4,2)},$
$\cancel{(4,3)}, \cancel{(4,4)}, (4,5), (4,6), (4,7), \cancel{(5,2)}, \cancel{(5,3)}, \cancel{(5,4)}, \cancel{(5,5)},$
$(5,6), (5,7), (5,8), \cancel{(6,3)}, \cancel{(6,4)}, \cancel{(6,5)}, \cancel{(6,6)}, (6,7), (6,8),$
$(6,9), \cancel{(7,4)}, \cancel{(7,5)}, \cancel{(7,6)}, \cancel{(7,7)}, (7,8), (7,9), (7,10), \cancel{(8,5)},$
$\cancel{(8,6)}, \cancel{(8,7)}, \cancel{(8,8)}, (8,9), (8,10), \cancel{(9,6)}, \cancel{(9,7)}, \cancel{(9,8)}, \cancel{(9,9)},$
$(9,10), \cancel{(10,7)}, \cancel{(10,8)}, \cancel{(10,9)}, \cancel{(10,10)} \}$



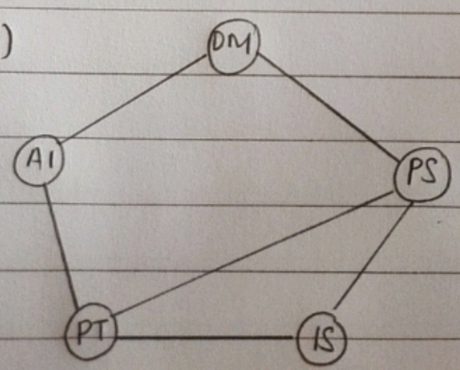
$e(G) = \{ e_1, e_2, e_3, e_4, e_5, e_6, e_7, e_8, e_9, e_{10}, e_{11}, e_{12}, e_{13}, e_{14}, e_{15}, e_{16},$
$e_{17}, e_{18}, e_{19}, e_{20}, e_{21}, e_{22}, e_{23}, e_{24} \}$

2. (a)



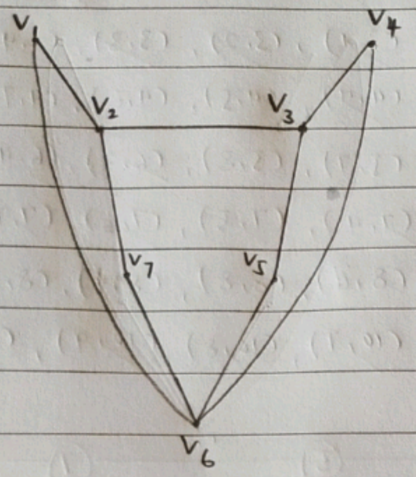
$$A_G = \begin{array}{c} \\ A \\ B \\ C \\ D \\ E \end{array} \begin{array}{c} \begin{array}{ccccc} A & B & C & D & E \end{array} \\ \begin{bmatrix} 0 & 1 & 0 & 1 & 0 \\ 1 & 0 & 1 & 1 & 1 \\ 0 & 1 & 0 & 1 & 1 \\ 1 & 1 & 1 & 0 & 1 \\ 0 & 1 & 1 & 1 & 0 \end{bmatrix} \end{array}$$

(b)



$$\begin{array}{c} \\ DM \\ PT \\ AI \\ PS \\ IS \end{array} \begin{array}{c} \begin{array}{ccccc} DM & PT & AI & PS & IS \end{array} \\ \begin{bmatrix} 0 & 0 & 1 & 1 & 0 \\ 0 & 0 & 1 & 1 & 1 \\ 1 & 1 & 0 & 0 & 0 \\ 1 & 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 1 & 0 \end{bmatrix} \end{array}$$

3. $\deg(V_1) = 2$; $\deg(V_2) = 3$; $\deg(V_3) = 3$; $\deg(V_4) = 2$; $\deg(V_5) = 2$;
$\deg(V_6) = 4$; $\deg(V_7) = 2$



4.

$$A_G = \begin{array}{c} \\ V_1 \\ V_2 \\ V_3 \\ V_4 \end{array} \begin{array}{c} \begin{array}{cccc} V_1 & V_2 & V_3 & V_4 \end{array} \\ \begin{bmatrix} 1 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{bmatrix} \end{array}$$

$$I_G = \begin{array}{c} \\ V_1 \\ V_2 \\ V_3 \\ V_4 \end{array} \begin{array}{c} \begin{array}{cccccc} e_1 & e_2 & e_3 & e_4 & e_5 & e_6 \end{array} \\ \begin{bmatrix} 1 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 1 & 0 \end{bmatrix} \end{array}$$

$$A_H = \begin{array}{c} \\ V_1 \\ V_2 \\ V_3 \\ V_4 \end{array} \begin{array}{c} \begin{array}{cccc} V_1 & V_2 & V_3 & V_4 \end{array} \\ \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 1 & 2 \\ 0 & 1 & 1 & 0 \\ 0 & 2 & 0 & 0 \end{bmatrix} \end{array}$$

$$\begin{array}{c} \\ V_1 \\ V_2 \\ V_3 \\ V_4 \end{array} \begin{array}{c} \begin{array}{cccccc} e_1 & e_2 & e_3 & e_4 & e_5 & e_6 \end{array} \\ \begin{bmatrix} 2 & 0 & 0 & 0 & 0 & 0 \\ 0 & 2 & 1 & 1 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 & 2 \\ 0 & 0 & 1 & 1 & 0 & 0 \end{bmatrix} \end{array}$$

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5. (a) - Both graphs have 5 vertices and 5 edges.

- Both are connected and simple graph.

$\deg(v_1)=2$; $\deg(v_2)=2$; $\deg(v_3)=3$; $\deg(v_4)=2$; $\deg(v_5)=1$

$\deg(u_1)=1$; $\deg(u_2)=3$; $\deg(u_3)=2$; $\deg(u_4)=2$; $\deg(u_5)=2$

- Both have 1 vertex with 1 edge, 1 vertex with 3 edges, 3 vertices with 2 edges.

- Define $f: G_1 \to G_2$, where $G_1 = \{v_1, v_2, v_3, v_4, v_5\}$ and $G_2 = \{u_1, u_2, u_3, u_4, u_5\}$

$f(v_1) = u_5$, $f(v_2) = u_3$, $f(v_3) = u_2$, $f(v_4) = u_4$, $f(v_5) = u_1$

$$A_{G_1} = \begin{array}{c} \\ v_1 \\ v_2 \\ v_3 \\ v_4 \\ v_5 \end{array} \begin{array}{c} \begin{array}{ccccc} v_1 & v_2 & v_3 & v_4 & v_5 \end{array} \\ \begin{bmatrix} 0 & 1 & 0 & 1 & 0 \\ 1 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 1 & 1 \\ 1 & 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \end{bmatrix} \end{array} \qquad A_{G_2} = \begin{array}{c} \\ u_5 \\ u_3 \\ u_2 \\ u_4 \\ u_1 \end{array} \begin{array}{c} \begin{array}{ccccc} u_5 & u_3 & u_2 & u_4 & u_1 \end{array} \\ \begin{bmatrix} 0 & 1 & 0 & 1 & 0 \\ 1 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 1 & 1 \\ 1 & 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \end{bmatrix} \end{array}$$

A_{G_1} and A_{G_2} are the same. G_1 and G_2 are isomorphic.

(b) - Both graphs have 5 vertices and 6 edges.

- Both graphs are connected.

- Both graphs have 1 loop and 2 parallel edges.

$\deg(a_1)=1$; $\deg(a_2)=1$; $\deg(a_3)=3$; $\deg(a_4)=2$; $\deg(a_5)=4$

$\deg(x_1)=3$; $\deg(x_2)=1$; $\deg(x_3)=2$; $\deg(x_4)=3$; $\deg(x_5)=2$

- Both graphs do not have the corresponding pair of vertices connected.

$\therefore$ H_1 and H_2 are not isomorphic.

6. (a) $v_0\, e_1\, v_1\, e_{10}\, v_5\, e_9\, v_2\, e_2\, v_1$

$=$ trail

(b) $v_4\, e_7\, v_2\, e_9\, v_5\, e_{10}\, v_1\, e_3\, v_2\, e_9\, v_5$

$=$ walk

(c) v_2

$=$ closed walk

(d) $v_5\, v_9\, v_2\, e_4\, v_3\, e_5\, v_4\, e_6\, v_4\, e_8\, v_5$

$=$ circuit

(e) $v_2\, e_4\, v_3\, e_5\, v_4\, e_8\, v_5\, e_9\, v_2\, e_7\, v_4\, e_5\, v_3\, e_4\, v_2$

$=$ closed walk

(f) $v_3\, e_5\, v_4\, e_8\, v_5\, e_{10}\, v_1\, e_3\, v_2$

$=$ walk

7. (a) $v_1 \to v_4$ path

(i) $v_1, e_1, v_2, e_2, v_3, e_5, v_4$

(ii) $v_1, e_1, v_2, e_3, v_3, e_5, v_4$

(iii) $v_1, e_1, v_2, e_4, v_3, e_5, v_4$

$\therefore$ Therefore, there are 3 paths from v_1 to v_4.

(b) $v_1 \to v_4$ trail

- $v_1, e_1, v_2, e_2, v_3, e_5, v_4$
- $v_1, e_1, v_2, e_3, v_3, e_5, v_4$
- $v_1, e_1, v_2, e_4, v_3, e_5, v_4$
- $v_1, e_1, v_2, e_2, v_3, e_4, v_2, e_3, v_3, e_5, v_4$
- $v_1, e_1, v_2, e_2, v_3, e_3, v_2, e_4, v_3, e_5, v_4$
- $v_1, e_1, v_2, e_3, v_3, e_2, v_2, e_4, v_3, e_5, v_4$
- $v_1, e_1, v_2, e_3, v_3, e_4, v_2, e_3, v_3, e_5, v_4$
- $v_1, e_1, v_2, e_4, v_3, e_3, v_2, e_2, v_3, e_5, v_4$
- $v_1, e_1, v_2, e_4, v_3, e_2, v_2, e_3, v_3, e_5, v_4$

$\therefore$ Therefore, there are 9 trails from v_1 to v_4

(c) $v_1 \to v_4$ walk

There are many finite alternating sequence of adjacent vertices and edges from v_1 to v_4.

Therefore, there are infinity of walks from v_1 to v_4.

8. (a)

Vertex	v_1	v_2	v_3	v_4	v_5
Degree	2	4	2	4	4

- Graph (a) is connected and every vertex has even degree.

Graph (a) has an Euler circuit.

$v_1, e_1, v_2, e_2, v_5, e_7, v_4, e_6, v_5, e_3, v_2, e_4, v_3, e_5, v_4, e_8, v_1$

There Euler circuit contains every vertex and every edges of (a).

It starts and ends at the same vertex, uses every vertex of (a) at least once, and uses every edge of (a) exactly once.

(b)

Vertex	v_0	v_1	v_2	v_3	v_4	v_5	v_6	v_7	v_8	v_9
Degree	2	5	2	2	2	4	2	3	3	3

Graph (b) does not has an Euler circuit. This is because vertex v_1, v_7, v_8, v_9 have odd degrees.

9. (a)

Vertex	u	v_0	v_1	v_2	v_3	v_4	v_5	v_6	v_7	w
Degree	3	2	2	4	2	4	2	4	2	3

V and W are the only vertices that have odd degrees in graph (a).

Therefore, it has Euler path.

Euler path: $u, v_1, v_0, v_7, u, v_2, v_3, v_4, v_2, v_6, v_4, w, v_6, v_5, w$

(b)

Vertex	u	a	b	c	d	e	f	g	h	w
Degree	3	2	2	2	2	3	4	2	3	3

Vertex e and h have odd degrees apart from U and W.

Therefore, there is no Euler path at graph (b).

10. There is no Hamiltonian circuit in both graph (a) and (b).

11. $m=3$, $n=100$

$$l = \frac{(m-1)n+1}{m}$$

$$= \frac{(3-1)100+1}{3}$$

$$= \frac{200+1}{3}$$

$$= \frac{201}{3} = 67$$

∴ A full 3-ary tree with 100 vertices have 67 leaves.

12. (a) root : a

(b) internal vertices : a, b, d, e, g, h, j

(c) leaves : c, k, l, m, f, r, s, o, i, p, q

(d) children of n : r, s

(e) parent of e : b

(f) siblings of k : l, m

(g) proper ancestors of q : j, d

(h) proper descendants of b : e, k, l, m, f, g, n, r, s

13. Preorder : a, b, e, k, l, m, f, g, n, r, s, c, d, h, o, i, j, p, q

Inorder : k, e, l, m, b, f, r, n, s, g, a, c, o, h, d, i, p, j, q

Posterder : k, l, m, e, f, r, s, n, g, b, c, o, h, i, p, q, j, d, a

14.

Edge	AB	GH	BC	AF	BF	CD	BG	CG	DH	DE	DG	FG	EH
weight	1	1	2	3	4	4	5	6	6	7	7	8	8

AB 1

GH 1

BC 2

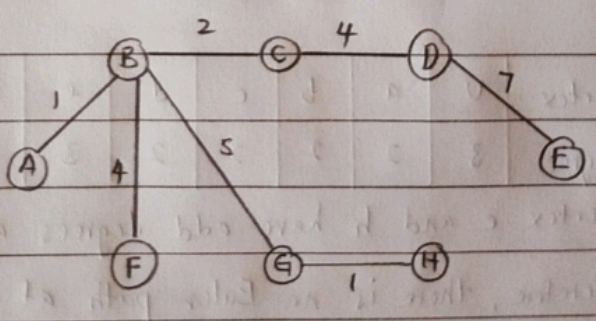
BF 4

CD 4

BG 5

DE 7

24



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$S = \phi \quad N = \{M, N, O, P, Q, R, S, T\}$

15.

No	S	N	L(M)	L(N)	L(O)	L(P)	L(Q)	L(R)	L(S)	L(T)
0	{ }	{M, N, O, P, Q, R, S, T}	0	∞	∞	∞	∞	∞	∞	∞
1	{M}	{N, O, P, Q, R, S, T}		4	∞	2	∞	5	∞	∞
2	{M, P}	{N, O, Q, R, S, T}		4	∞		6	5	5	∞
3.	{M, P, N}	{O, Q, R, S, T}			10		6	5	5	∞
4.	{M, P, N, R}	{O, Q, S, T}			7		6		5	6
5	{M, P, N, R, S}	{O, Q, T}			7		6			6
6	{M, P, N, R, S, Q}	{O, T}			7					6
7.	{M, P, N, R, S, Q, T}	{O}			7					6

The shortest path from M to T is M, R, T. The length is 6.