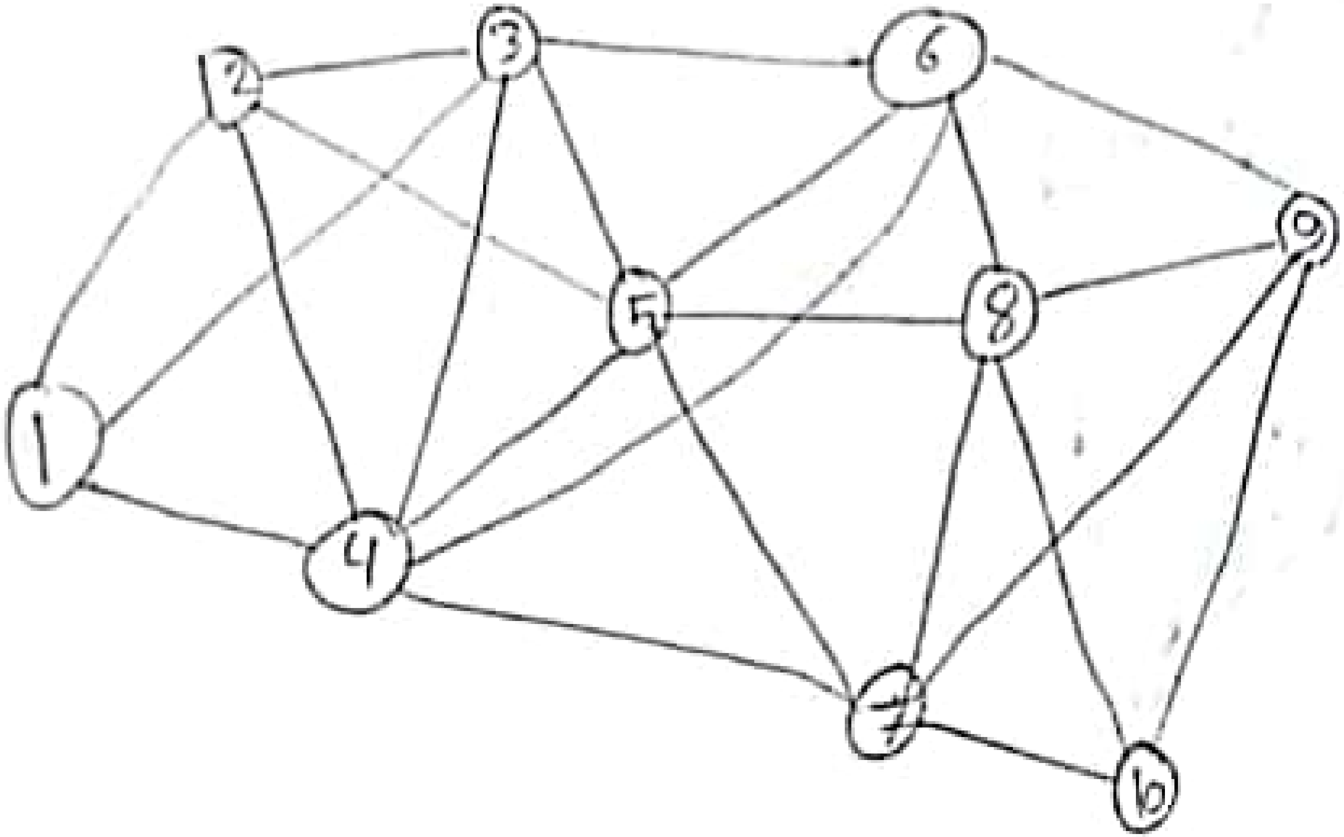
Problem-1

$V(G) = \{1, 2, \ldots, 10\}$ v and w are adjacent $|v - w| \leq 3$

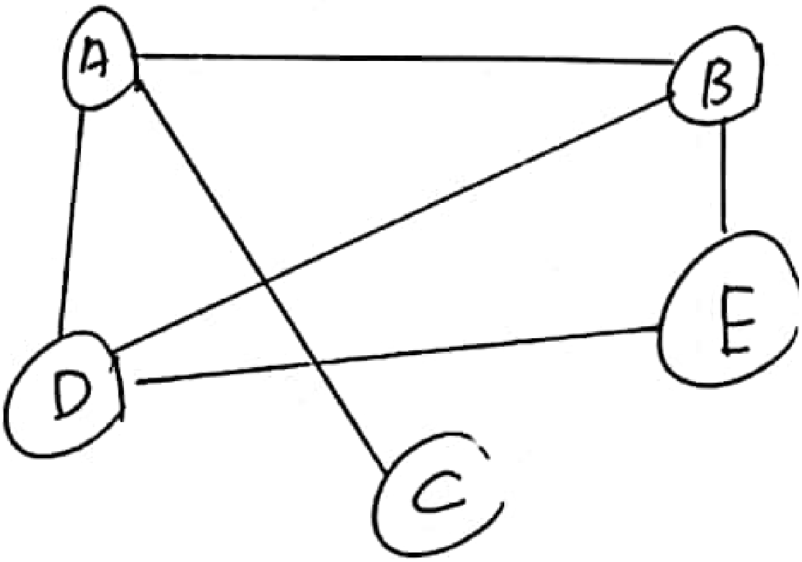
Draw graph G, determine the number of edges $e(G)$

	1	2	3	4	5	6	7	8	9	10
1	0	1	1	1	0	0	0	0	0	0
2	1	0	1	1	1	0	0	0	0	0
3	1	1	0	1	1	1	0	0	0	0
4	1	1	1	0	1	1	1	0	0	0
5	0	1	1	1	0	1	1	1	0	0
6	0	0	1	1	1	0	1	1	1	0
7	0	0	0	1	1	1	0	1	1	1
8	0	0	0	0	1	1	1	0	1	1
9	0	0	0	0	0	1	1	1	0	1
10	0	0	0	0	0	0	1	1	1	0



$\varphi(g)=48$

2) a) A : Ahmad
B : Baki
C : Chong
D : David
E : Ehsan



	A	B	C	D	E
A	0	1	1	1	0
B	1	0	0	1	1
C	1	0	0	0	0
D	1	1	0	0	1
E	0	1	0	1	0

(b) 5 subjects Dm, PT, AI, PS, IS

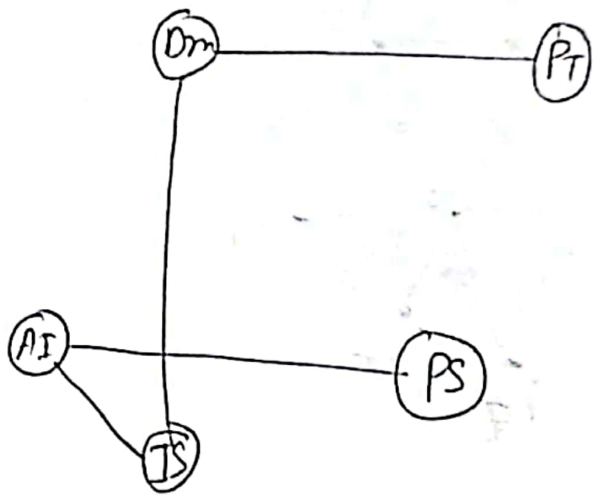
cannot be in the same

Dm & IS

Dm & PT

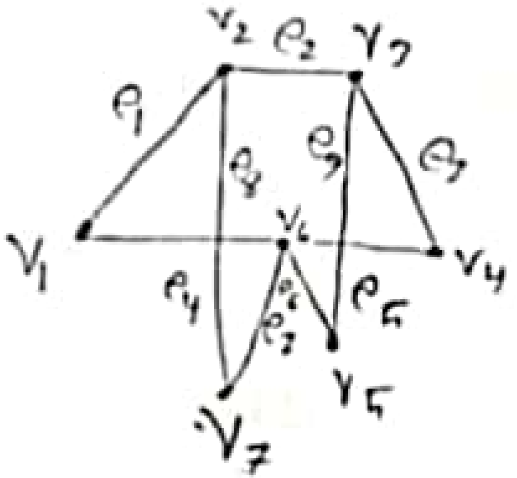
AI & PS

IS & AI



$$\begin{array}{c|ccccc|}
 & Dm & IS & PT & AI & Ps \\
Dm & 0 & 1 & 1 & 0 & 0 \\
IS & 1 & 0 & 0 & 1 & 0 \\
Pt & 0 & 0 & 0 & 0 & 0 \\
AI & 0 & 1 & 0 & 0 & 1 \\
PS & 1 & 0 & 0 & 0 & 0
\end{array}$$

Ans. no-3



7 vertices and 9 edges

all the vertices have 3 degree v_6 have 4 degree

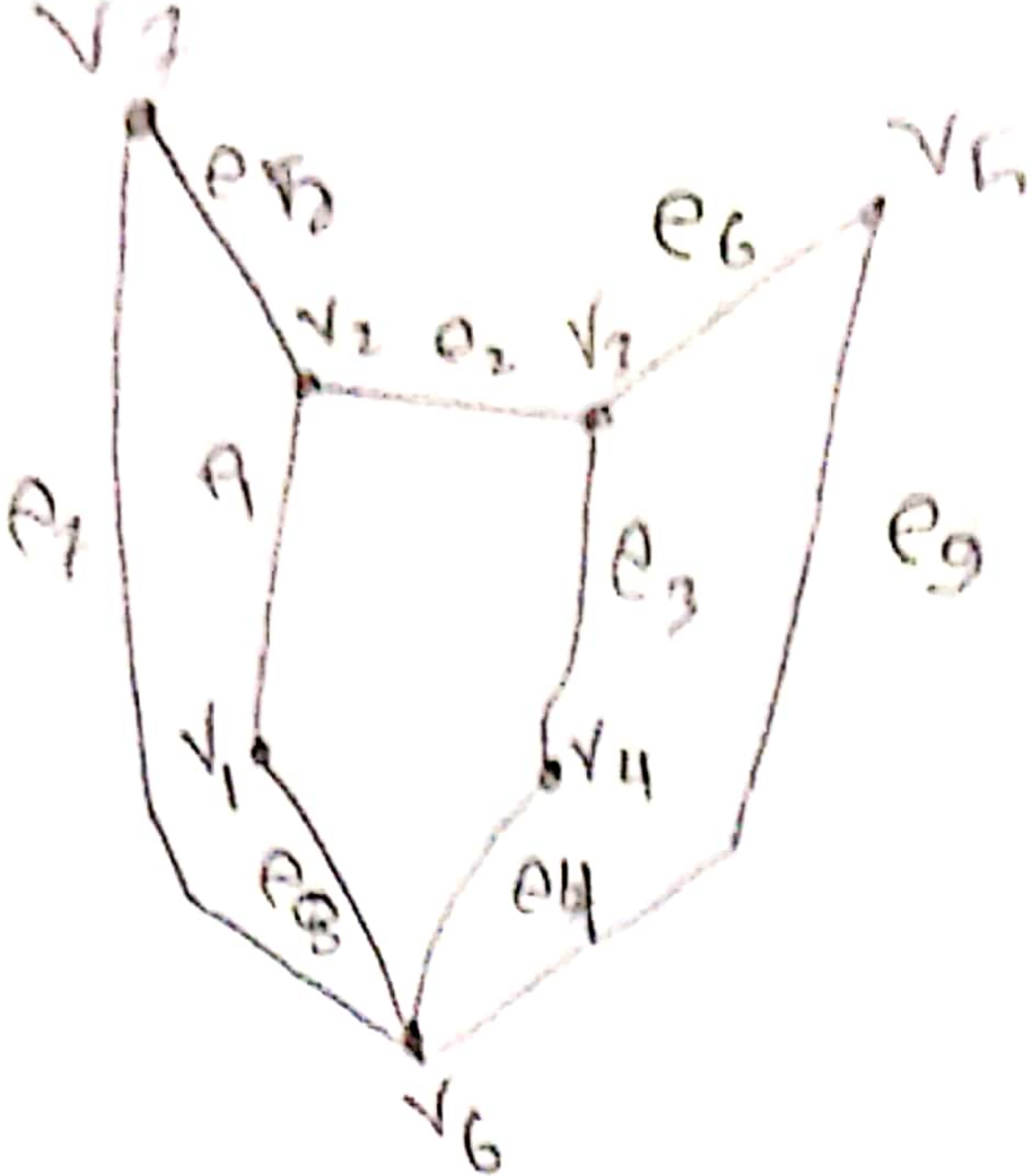
all adjacent of v_6.

deg $v_1 = 2$

deg $v_4 = 2$

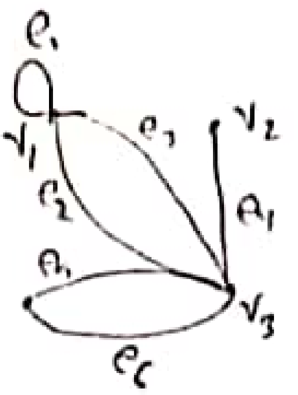
deg $v_5 = 2$

deg $v_7 = 2$


v2
e2
e6
v5
e1
e3
e9
v1
v4
e8
e4
v6

Ans. no - 4

G



Adjacency

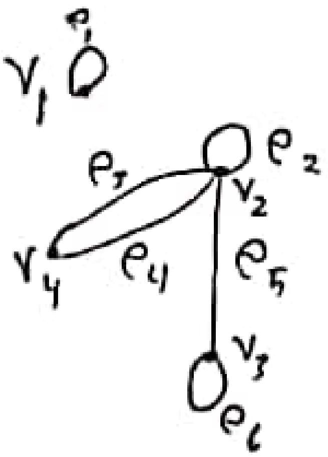
$$\begin{array}{c|cccc|} & v_1 & v_2 & v_3 & v_4 \\ v_1 & 1 & 0 & 1 & 0 \\ v_2 & 0 & 0 & 1 & 0 \\ v_3 & 1 & 0 & 0 & 1 \\ v_4 & 0 & 0 & 1 & 0 \end{array}$$

~~Go:~~

incidence:-

$$\begin{array}{c|cccccc|} & e_1 & e_2 & e_3 & e_4 & e_5 & e_6 \\ v_1 & 1 & 0 & 1 & 0 & 0 & 0 \\ v_2 & 0 & 0 & 0 & 1 & 0 & 0 \\ v_3 & 0 & 1 & 0 & 0 & 0 & 1 \\ v_4 & 0 & 0 & 0 & 0 & 1 & 0 \end{array}$$

H :



adjacency

$$\begin{array}{c|cccc|} & v_1 & v_2 & v_3 & v_4 \\ v_1 & 1 & 0 & 0 & 0 \\ v_2 & 0 & 1 & 1 & 1 \\ v_3 & 0 & 1 & 1 & 0 \\ v_4 & 0 & 1 & 0 & 0 \end{array}$$

incidence

$$\begin{array}{c|cccccc|} & e_1 & e_2 & e_3 & e_4 & e_5 & e_6 \\ v_1 & 1 & 0 & 0 & 0 & 0 & 0 \\ v_2 & 0 & 1 & 1 & 1 & 1 & 0 \\ v_3 & 0 & 0 & 0 & 0 & 1 & 1 \\ v_4 & 0 & 0 & 1 & 1 & 0 & 0 \end{array}$$

Ans. no-5

ⓐ

vertices	edges	degree
$G_1 = 5$	$G_1 = 5$	G_1 = has degree of 2
$G_2 = 5$	$G_2 = 5$	G_2 = has degree of 2, 3

These graph are not isomorphic.

ⓑ

vertices	edges	degree
$H_1 = 5$	$H_1 = 6$	H_1 = have degree of 1, 2
$H_2 = 5$	$H_2 = 6$	H_2 = have degree of 2, 3

These graph are not isomorphic.

6) a) Trails
b) walk and closed walk
c) walk
d) circuits / cycle
e) closed walk
f) path

7) a) 3 path : $v_1 e_1 v_2 e_2 v_3 e_5 v_4$
$v_1 e_1 v_2 e_3 v_3 e_5 v_4$
$v_1 e_1 v_2 e_4 v_3 e_5 v_4$

b) 9 trails : $v_1 e_1 v_2 e_2 v_3 e_5 e_4$
$v_1 e_1 v_2 e_3 v_3 e_5 e_4$
$v_1 e_1 v_2 e_4 v_3 e_5 v_4$
$v_1 e_1 v_2 e_2 v_3 e_2 v_2 e_4 v_3 e_5 v_4$
$v_1 e_1 v_2 e_2 v_3 e_4 v_2 e_3 v_3 e_5 v_4$
$v_1 e_1 v_2 e_3 v_3 e_2 v_2 e_4 v_3 e_5 v_4$
$v_1 e_1 v_2 e_3 v_3 e_4 v_2 e_2 v_3 e_5 v_4$
$v_1 e_1 v_2 e_4 v_3 e_2 v_2 e_3 v_3 e_5 v_4$
$v_1 e_1 v_2 e_4 v_3 e_3 v_2 e_3 v_3 e_5 v_4$

c) unlimited number of walk, because there is no limit for length of walk.

8) a)

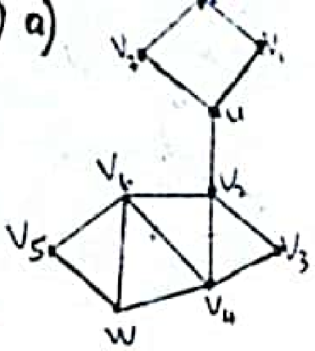
vertex	v_1	v_2	v_3	v_4	v_5
degree	2	4	2	4	4

- Graph in (a) has Euler circuit since all vertices have even degree.
- It is possible to go through all of the edges exactly once and return to the starting point.
- For example : $v_2 - e_2 - v_5 - e_7 - v_4 - e_6 - v_5 - e_3 - v_2 - e_4 - v_3 - e_5 - 1$
$- e_8 - v_1 - e_1 - v_2$

b)

vertex	v_0	v_1	v_2	v_3	v_4	v_5	v_6	v_7	v_8	v_9
degree	2	5	2	2	2	4	2	3	3	3

- Graph in (b) does not have Euler circuit since 4 vertices (vertex v_1, v_7, v_8, v_9) have odd degree

9) a) 

Yes, have an Euler path from u to w.

$u - v_1 - v_0 - v_7 - u - v_2 - v_3 - v_4 - v_2 - v_6 - v_4 - w - v_6 - v_5 - w$

b) Does not have Euler path from u to w.

10) a) Does not have Hamiltonian circuit

b). Does not have Hamiltonian circuit

- But have a Hamiltonian path (c-b-a-u-f-d-e-w-g-h)

11 How many leaves $\frac{(m-1)n+1}{m}$

$$l = \frac{(3-1)\times 100+1}{3} = 67 \text{ leaves}$$

12 a) a
b) b, e, g, n, c, d, h, j
c) k, l, m, f, r, s, o, i, p, q
d) r, s
e) b
f) l, m
g) j, d, a
h) e, f, g, k, l, m, n, r, s

13 Preorder

a, b, e, k, l, m, f, g, n, r, s, c, d, h, o, i, j, p, q

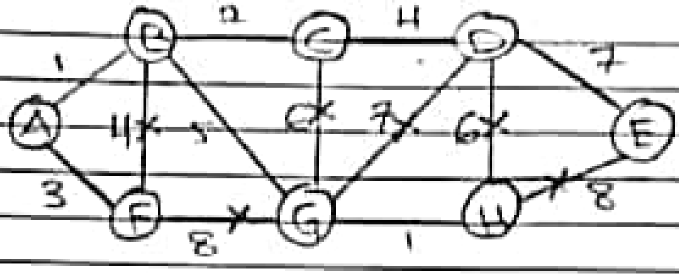
Postorder

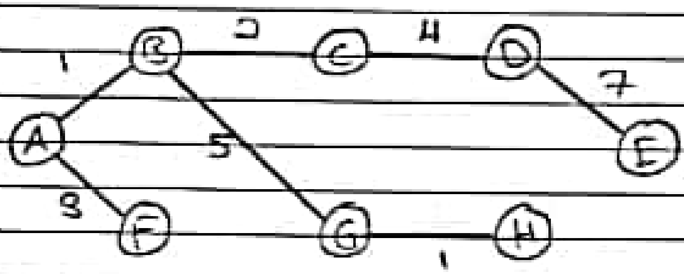
k, l, m, e, f, r, s, n, g, b, c, o, h, i, p, q, j, d, a

Inorder

k, e, l, m, b, f, g, r, n, s, a, c, o, h, d, i, p, j, q

14





min = 1 + 3 + 2 + 5 + 1 + 4 + 7
= 23

No. :

Date :

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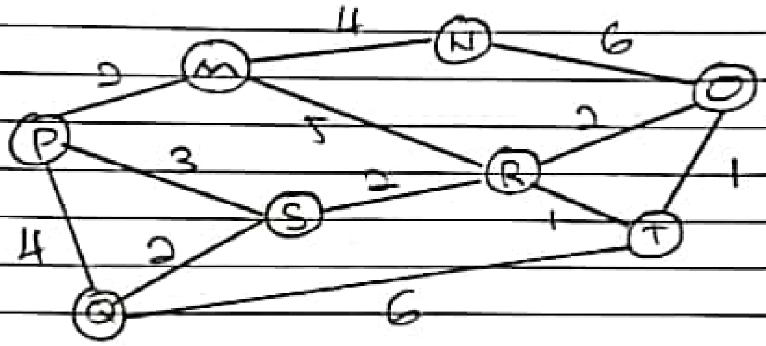

i	S	N	L(M)	L(N)	L(O)	L(P)	L(Q)	L(R)	L(S)	L(T)
0	∅	{M,N,O,P,Q,R,S,T}	∞	∞	∞	∞	∞	∞	∞	∞
1	{M}	{N,O,P,Q,R,S,T}	0	4	∞	2	∞	5	∞	∞
2	{M,P}	{N,O,Q,R,S,T}	0	4	∞	2	6	5	5	∞
3	{M,P,S}	{N,O,Q,R,T}	0	4	∞	2	6	5	5	12
4	{M,P,S,R}	{N,O,Q,T}	0	4	7	2	6	5	5	6
5	{M,P,S,R,T}	{N,O,Q}	0	4	7	2	6	5	5	6

∴ Minimum distance from M to T is 6

∴ the shortest path is M — R — T.