DISCRETE STRUCTURE - ASSIGNMENT 4

(GROUP 6)

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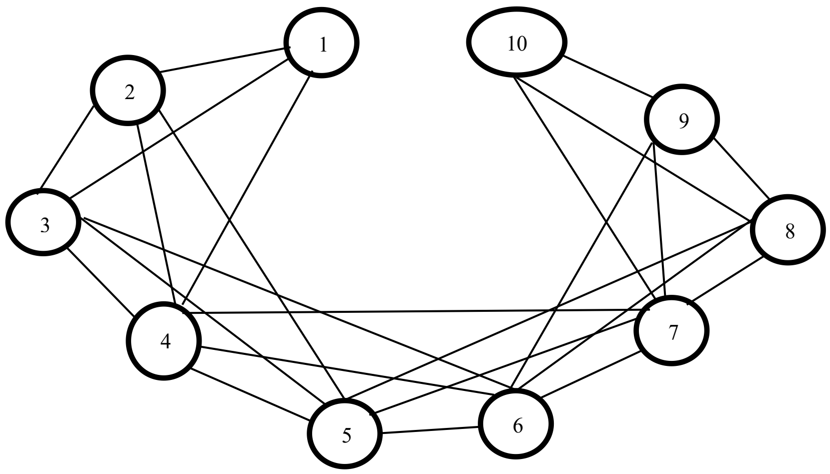
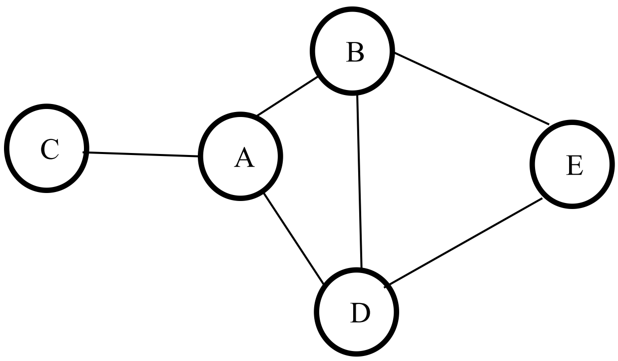
1. Let G be a graph with $V(G) = \{1,2,\ldots,10\}$, such that two numbers 'v' and 'w' in $V(G)$ are adjacent if and only if $|v - w| \leq 3$. Draw the graph G and determine the numbers of edges, $e(G)$.



e(G)=24

Edges List: (1,2), (1,3), (1,4), (2,3), (2,4), (2,5), (3,4), (3,5), (3,6), (4,5), (4,6), (4,7), (5,6), (5,7), (5,8), (6,7), (6,8), (6,9), (7,8), (7,9), (7,10), (8,9), (8,10), (9,10)

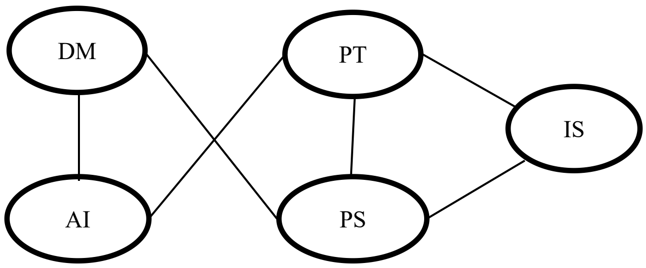
2. Model the following situation as graphs, draw each graphs and gives the corresponding adjacency matrix.
   (a) Ahmad and Bakri are friends. Ahmad is also friends with David and Chong. David, Bakri and Ehsan all friends.
   (Note that you may use the representation of A= Ahmad; B = Bakri; C = Chong; D = David; E= Ehsan)



$$A_G = \begin{array}{c} \\ A \\ B \\ C \\ D \\ E \end{array} \begin{array}{c} \begin{array}{ccccc} A & B & C & D & E \end{array} \\ \begin{bmatrix} 0 & 1 & 1 & 1 & 0 \\ 1 & 0 & 0 & 1 & 1 \\ 1 & 0 & 0 & 0 & 0 \\ 1 & 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 1 & 0 \end{bmatrix} \end{array}$$

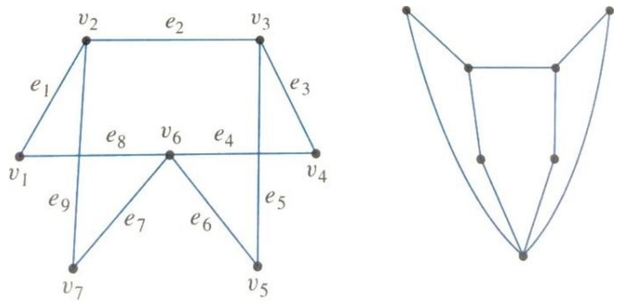
(b) There are 5 subjects to be scheduled in the exam week: Discrete Mathematics (DM), Programming Technique (PT), Artificial Intelligence (AI), Probability Statistic (PS) and Information System (IS). The following subjects cannot be scheduled in the same time slot:

i. DM and IS
ii. DM and PT
iii. AI and PS
iv. IS and AI

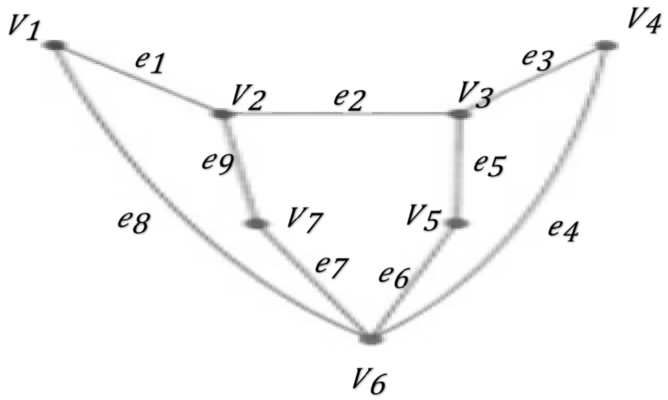


$$A_G = \begin{array}{c} \\ DM \\ PT \\ AI \\ PS \\ IS \end{array} \begin{array}{c} \begin{array}{ccccc} DM & PT & AI & PS & IS \end{array} \\ \begin{bmatrix} 0 & 0 & 1 & 1 & 0 \\ 0 & 0 & 1 & 1 & 1 \\ 1 & 1 & 0 & 0 & 0 \\ 1 & 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 1 & 0 \end{bmatrix} \end{array}$$

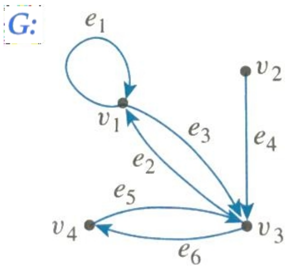
3. Show that the two drawing represent the same graph by labeling the vertices and edges of the right-hand drawing to correspond to left-hand drawing.

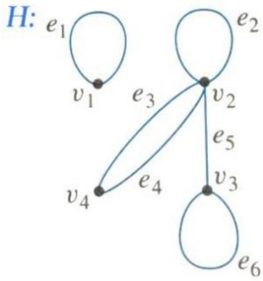


Answer:



4. Find the adjacency and incidence matrices for the following graphs.





Directed graph

$$A_G = \begin{array}{c} \\ V_1 \\ V_2 \\ V_3 \\ V_4 \end{array} \begin{array}{c} \begin{array}{cccc} V_1 & V_2 & V_3 & V_4 \end{array} \\ \begin{bmatrix} 1 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{bmatrix} \end{array}$$

$$I_G = \begin{array}{c} \\ V_1 \\ V_2 \\ V_3 \\ V_4 \end{array} \begin{array}{c} \begin{array}{cccccc} e_1 & e_2 & e_3 & e_4 & e_5 & e_6 \end{array} \\ \begin{bmatrix} 2 & 1 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 1 & 1 & 1 & 1 & 1 \\ 0 & 0 & 0 & 0 & 1 & 1 \end{bmatrix} \end{array}$$

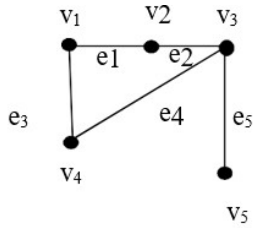
Undirected graph

$$A_H = \begin{array}{c} \\ V_1 \\ V_2 \\ V_3 \\ V_4 \end{array} \begin{array}{c} \begin{array}{cccc} V_1 & V_2 & V_3 & V_4 \end{array} \\ \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 1 & 2 \\ 0 & 1 & 1 & 0 \\ 0 & 2 & 0 & 0 \end{bmatrix} \end{array}$$

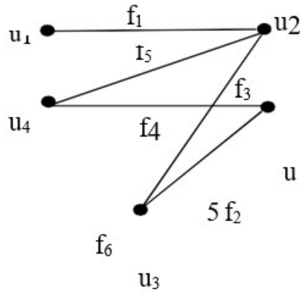
$$I_H = \begin{array}{c} \\ V_1 \\ V_2 \\ V_3 \\ V_4 \end{array} \begin{array}{c} \begin{array}{cccccc} e_1 & e_2 & e_3 & e_4 & e_5 & e_6 \end{array} \\ \begin{bmatrix} 2 & 0 & 0 & 0 & 0 & 0 \\ 0 & 2 & 1 & 1 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 & 2 \\ 0 & 0 & 1 & 1 & 0 & 0 \end{bmatrix} \end{array}$$

5. Determine whether the following graphs are isomorphic.

a)

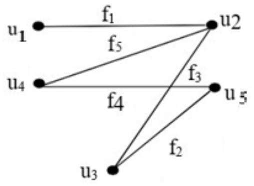


G1



G2

Edited image for G2



G2

- Number of vertices: Both 5
- Number of edges: Both 5
- Degrees of corresponding vertices:

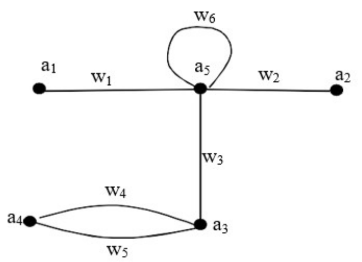
  G1: 3 vertices with degree 2, 1 vertex with degree 3, 1 vertex with degree 1

  G2: 3 vertices with degree 2, 1 vertex with degree 3, 1 vertex with degree 1

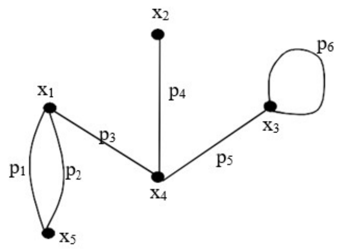
$$A_{G1} = \begin{array}{c} \\ V_1 \\ V_2 \\ V_3 \\ V_4 \\ V_5 \end{array} \begin{array}{c} \begin{array}{ccccc} V_1 & V_2 & V_3 & V_4 & V_5 \end{array} \\ \begin{bmatrix} 0 & 1 & 0 & 1 & 0 \\ 1 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 1 & 1 \\ 1 & 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \end{bmatrix} \end{array} \quad A_{G2} = \begin{array}{c} \\ u_5 \\ u_4 \\ u_2 \\ u_3 \\ u_1 \end{array} \begin{array}{c} \begin{array}{ccccc} u_5 & u_4 & u_2 & u_3 & u_1 \end{array} \\ \begin{bmatrix} 0 & 1 & 0 & 1 & 0 \\ 1 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 1 & 1 \\ 1 & 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \end{bmatrix} \end{array}$$

$\therefore A_{G1}\ and\ A_{G2}\ are\ same.\ G_1\ and\ G_2\ are\ isomorphic.$

b)



H1



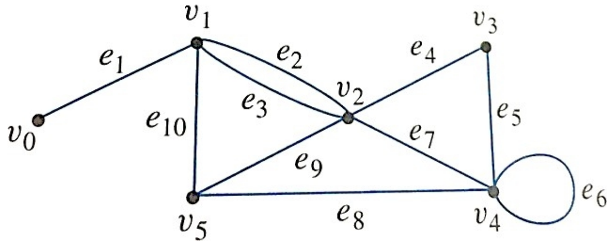
H2

Number of vertices: Both 5
Number of edges: Both 6
Degrees of corresponding vertices: H_1= 1 vertex with degree 5, 1 vertex with degree 3, 1 vertex with degree 2, 2 vertices with degree 1

H_2= 3 vertices with degree 3, 1 vertex with degree 2, 1 vertex with degree 1

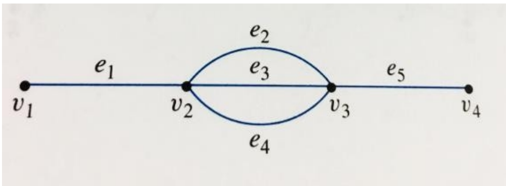
$\therefore Degrees\ of\ corresponding\ vertices\ of\ H_1\ and\ H_2\ are\ not\ same.\ H_1\ and\ H_2\ are\ not\ isomorphic.$

6. In the graph below, determine whether the following walks are trails, paths, closed walks, circuits/cycles, simple circuits or just walks.



a) $v_0e_1v_1e_{10}v_5e_9v_2e_2v_1$ = trail (no repeated edge but repeated vertex)
b) $v_4e_7v_2e_9v_5e_{10}v_1e_3v_2e_9v_5$ = walk (repeated edge and vertices)
c) v_2 = walk (only one vertex)
d) $v_5e_9v_2e_4v_3e_5v_4e_6v_4e_8v_5$ = closed trail (repeated vertices but no repeated edge and closed)
e) $v_2e_4v_3e_5v_4e_8v_5e_9v_2e_7v_4e_5v_3e_4v_2$ = closed walk (repeated vertices and edge and closed)
f) $v_3e_5v_4e_8v_5e_{10}v_1e_3v_2$ = path (no repeated vertex and edge)

7. Consider the following graph.



a) How many paths are there from v_1 to v_4?
3 paths
$v_1e_1v_2e_2v_3e_5v_4$, $v_1e_1v_2e_3v_3e_5v_4$, $v_1e_1v_2e_4v_3e_5v_4$

b) How many trails are there from v_1 to v_4?
3×2×1+3= 9
$v_1e_1v_2e_2v_3e_5v_4$
$v_1e_1v_2e_3v_3e_5v_4$
$v_1e_1v_2e_4v_3e_5v_4$

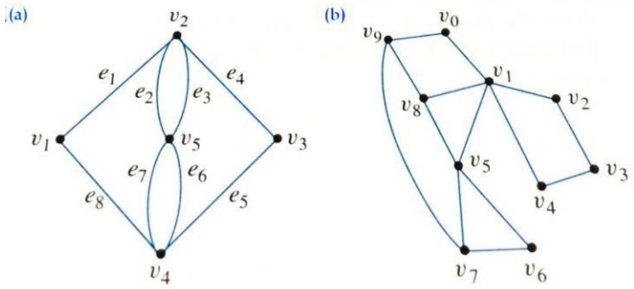
$v_1e_1v_2e_2v_3e_3v_2e_4v_3e_5v_4$
$v_1e_1v_2e_2v_3e_4v_2e_3v_3e_5v_4$

$v_1e_1v_2e_3v_3e_2v_2e_4v_3e_5v_4$
$v_1e_1v_2e_3v_3e_4v_2e_2v_3e_5v_4$

$v_1e_1v_2e_4v_3e_2v_2e_3v_3e_5v_4$
$v_1e_1v_2e_4v_3e_3v_2e_2v_3e_5v_3$

c) How many walks are there from v_1 to v_4?
Infinite,∞. Uncountable.

8. Determine which of the graphs in (a) – (b) have Euler circuits. If the graph does not have a Euler circuit, explain why not. If it does have a Euler circuit, describe one.



a.

Vertex	v_1	v_2	v_3	v_4	v_5
Degree	2	4	2	4	4

Euler circuit because every vertex has even degree.

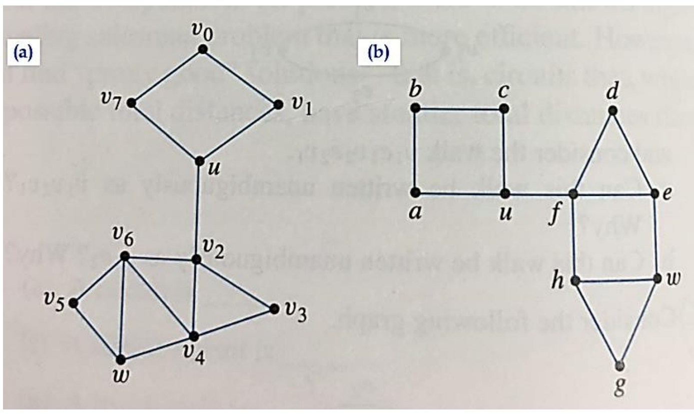
$(v_2, e_2, v_5, e_7, v_4, e_6, v_5, e_3, v_2, e_4, v_3, e_5, v_4, e_8, v_1, e_1, v_2)$

b.

Vertex	v_0	v_1	v_2	v_3	v_4	v_5	v_6	v_7	v_8	v_9
Degree	2	5	2	2	2	4	2	3	3	3

Don't have Euler circuit because there are 4 vertices have odd degree.

9. For each of graph in (a) – (b), determine whether there is an Euler path from u to w. If there is, find such a path.



a.

Vertex	u	v_0	v_1	v_2	v_3	v_4	v_5	v_6	v_7	w
Degree	3	2	2	4	2	4	2	4	2	3

Euler path because there have 2 vertices with odd degree.

$(u, v_7, v_0, v_1, u, v_2, v_6, v_5, w, v_6, v_4, v_2, v_3, v_4, w)$

b.

Vertex	u	a	b	c	d	e	f	g	h	w
Degree	3	2	2	2	2	3	4	2	3	3

No Euler path because there have 4 vertices with odd degree.

10. For each of graph in (a) – (b), determine whether there is Hamiltonian circuit. If there is, exhibit one.
Both are not Hamiltonian circuit. In a, the path needs to pass through twice at vertex u and v_2 while in b, the path needs to pass through twice at vertex u and f.

11. How many leaves does a full *3-ary* tree with 100 vertices have?

$$l = \frac{[(3-1)100+1]}{3} = 67$$

12. Find the following vertex/vertices in the rooted tree illustrated below.

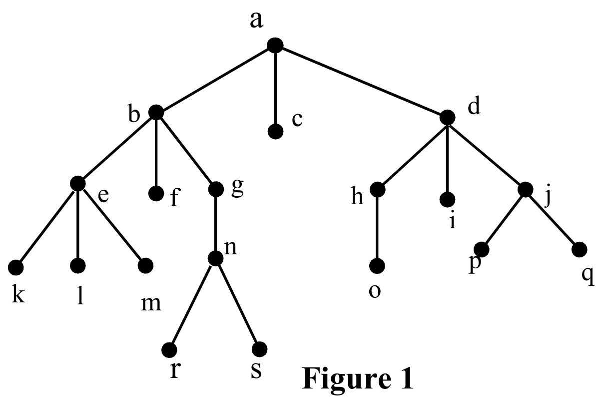
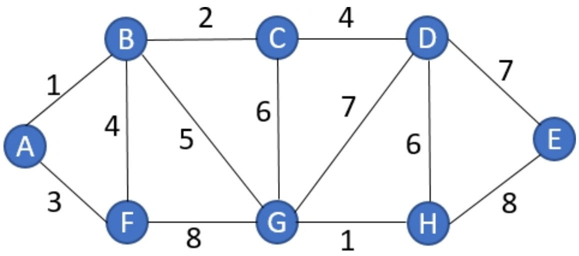


Figure 1

a) Root = a
b) Internal vertices = a, b, d, e, g, h, j, n
c) Leaves = c, f, i, k, l, m, r, s, o, p, q
d) Children of *n* = r and s
e) Parent of *e*= b
f) Siblings of *k* = l and m
g) Proper ancestors of *q* = a, d, j
h) Proper descendants of *b*= e, k, l, m, f, g, n, r, s

13. In which order are the vertices of ordered rooted tree in **Figure 1** is visited using *preorder, inorder* and
postorder.
Preorder = a, b, e, k, l, m, f, g, n, r, s, c, d, h, o, i, j, p, q
Inorder = k, e, l, m, b, f, g, r, n, s, a, c, o, h, d, i, p, j, q
Postoder = k, l, m, e, f, r, s, n, g, b, c, o, h, i, p, q, j, d, a

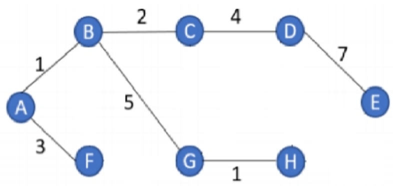
14. Find the minimum spanning tree for the following graph using Kruskal's algorithm.



AB 1
GH 1
BC 2
AF 3
BF 4(cancel)
CD 4
BG 5
CG 6(cancel)
DH 6(cancel)
DE 7
DG 7(cancel)
GF 8 (cancel)
EH 8(cancel)

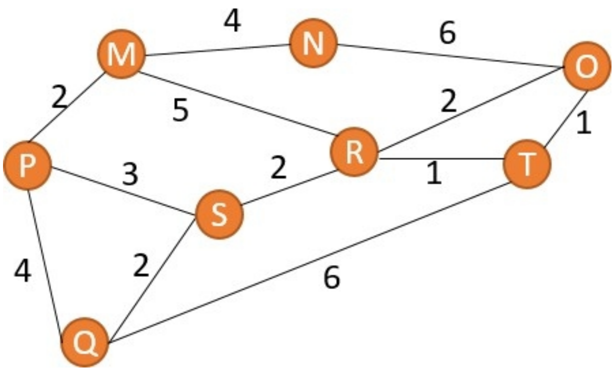
→

AB 1
GH 1
BC 2
AF 3
CD 4
BG 5
DE 7



∴MST = 1+1+2+3+4+5+7
= 23

15. Use Dijsktra's algorithm to find the shortest path from **M** to **T** for the following graph.



Iteration	S	N	L(M)	L(N)	L(O)	L(P)	L(Q)	L(R)	L(S)	L(T)
0	{}	{M, N, O, P, Q, R, S, T}	0	∞	∞	∞	∞	∞	∞	∞
1	{M}	{N, O, P, Q, R, S, T}	0	4	∞	2	∞	5	∞	∞
2	{M, P}	{N, O, Q, R, S, T}	0	4	∞	2	6	5	5	∞
3	{M, P, N}	{O, Q, R, S, T}	0	4	10	2	6	5	5	∞
4	{M, P, N, R}	{O, Q, S, T}	0	4	7	2	6	5	5	6
5	{M, P, N, R, S}	{O, Q, T}	0	4	7	2	6	5	5	6
6	{M, P, N, R, S, Q}	{O, T}	0	4	7	2	6	5	5	6
7	{M, P, N, R, S, Q, T}	{O}	0	4	7	2	6	5	5	6

∴Shortest length = 6, Shortest path = M→R→T