ASSIGNMENT 3
SECI1013: DISCRETE STRUCTURE
Group No: 6
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Question 1

a. i. $A - B = \{1,3,4,6,7,8\}$
ii. $(A \cap B) \cup C = \{2,5,a,b\}$
iii. $A \cap B \cap C = \emptyset$
iv. $B \times C = \{(2,a),(2,b),(5,a),(5,b),(9,a),(9,b)\}$
v. $P(C) = \{\emptyset, \{a\}, \{b\}, \{a,b\}\}$

b. $\left(P \cap \left((P' \cup Q)'\right)\right) \cup (P \cap Q)$
$= \left(P \cap (P'' \cap Q')\right) \cup (P \cap Q)$ *De Morgan's Law*
$= \left(P \cap (P \cap Q')\right) \cup (P \cap Q)$ *Double complement law*
$= ((P \cap P)Q') \cup (P \cap Q)$ *Associative law*
$= (P \cap Q') \cup (P \cap Q)$ *Idempotent laws*
$= P \cap (Q' \cup Q)$ *Distributive laws*
$= P \cap U$ *Complement laws*
$= P$ *Universal law*

c.

p	q	¬p	¬p ∨ q	(q → p)	(¬p ∨ q) ↔ (q → p)
T	T	F	T	T	T
T	F	F	F	T	F
F	T	T	T	F	F
F	F	T	T	T	T

d. p: x is odd, q: $(x+2)^2$ is odd
Assume p → q : TRUE
Let x is odd integer.
$x = 2n + 1$, where n is any integer
$(x+2)^2 = (2n+1+2)^2$
$= (2n+3)^2$
$= (2n+3)(2n+3)$
$= 4n^2 + 12n + 9$
$= 4n^2 + 12n + 8 + 1$
$= 2(2n^2 + 6n + 4) + 1$, *where* $m = 2n^2 + 6n + 4$, *m is an integer*
$= 2m + 1$ *is odd integer (Proven)*
p → q is TRUE

e. Let $x = 0, y = 0 \quad x \geq y \; True$
$x = 0, y = 1 \quad x \geq y \; False$
$x = 1, y = 0 \quad x \geq y \; True$
$x = 2, y = 3 \quad x \geq y \; False$

i. ∃x ∃y P(x, y) *is TRUE.*
ii. ∀x ∀y P(x, y) *is FALSE. Counterexamples are* $x = 0, y = 1$ *and* $x = 2$, $y = 3$ *that makes the proposition false.*

Question 2

a. i. Domain = {1,2,3}
Range = {1,2}

ii. Determine whether the relation is irreflexive and/or antisymmetric. Justify your answer.

- R is not irreflexive because the value of diagonal is not all 0.
- R is antisymmetric because :
- (a, b)∈R ∧ (b, a)∈R→ a=b
- (1,2)∈R but (2,1)∈R, and (1,1)∈R implies a=b

b. i. $S = \{(4,5), (5,4), (5,5)\}$

ii. **Is S reflexive, symmetric, transitive, and/or an equivalence relation? Justify your answer.**

- S is not reflexive because the value of diagonal is not 1.

$$M_R = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 1 \end{bmatrix} \qquad M_R^T = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 1 \end{bmatrix}$$

- $M_R = M_R^T$
- S is symmetric.

$$M_R \otimes M_R = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 1 \end{bmatrix} \otimes \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 1 \end{bmatrix} \neq \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 1 & 1 \end{bmatrix}$$

- $M_R \otimes M_R \neq M_R$
- S is not transitive.

∴ S is not equivalence relation because S is symmetric, not transitive and not reflexive.

c. i. f={(1,1), (2,2), (3,3)}
ii. g={(1,1), (2,2), (3,3), (4,3)}
iii.h={(1,1), (2,2), (3,2)}

d. i. $\mathrm{m}(x) = y$
$y = 4x + 3$
$y - 3 = 4x$
$x = \frac{y-3}{4}$
$m^{-1}(y) = \frac{y-3}{4}$
$\therefore m^{-1}(x) = \frac{x-3}{4}$

ii. $nm(x) = n(4x + 3)$
$= 2(4x + 3) - 4$
$= 8x + 6 - 4$
$= 8x + 2$

Question 3

a. i. $a_1 = 1$
$a_2 = 1 + 2(2) = 5$
$a_3 = 5 + 2(3) = 11$
$a_4 = 11 + 2(4) = 19$

ii. Input: k, k≥2
Output: f(k)

```
a(k) {
 if(k=1)
     return 1
return a(k-1) + 2(k)
```

b. $r_1 = 7$
$r_2 = 2(2 - 1)$
$= 2$
$r_k = 2r_{k-1}, k \geq 2$

c. S (1) = 5
S (2) = 5 × 5 =25
S (3) = 5 × 25 =125
S (4) = 5 × 125 =625

Question 4

a. first digit = 3, 4, 5, 6, 7, 8, 9, A, B = 9 ways
Second digit = 16 digits = 16 ways
Third digit = 16 digits = 16 ways
Fourth = 5, 6, 7, 8, 9, A, B, C, D, E, F = 11 ways
9 × 16 × 16 × 11= 25344

b. 1 26 26 26 9 10 1
26 × 26 × 26 × 9 × 10 = 1581840

c. Row 1 = 8
Row 2 = 8 × 7 = 56
Row 3 = 8 × 7 × 6 = 336
8 + 56 + 336 = 400

d. C (7,4) × C (6,3) = 700

e. $\frac{11!}{2!2!} = 9979200$

f. C (6+10-1,10) = C (15,10) = 3003

Question 5

a. |n| = 18 persons
k = {(Ali, Daud), (Ali, Elyas), (Bahar, Daud), (Bahar, Elyas), (Carlie, Daud), (Carlie, Elyas)}
|k| = 3 × 2
= 6 combinations

$$m = \left\lceil \frac{n}{k} \right\rceil$$
$$= \left\lceil \frac{18}{6} \right\rceil$$
$$= \lceil 3 \rceil$$
$$= 3$$

b. 10 odd integer 10 even integer
Pick at least 1 odd integer = 10 + 1 = 11

c. Integer that divisible by 5 = 20
Integer that not divisible by 5 = 80
Getting one that is divisible by 5 = 80 + 1 = 81