DISCRETE STRUCTURE ASSIGNMENT 1
Group No: 6
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Question 1

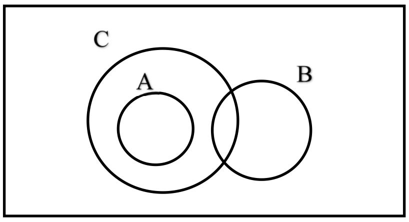
a. $A \cup C = \{x \in R|\ 0 < x \leq 2\ or\ 3 \leq x\ < 9\}$

b. $(A \cup B)' = \{\ \mathrm{x} \in R|\ 0 < x \leq 2\ or\ 1 \leq x\ < 4\}$
$= \{\ \mathrm{x} \in R|\ \neg(\ 0 < x < 4)\}$
$= \{\ \mathrm{x} \in R|\ x \leq\ 0\ or\ x\ \geq 4\}$

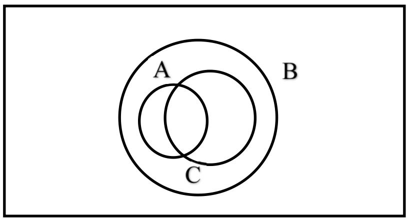
c. $A' \cup B' = \mathrm{x} \in R|\ \neg(\ 0 < x \leq 2\ )\ or\ \ \neg(1 \leq\ x\ < 4)$
$= x \in R|\ (x \leq 0\ or\ x > 2\ )or\ (x < 1\ or\ x \geq 4\ )\}$
$= \{\ \mathrm{x} \in R|x \leq 0\ or\ x > 2\}$

Question 2

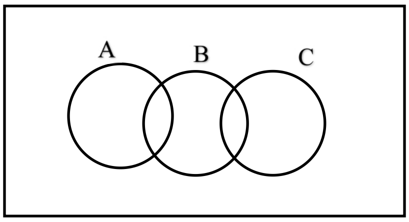
a. ∪



b. ∪



c. ∪



Question 3

$A \times B = \{(-1,1), (-1,2), (1,1), (1,2), (2,1), (2,2), (4,1), (4,2)\}$
$S = \{(-1,1), (1,1), (2,2)\}$
$T = \{(-1,1), (1,1), (2,2), (4,2)\}$
$S \cap T = \ \{(-1,1), (1,1), (2,2)\}$
$S \cup T = \{(-1,1), (1,1), (2,2), (4,2)\}$

Question 4

$\neg((\neg p \wedge q) \vee (\neg p \wedge \neg q)) \vee (p \wedge q)$

$= \neg(\neg p \wedge (q \vee \neg q)) \vee (p \wedge q)$ (Distributive law)

$= \neg(\neg p) \vee \neg(q \vee \neg q) \vee (p \wedge q)$ (De Morgan's law)

$= p \vee (\neg q \wedge q) \vee (p \wedge q)$ (Double negation law)

$= (p \vee \phi) \vee (p \wedge q)$ (Complement law)

$= p \vee (p \wedge q)$ (Properties of empty set)

$= p$ (Absorption law)

$\therefore \neg((\neg p \wedge q) \vee (\neg p \wedge \neg q) \vee (p \wedge q) \equiv q$

Question 5

a. $R_1 = \{(1,1),(1,2),(1,3),(1,4),(1,5),(2,1),(2,2),(2,3),(2,4),(3,1),(3,2),(3,3),(4,1),(4,2),(5,1)\}$

$$A_1 = \begin{bmatrix} 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 0 \\ 1 & 1 & 1 & 0 & 0 \\ 1 & 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 \end{bmatrix}$$

$b. R_2 = \{(2,1), (3,1), (3,2), (4,1), (4,2), (4,3), (5,1), (5,2), (5,3), (5,4)\}$

$$A_2 = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 \\ 1 & 1 & 0 & 0 & 0 \\ 1 & 1 & 1 & 0 & 0 \\ 1 & 1 & 1 & 1 & 0 \end{bmatrix}$$

c. Not reflexive. $\exists (x,x) \in \ \neq R_1, \forall x \in X, \forall x \in Y$.

Symmetric. $\forall x, y \ \in Y, Z \ , (x,y) \in R_1 \rightarrow (y,x) \in R_1$

Not transitive. $A_1 = \begin{bmatrix} 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 0 \\ 1 & 1 & 1 & 0 & 0 \\ 1 & 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 \end{bmatrix} \times \begin{bmatrix} 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 0 \\ 1 & 1 & 1 & 0 & 0 \\ 1 & 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 \end{bmatrix}$. $A_1 \times A_1 \neq A_1$.

d. Irreflexive. $\forall (x,x) \in \ \neq R_2 \, , \forall x \in X, \forall x \in Y$.

Antisymmetric. $(2,1) \in R_2 \ but \ (1,2) \notin R_2, (3,1) \in R_2 \ but \ (1,3) \notin R_2,$

$(4,1) \in R_2 \ but \ (1,4) \notin R_2, (5,1) \in R_2 \ but \ (1,5) \notin R_2, (3,2) \in R_2 \ but \ (2,3) \notin R_2.$

Not transitive. $A_2 = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 \\ 1 & 1 & 0 & 0 & 0 \\ 1 & 1 & 1 & 0 & 0 \\ 1 & 1 & 1 & 1 & 0 \end{bmatrix} \times \begin{bmatrix} 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 \\ 1 & 1 & 0 & 0 & 0 \\ 1 & 1 & 1 & 0 & 0 \\ 1 & 1 & 1 & 1 & 0 \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 \\ 1 & 1 & 0 & 0 & 0 \\ 1 & 1 & 1 & 0 & 0 \end{bmatrix}$. $A_2 \times A_2 \neq A_2$.

Question 6

$R_1 = \{(1,1), (2,2), (2,3), (3,1), (3,3)\}$
$R_2 = \{(1,2), (2,2), (3,1), (3,3)\}$

a. $M_{R_1 \cup R_2} = \begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & 1 \\ 1 & 0 & 1 \end{bmatrix}$

b. $M_{R_1 \cap R_2} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 1 & 0 \\ 1 & 0 & 1 \end{bmatrix}$

Question 7

Not True. Let f(R) = R, g(R) = -R, f (R) + g(R) = 0. Both only have image 0. Thus, it is not onto.

Question 8

$c_1 = 1$
$c_2 = 1$

$$c_n = c_{n-1} + c_{n-2}, n \geq 3$$

Question 9

a. $t_0 = 0$
$t_1 = 1$
$t_2 = 1$
$t_3 = t_2 + t_1 + t_0 = 1 + 1 + 1 = 2$
$t_4 = t_3 + t_2 + t_1 = 2 + 1 + 1 = 4$
$t_5 = t_4 + t_3 + t_2 = 4 + 2 + 1 = 7$
$t_6 = t_5 + t_4 + t_3 = 7 + 4 + 2 = 13$
$t_7 = t_6 + t_5 + t_4 = 13 + 7 + 4 = 24$

b. Input=n, n>=0

```
Output= t_n
t(n)
{
  if (n=1 or n=2)
   return 1
   else if (n=0)
   return 0
   return t_{n-1} + t_{n-2} + t_{n-3}
}
```