



**UTM**  
UNIVERSITI TEKNOLOGI MALAYSIA

FACULTY OF SOCIAL SCIENCES AND  
HUMANITIES

**DISCRETE STRUCTURE (SECI 1013)**

**2020/2021 – SEMESTER 1**

**ASSIGNMENT# 1**

**GROUP 2**

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**DISERAHKAN KEPADA:**

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1. Let the universal set be the set  $\mathbf{R}$  of all real numbers and let  $A=\{x \in \mathbf{R} \mid 0 < x \leq 2\}$ ,  $B=\{x \in \mathbf{R} \mid 1 \leq x < 4\}$  and  $C=\{x \in \mathbf{R} \mid 3 \leq x < 9\}$ . Find each of the following:

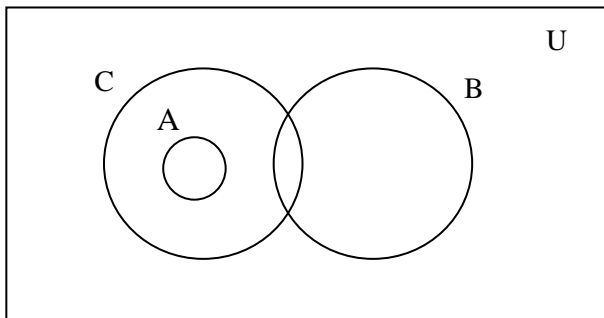
a)  $A \cup C = \{x \in \mathbf{R} \mid 0 < x < 9\}$ .

b)  $(A \cup B)' = \{x \in \mathbf{R} \mid 0 \leq x \text{ and } x > 3\}$

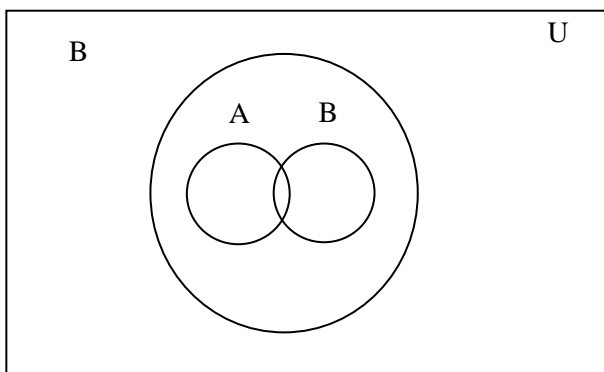
c)  $A' \cup B' = \{x \in \mathbf{R} \mid 2 \leq x \text{ and } x > 3\}$

2. Draw Venn diagrams to describe sets  $A$ ,  $B$ , and  $C$  that satisfy the given conditions.

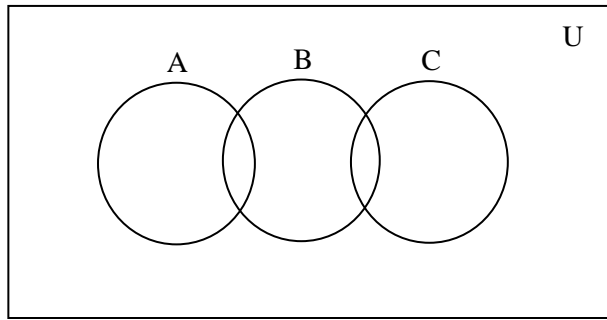
a)  $A \cap B = \emptyset, A \subseteq C, C \cap B \neq \emptyset$



b)  $A \subseteq B, C \subseteq B, A \cap C \neq \emptyset$



c)  $A \cap B \neq \emptyset, B \cap C \neq \emptyset, A \cap C = \emptyset, A \not\subseteq B, C \not\subseteq B$



3. Given two relations  $S$  and  $T$  from  $A$  to  $B$ ,

$$S \cap T = \{(x,y) \in A \times B \mid (x,y) \in S \text{ and } (x,y) \in T\}$$

$$S \cup T = \{(x,y) \in A \times B \mid (x,y) \in S \text{ or } (x,y) \in T\}$$

Let  $A = \{-1, 1, 2, 4\}$  and  $B = \{1, 2\}$  and defined binary relations  $S$  and  $T$  from  $A$  to  $B$  as follows:

For all  $(x,y) \in A \times B$ ,  $x S y \leftrightarrow |x| = |y|$

For all  $(x,y) \in A \times B$ ,  $x T y \leftrightarrow x - y$  is even

State explicitly which ordered pairs are in  $A \times B$ ,  $S$ ,  $T$ ,  $S \cap T$ , and  $S \cup T$ .

- $A \times B = \{(-1, 1), (-1, 2), (1, 1), (1, 2), (2, 1), (2, 2), (4, 1), (4, 2)\}$
- $S = \{(-1, 1), (1, 1), (2, 2)\}$
- $T = \{(-1, 1), (1, 1), (2, 2), (4, 2)\}$
- $S \cap T = \{(-1, 1), (1, 1), (2, 2)\}$
- $S \cup T = \{(-1, 1), (1, 1), (2, 2), (4, 2)\}$

4.

Show that  $\neg((\neg p \wedge q) \vee (\neg p \wedge \neg q)) \vee (p \wedge q) \equiv p$ . State carefully which of the laws are used at each stage.

From LHS:

$$\neg((\neg p \wedge q) \vee (\neg p \wedge \neg q)) \vee (p \wedge q) \equiv p$$

$$= (\neg(\neg p \wedge q) \wedge \neg(\neg p \wedge \neg q)) \vee (p \wedge q)$$

$$= (\neg\neg p \vee \neg q) \vee (\neg\neg p \vee \neg\neg q) \vee (p \wedge q)$$

$$= ((p \vee \neg q) \vee (p \vee q)) \vee (p \wedge q)$$

$$= p \vee (\neg q \vee q) \vee (p \wedge q)$$

$$= p \vee (p \wedge q)$$

$$= p$$

} De' Morgan's  
Law

→ Double negation

→ Distributive Law,  
complement law

→ Absorption Law

**5. a) the matrix  $A_1$  of the relation  $R_1$**

$$A_1 = \{(1,1), (1,2), (1,3), (1,4), (1,5), (2,1), (2,2), (2,3), (2,4), (3,1), (3,2), (3,3), (4,1), (4,2), (5,1)\}$$

$$A_1 = \begin{bmatrix} 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 0 \\ 1 & 1 & 1 & 0 & 0 \\ 1 & 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 \end{bmatrix}$$

**b) the matrix  $A_2$  of the relation  $R_2$**

$$A_2 = \{(2,1), (3,1), (3,2), (4,1), (4,2), (4,3), (5,1), (5,2), (5,3), (5,4)\}$$

$$A_2 = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 \\ 1 & 1 & 0 & 0 & 0 \\ 1 & 1 & 1 & 0 & 0 \\ 1 & 1 & 1 & 1 & 0 \end{bmatrix}$$

**c) is  $R_1$  reflexive, symmetrix, transitive and/or equivalence relation?**

-it's not reflexive because the diagonal does not all have value 1.

- its symmetrix for  $(x, y) \in R$ ,  $(y, x) \in R$ .

$$\begin{bmatrix} 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 0 \\ 1 & 1 & 1 & 0 & 0 \\ 1 & 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 \end{bmatrix} \times \begin{bmatrix} 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 0 \\ 1 & 1 & 1 & 0 & 0 \\ 1 & 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 \end{bmatrix}$$

$M_R \neq M_R$  so it is not transitive.

$\therefore$  it is not equivalence relation because it is not reflexive and not transitive.

**d) is reflexive , antisymmetric, transitive and/or a partial order relation.**

$\therefore$  it is not reflexive because the diagonal does bot all have value 1

$\therefore$  it is . antisymmetric for  $a \neq b$  ( $b, a$ ) *not belong to*  $R$

$$\begin{bmatrix} 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 \\ 1 & 1 & 0 & 0 & 0 \\ 1 & 1 & 1 & 0 & 0 \\ 1 & 1 & 1 & 1 & 0 \end{bmatrix} \times \begin{bmatrix} 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 \\ 1 & 1 & 0 & 0 & 0 \\ 1 & 1 & 1 & 0 & 0 \\ 1 & 1 & 1 & 1 & 0 \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 \\ 1 & 1 & 0 & 0 & 0 \\ 1 & 1 & 1 & 0 & 0 \end{bmatrix}$$

$\therefore$  it is not transitive because  $M_R \neq M_R$ .

$\therefore$  it is not partial order relation because it is not reflexive and not transitive

6.

$$R_1 = \begin{matrix} & \begin{matrix} 1 & 2 & 3 \end{matrix} \\ \begin{matrix} 1 \\ 2 \\ 3 \end{matrix} & \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 1 \\ 1 & 0 & 1 \end{bmatrix} \end{matrix} \quad R_1 = \{(1,1), (2,2), (2,3), (3,1), (3,3)\}$$

$$R_2 = \begin{matrix} & \begin{matrix} 1 & 2 & 3 \end{matrix} \\ \begin{matrix} 1 \\ 2 \\ 3 \end{matrix} & \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 1 \\ 1 & 0 & 1 \end{bmatrix} \end{matrix} \quad R_2 = \{(1,1), (2,2), (2,3), (3,1), (3,1)\}$$

(a) the matrix of relation  $R_1 \cup R_2$

$$R_1 \cup R_2 = \{(1,1), (1,2), (2,2), (2,3), (3,1), (3,3)\}$$

$$\begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & 1 \\ 1 & 0 & 1 \end{bmatrix}$$

(b) the matrix of relation  $R_1 \cap R_2$

$$R_1 \cap R_2 = \{(1,1), (2,2), (3,1), (3,3)\}$$

$$\begin{bmatrix} 0 & 0 & 0 \\ 0 & 1 & 0 \\ 1 & 0 & 1 \end{bmatrix}$$

7. If  $f: \mathbf{R} \rightarrow \mathbf{R}$  and  $g: \mathbf{R} \rightarrow \mathbf{R}$  are both one-to-one, is  $f + g$  also one-to-one? Justify your answer.

**Solution:**

$$(f + g)(x) = f(x) + g(x)$$

Let us assume;

$$(f + g)(a) = (f + g)(b) \Rightarrow \text{for some arbitrary } a, b \in \mathbf{R}$$

$$f(a) + g(a) = f(b) + g(b)$$

$$f(a) = f(b) \Rightarrow \text{because } f \text{ is one-to-one}$$

$$g(a) = g(b) \Rightarrow \text{because } g \text{ is one-to-one}$$

$$a = b$$

**$\therefore$  Since  $a, b \in \mathbf{R}$  were arbitrary, this means  $\forall a, b \in \mathbf{R}$  and  $a = b$  are proven.**

Therefore,  $f+g$  is one-to-one

8. With each step you take when climbing a staircase, you can move up either one stair or two stairs. As a result, you can climb the entire staircase taking one stair at a time, taking two at a time, or taking a combination of one- or two-stair increments. For each integer  $n \geq 1$ , if the staircase consists of  $n$  stairs, let  $c_n$  be the number of different ways to climb the staircase. Find a recurrence relation for  $c_1, c_2, \dots, c_n$ .

**Solution:**

Let  $c_n$  = number of different ways

$\Rightarrow$  When  $n = 1$ , move up 1 stair

$\Rightarrow$  When  $n = 2$ , move up 2 stairs

$\Rightarrow$  When  $n \geq 3$ , move up more than 2 steps so use both 1 and 2 stairs

Let when last step is 1 stair, then  $c_{n-1}$  ways to arrive

Let when last step is 2 stairs, then  $c_{n-2}$  ways to arrive

Since  $c_n$  is number of different ways,

$$c_1 = 1, c_2 = 2$$

$$c_n = c_{n-1} + c_{n-2}, \text{ when } n \geq 3$$

9. The Tribonacci sequence ( $t_n$ ) is defined by the equations,

$$t_0 = 0,$$

$$t_1 = t_2 = 1,$$

$$t_n = t_{n-1} + t_{n-2} + t_{n-3} \text{ for all } n \geq 3.$$

(a) Find  $t_7$

**Solution:**

$$t_3 = t_2 + t_1 + t_0$$

$$= 1 + 1 + 0$$

$$= 2$$

$$t_4 = t_3 + t_2 + t_1$$

$$= 2 + 1 + 1$$

$$= 4$$

$$t_5 = t_4 + t_3 + t_2$$

$$= 4 + 2 + 1$$

$$= 7$$

$$t_6 = t_5 + t_4 + t_3$$

$$= 7 + 4 + 2$$

$$= 13$$

$$\therefore t_7 = t_6 + t_5 + t_4$$

$$= 13 + 7 + 4$$

$$= 24$$

b) Write a recursive algorithm to compute  $t_n$ ,  $n \geq 3$ .

**Solution:**

Input :  $n$  , Output :  $t(n)$

$t(n)$

{ if ( $n = 1$  or  $n = 2$ )

    return 1

    return  $t(n-1) + t(n-2) + t(n-3)$ }



