



DISCRETE STRUCTURE (SECI 1013)

2020/2021 – SEMESTER 1

ASSIGNMENT# 1

GROUP 2

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1. Let the universal set be the set \mathbf{R} of all real numbers and let $A=\{x \in \mathbf{R} \mid 0 < x \leq 2\}$, $B=\{x \in \mathbf{R} \mid 1 \leq x < 4\}$ and $C=\{x \in \mathbf{R} \mid 3 \leq x < 9\}$. Find each of the following:

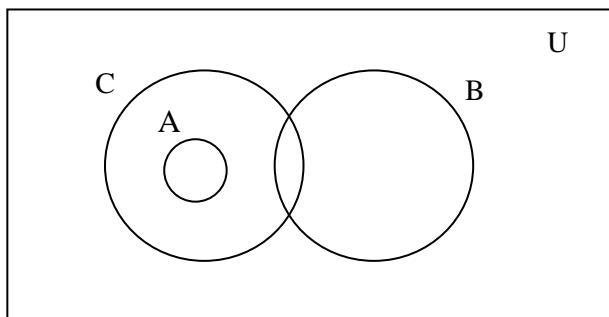
a) $A \cup C = \{x \in \mathbf{R} \mid 0 < x < 9\}$.

b) $(A \cup B)' = \{x \in \mathbf{R} \mid 0 \leq x \text{ and } x > 3\}$

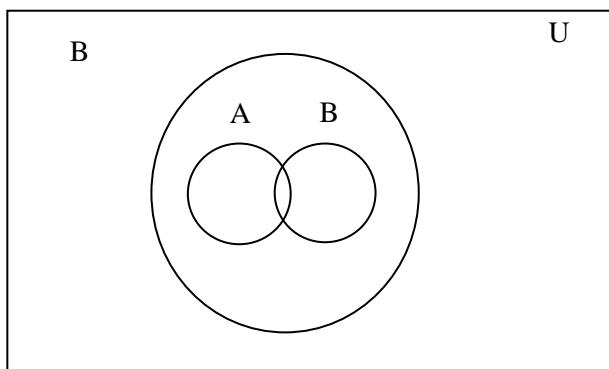
c) $A' \cup B' = \{x \in \mathbf{R} \mid 2 \leq x \text{ and } x > 3\}$

2. Draw Venn diagrams to describe sets A , B , and C that satisfy the given conditions.

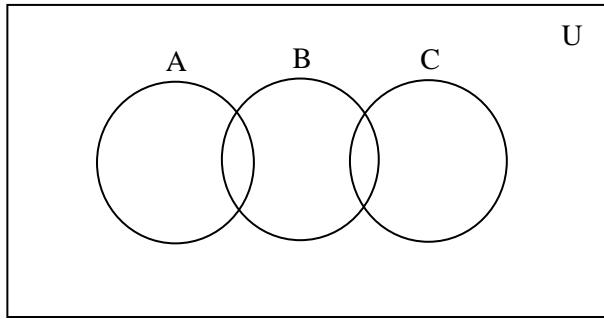
a) $A \cap B = \emptyset$, $A \subseteq C$, $C \cap B \neq \emptyset$



b) $A \subseteq B$, $C \subseteq B$, $A \cap C \neq \emptyset$



c) $A \cap B \neq \emptyset$, $B \cap C \neq \emptyset$, $A \cap C = \emptyset$, $A \not\subseteq B$, $C \not\subseteq B$



3. Given two relations S and T from A to B ,

$$S \cap T = \{(x,y) \in A \times B \mid (x,y) \in S \text{ and } (x,y) \in T\}$$

$$S \cup T = \{(x,y) \in A \times B \mid (x,y) \in S \text{ or } (x,y) \in T\}$$

Let $A = \{-1, 1, 2, 4\}$ and $B = \{1, 2\}$ and defined binary relations S and T from A to B as follows:

For all $(x,y) \in A \times B$, $x S y \leftrightarrow |x| = |y|$

For all $(x,y) \in A \times B$, $x T y \leftrightarrow x - y \text{ is even}$

State explicitly which ordered pairs are in $A \times B$, S , T , $S \cap T$, and $S \cup T$.

- $A \times B = \{(-1, 1), (-1, 2), (1, 1), (1, 2), (2, 1), (2, 2), (4, 1), (4, 2)\}$
- $S = \{(-1, 1), (1, 1), (2, 2)\}$
- $T = \{(-1, 1), (1, 1), (2, 2), (4, 2)\}$

- $S \cap T = \{(-1, 1), (1, 1), (2, 2)\}$
- $S \cup T = \{(-1, 1), (1, 1), (2, 2), (4, 2)\}$

4.

Show that $\neg((\neg p \wedge q) \vee (\neg p \wedge \neg q)) \vee (p \wedge q) \equiv p$. State carefully which of the laws are used at each stage.

From LHS:

$$\begin{aligned} & \neg((\neg p \wedge q) \vee \neg(\neg p \wedge \neg q)) \vee (p \wedge q) \equiv p \\ &= (\neg(\neg p \wedge q) \wedge \neg(\neg p \wedge \neg q)) \vee (p \wedge q) \\ &= (\neg\neg p \vee \neg q) \vee (\neg\neg p \vee \neg\neg q) \vee (p \wedge q) && \left. \begin{array}{l} \\ \\ \end{array} \right\} \text{De' Morgan's Law} \\ &= ((p \vee \neg q) \vee (p \vee q)) \vee (p \wedge q) && \xrightarrow{\quad} \text{Double negation} \\ &= p \vee (\neg q \vee q) \vee (p \wedge q) && \xrightarrow{\quad} \text{Distributive Law, complement law} \\ &= p \vee (p \wedge q) && \xrightarrow{\quad} \text{Absorption Law} \\ &= p \end{aligned}$$

5. a) the matrix A_1 of the relation R_1

$A_1 = \{(1,1), (1,2), (1,3), (1,4), (1,5), (2,1), (2,2), (2,3), (2,4), (3,1), (3,2), (3,3), (4,1), (4,2), (5,1)\}$

$$A_1 = \begin{bmatrix} 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 0 \\ 1 & 1 & 1 & 0 & 0 \\ 1 & 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 \end{bmatrix}$$

b) the matrix A_2 of the relation R_2

$A_2 = \{(2,1), (3,1), (3,2), (4,1), (4,2), (4,3), (5,1), (5,2), (5,3), (5,4)\}$

$$A_2 = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 \\ 1 & 1 & 0 & 0 & 0 \\ 1 & 1 & 1 & 0 & 0 \\ 1 & 1 & 1 & 1 & 0 \end{bmatrix}$$

c) is R_1 reflexive, symmetrix, transitive and/or equivalence relation?

-it's not reflexive because the diagonal does not all have value 1.

- its symmetrix for $(x, y) \in R, (y, x) \in R$.

$$\begin{bmatrix} 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 0 \\ 1 & 1 & 1 & 0 & 0 \\ 1 & 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 \end{bmatrix} \times \begin{bmatrix} 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 0 \\ 1 & 1 & 1 & 0 & 0 \\ 1 & 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 \end{bmatrix}$$

$M_R \neq M_R$ so it is not transitive.

\therefore it is not equivalence relation because it is not reflexive and not transitive.

d) is reflexive , antisymmetric, transitive and/or a partial order relation.

\therefore it is not reflexive because the diagonal does not all have value 1

\therefore it is . antisymmetric for $a \neq b$ (b, a) not belong to R

$$\begin{bmatrix} 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 \\ 1 & 1 & 0 & 0 & 0 \\ 1 & 1 & 1 & 0 & 0 \\ 1 & 1 & 1 & 1 & 0 \end{bmatrix} \times \begin{bmatrix} 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 \\ 1 & 1 & 0 & 0 & 0 \\ 1 & 1 & 1 & 0 & 0 \\ 1 & 1 & 1 & 1 & 0 \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 \\ 1 & 1 & 0 & 0 & 0 \\ 1 & 1 & 1 & 0 & 0 \end{bmatrix}$$

\therefore it is not transitive because $M_R \neq M_R$.

\therefore it is not partial order relation because it is not reflexive and not transitive

6.

$$R_1 = \begin{matrix} & \begin{matrix} 1 & 2 & 3 \end{matrix} \\ \begin{matrix} 1 \\ 2 \\ 3 \end{matrix} & \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 1 \\ 1 & 0 & 1 \end{bmatrix} \end{matrix} \quad R_1 = \{(1,1), (2,2), (2,3), (3,1), (3,3)\}$$

$$\begin{matrix} & \begin{matrix} 1 & 2 & 3 \end{matrix} \\ \begin{matrix} 1 \\ 2 \\ 3 \end{matrix} & \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 1 \\ 1 & 0 & 1 \end{bmatrix} \end{matrix} \quad R_2 = \{(1,1), (2,2), (2,3), (3,1), (3,1)\}$$

(a) the matrix of relation $R_1 \cup R_2$

$$R_1 \cup R_2 = \{(1,1), (1,2), (2,2), (2,3), (3,1), (3,3)\}$$

$$\begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & 1 \\ 1 & 0 & 1 \end{bmatrix}$$

(b) the matrix of relation $R_1 \cap R_2$

$$R_1 \cap R_2 = \{(1,1), (2,2), (3,1), (3,3)\}$$

$$\begin{bmatrix} 0 & 0 & 0 \\ 0 & 1 & 0 \\ 1 & 0 & 1 \end{bmatrix}$$

7. If $f: \mathbf{R} \rightarrow \mathbf{R}$ and $g: \mathbf{R} \rightarrow \mathbf{R}$ are both one-to-one, is $f + g$ also one-to-one? Justify your answer.

Solution:

$$(f + g)(x) = f(x) + g(x)$$

Let us assume;

$$(f + g)(a) = (f + g)(b) \Rightarrow \text{for some arbitrary } a, b \in \mathbf{R}$$

$$f(a) + g(a) = f(b) + g(b)$$

$$f(a) = f(b) \Rightarrow \text{because } f \text{ is one-to-one}$$

$$g(a) = g(b) \Rightarrow \text{because } g \text{ is one-to-one}$$

$$a = b$$

∴ Since $a, b \in \mathbf{R}$ were arbitrary, this means $\forall a, b \in \mathbf{R}$ and $a = b$ are proven.

Therefore, $f+g$ is one-to-one

8. With each step you take when climbing a staircase, you can move up either one stair or two stairs. As a result, you can climb the entire staircase taking one stair at a time, taking two at a time, or taking a combination of one- or two-stair increments. For each integer $n \geq 1$, if the staircase consists of n stairs, let c_n be the number of different ways to climb the staircase.

Find a recurrence relation for c_1, c_2, \dots, c_n .

Solution:

Let c_n = number of different ways

- ⇒ When $n = 1$, move up 1 stair
- ⇒ When $n = 2$, move up 2 stairs
- ⇒ When $n \geq 3$, move up more than 2 steps so use both 1 and 2 stairs

Let when last step is 1 stair, then c_{n-1} ways to arrive

Let when last step is 2 stairs, then c_{n-2} ways to arrive

Since c_n is number of different ways,

$$c_1 = 1, c_2 = 2$$

$$c_n = c_{n-1} + c_{n-2}, \text{ when } n \geq 3$$

9. The Tribonacci sequence (t_n) is defined by the equations,
 $t_0 = 0$,

$$t_1 = t_2 = 1,$$

$$t_n = t_{n-1} + t_{n-2} + t_{n-3} \text{ for all } n \geq 3.$$

(a) Find t_7

Solution:

$$t_3 = t_2 + t_1 + t_0$$

$$= 1 + 1 + 0$$

$$= 2$$

$$t_4 = t_3 + t_2 + t_1$$

$$= 2 + 1 + 1$$

$$= 4$$

$$t_5 = t_4 + t_3 + t_2$$

$$= 4 + 2 + 1$$

$$= 7$$

$$t_6 = t_5 + t_4 + t_3$$

$$= 7 + 4 + 2$$

$$= 13$$

$$\therefore t_7 = t_6 + t_5 + t_4$$

$$= 13 + 7 + 4$$

$$= 24$$

b) Write a recursive algorithm to compute t_n , $n \geq 3$.

Solution:

Input : n , Output : $t(n)$

$t(n)$

{if ($n = 1$ or $n = 2$)

 return 1

 return $t(n-1) + t(n-2) + t(n-3)$ }

