UNIVERSITI TEKNOLOGI MALAYSIA

Faculty of engineering (School of computing)
Discrete structure (SECI1013-10)

Topic: Assignment 3

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Question 1

a) $A = \{1, 2, 3, 4, 5, 6, 7, 8\}$

$B = \{2, 5, 9\}$

$C = \{a, b\}$

i) $= A - B = \{1, 2, 3, 4, 5, 6, 7, 8\} - \{2, 5, 9\}$

$= \{1, 3, 4, 6, 7, 8\}$

ii) $(A \cap B) \cup C$

$A \cap B = \{2, 5\}$

$(A \cap B) \cup C = \{a, b, 2, 5\}$

iii) $A \cap B \cap C$

$A \cap B \cap C = \emptyset$

iv) $B \times C = \{2, 5, 9\} \times \{a, b\}$

$= \{(2, a), (2, b), (5, a), (5, b), (9, a), (a, b)\}$

v) $P(C) = \{\emptyset, \{a\}, \{b\}, \{a, b\}\}$

b) $(P \cap ((P' \cup Q')') \cup (P \cap Q)$

$= (P \cap (P'' \cup Q'')) \cup (P \cap Q)$ — Double complement law

$= (P \cap (P \cup Q'')) \cup (P \cap Q)$ — absorption law

$= P \cup (P \cap Q)$ — absorption law

$\therefore = P$ shown//

c) $A = (\neg p \vee q) \leftrightarrow (q \to p)$

p	q	$\neg p$	$\neg p \vee q$	$q \to p$	$(\neg p \vee q) \leftrightarrow (q \to p)$
T	T	f	T	T	T
T	f	f	f	T	f
f	T	T	T	f	f
f	f	T	T	T	T

d) $P(x) = x$ is odd

$Q(x) = (x+2)^2$ is odd

$\forall x = (P(x) \to Q(x))$

$x = 2k+1$

$(x+2)^2 = (2k+3)^2$

$= 4k^2 + 12k + 9$

$2l + 1 = 4k^2 + 12k + 9$

$2l = 4k^2 + 12k + 8$

$l = 2k^2 + 6k + 4$

$\therefore 2l + 1 = 2(2k^2 + 6k + 4) + 1$

$= 2m + 1$, where $m = 2k^2 + 6k + 4$

If x is odd, then $(x+2)^2$ is odd

e) i) $\exists x \exists y \, P(x, y)$

let $x = 3$, $y = 1$

$3 \geq 1$ true

$\therefore$ This statement is true because there exist a set of values to prove the function $P(x, y)$

ii) $\forall x \forall y \, P(x, y)$

let $x = 3$, $y = 5$

$3 \geq 5$ false

$\therefore$ This statement is false because all set of values should prove the function $P(x, y)$

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Q2

a) $R\{(1,1),(1,2),(2,2),(3,1)\}$

i)

Domain $= \{1,2,3\}$

Range $= \{1,2\}$

ii) As $(1,1) \in R$

there is an element related to itself

R is not irreflexive

As we can see $(1,1)$ and $(2,2)$ in R. $(x,y) \in R$, $(y,x) \in R$

then $x = y$ is satisfied so R is antisymmetric

Q2 b)

i) $S = \{(4,5)\ (5,4)\ (5,5)\}$

ii)

$$M_s = \begin{array}{c|cccc} & 2 & 3 & 4 & 5 \\ \hline 2 & 0 & 0 & 0 & 0 \\ 3 & 0 & 0 & 0 & 0 \\ 4 & 0 & 0 & 0 & 1 \\ 5 & 0 & 0 & 1 & 1 \end{array}$$

Not all main diagonal mtrix elomens are 1 and the matrix is irreflexire

$$M_s^T = \begin{matrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 1 \end{matrix}$$

$M_s = M_s^T$ So S is symmetric

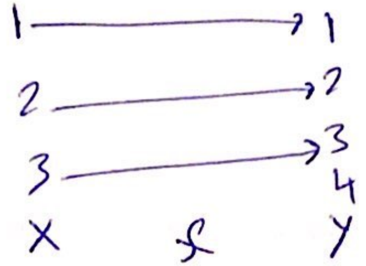
$$\begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 1 \end{bmatrix} \otimes \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 1 \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 1 \end{bmatrix}$$

The product of boolean show that the matrix is transitive.

S is not equiralence relation

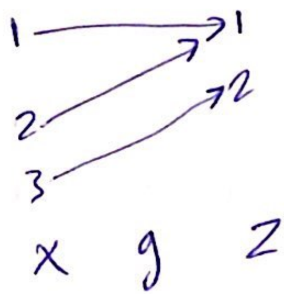
Q2

c) i)



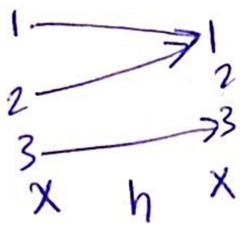
Define f b $f(x) = x$ f is clearly one to one, but it is not onto as it does not tak the value 4.

ii)



g is onto because the value 1 and 2 is not one to one as $g(1)=1$, $g(2)=1$ $g(3)=2$

iii)



h is not one to one as $h(1)=h(2)$ and it is not onto as it does not tak value 2.

Q2 d)

i) $m(x) = 4x + 3$

Let $(y, x) \in m^{-1}$; $m^{-1}(y) = x$

$(x, y) \in m$ $\quad y = 4x + 3$

$$x = \frac{y-3}{4}$$

$$m^{-1}(y) = \frac{y-3}{4}$$

ii) $n \circ m(x) = n(m(x))$

$\cancel{n(m(x)) = 4(2x - 4) + 3}$

$2(4x + 3) - 4$

$= 8x + 6 - 4$

$= 8x - 2$

Question 3

a) $a_k = a_{k-1} + 2k$, for all integers $k \geq 2$, $a_1 = 1$

i) $a_1 = 1$

$a_2 = 1 + 2 \cdot 2 = 5$

$a_3 = 5 + 2 \cdot 3 = 11$

$a_4 = 11 + 2 \cdot 4 = 19$

ii) recussive

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if (k == 1)
    return 0);
else
    return (arg(k-1) + 2k);
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b) r_k = number of operations when run by input of size k

- Input size 1, $r_1 = 5$

- input size x is 2 x number of size k-1, $r_k = 2r$

$k-1 \geq 1$ or $x \geq 2$

$r_k = 2r_k = 2 \cdot 2 r_{k-2}$

$= 2 \cdot 2 \cdot 2 \, r_{k-3}$

$2^3 \cdot r_{k-3}$

$r_k = 2^{x-1} \cdot r_1$

$r_k = 2^{x-1} \cdot r_1$

$r_k = 5 \cdot 2^{x-1}$

recussance relation

() return $4 \times 5(3) = 72$

return $3 \times 5(2) = 11$ $\Rightarrow 3 \cdot 2 \cdot 3 \cdot 4 = 72$

return $2 \times (1) = 6$

return $= 3$

No:

Date: ....

Q4

a) • Begin with one of the digits 3 through B
• End with one of the digits 5 through F
• 4 digits long

First digit = 9 ways (3, 4, 5, 6, 7, 8, 9, A, B)
Second digit = 16 ways (all the sixteen digits)
Third digit = 16 ways (all the sixteen digits)
Fourth digit = 11 ways (5, 6, 7, 8, 9, A, B, C, D, E, F)

$9 \times 16 \times 16 \times 11 = 25344$ hexadecimal numbers

b) • Four letters ; first A
• Three digits ; end 0

A 6 6 6 10 10 0

$6 \times 6 \times 6 \times 10 \times 10 = 21600$

c) • Computer
• no more than 3 letter
• no repetitions.

One letter word $= 8 \qquad = 8$
Two letter word $= 8 \times 7 \qquad = 56$
Three letter word $= 8 \times 7 \times 6 \qquad = 336$

$8 + 56 + 336 = 400$ arrangements

d) • 7 women, 6 men
• make group of 7 - 4 women & 3 men

${}^{7}C_4 \times {}^{6}C_3 = 700$ groups

e) • PROBABILITY = 11 letters

$$\frac{11!}{(2! \times 2!)} = 9979200 \text{ ways}$$

f) • Produce six kinds.
• Selection of ten patries.

$r = 10$

$n = 6$

$$= \frac{(n+r-1)!}{r!(n-1)!}$$

$$= \frac{(6+10-1)!}{10!(6-1)!}$$

$$= \frac{15!}{10!(5!)}$$

$$= 3003$$

Q 5

a) When n pegions are placed into K pegionhol then the exists a pegionhole with at least $\frac{n}{K}$ pegio

K= Number of combinations of first names and last names combinations.

$K = 2 \times 3 = 6$, there are 6 different first and second name

$n = 18$ person

$\frac{18}{6} = 3$ there are at least 3 people with first and last name.

b) odd integers = 1, 3, 5, 7, 9

number of that integers ~~tha~~ that is odd = 8

The probability of picking an odd integer is 0.5 in every pick. Hence if you pick 10 integers randomly there is a chance that all of them are even. So you have pick 11 integers to be sure that at least one of them is odd.

c) divisible by 5 = 5, 10, 15, 20, 25, 30, 35, 40, 45, 50, 55, 60, 65, 70, 75, 80, 85, 90, 95, 100

probability of integers that is odd = 20