SECI 1013

DISCRETE STRUCTURE

TUTORIAL 4

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NO.: Date:

1. condition: $|v-w| \le 3$

for 1 = $\{(1,2),(1,3),(1,4),(1,1)\} = 4$

for 10 = $\{10,10\} = 1$

for 2 = $\{(2,3),(2,4),(2,5),(2,2)\} = 4$

for 3 = $\{(3,4),(3,5),(3,6),(3,3)\} = 4$

for 4 = $\{(4,5),(4,6),(4,7),(4,4)\} = 4$

for 5 = $\{(5,6),(5,7),(5,8),(5,5)\} = 4$

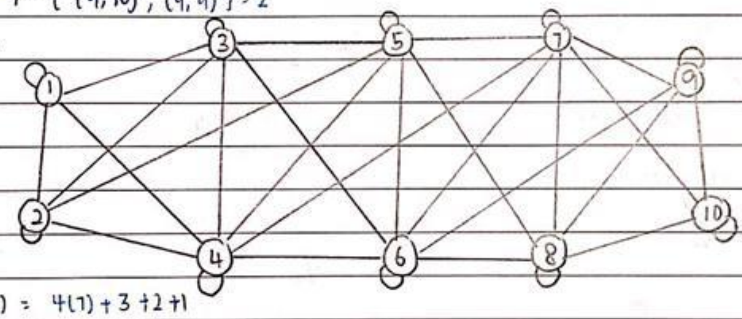
for 6 = $\{(6,7),(6,8),(6,9),(6,6)\} = 4$

for 7 = $\{(7,8),(7,9),(7,10),(7,7)\} = 4$

for 8 = $\{(8,9),(8,10),(8,8)\} = 3$

for 9 = $\{(9,10),(9,9)\} = 2$

Graph:



$e(G) = 4(7)+3+2+1$

$= 34$

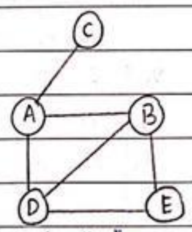
2. (a) A = Ahmad

B = Bakri

C = Chong

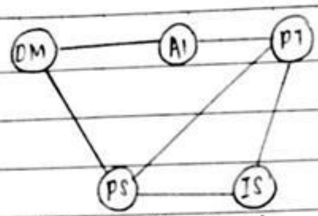
D = David

E = Ehsan



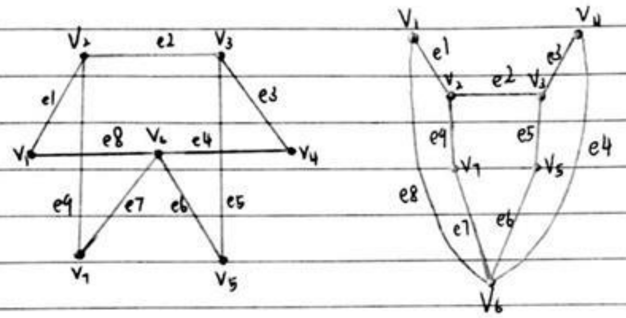
$$A_G = \begin{array}{c} \\ A \\ B \\ C \\ D \\ E \end{array} \begin{array}{c} \begin{array}{ccccc} A & B & C & D & E \end{array} \\ \begin{bmatrix} 0 & 1 & 1 & 1 & 0 \\ 1 & 0 & 0 & 1 & 1 \\ 1 & 0 & 0 & 0 & 0 \\ 1 & 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 1 & 0 \end{bmatrix} \end{array}$$

2. (b)



$$A_G = \begin{array}{c} \\ DM \\ AI \\ PT \\ PS \\ IS \end{array} \begin{array}{c} \begin{array}{ccccc} DM & AI & PT & PS & IS \end{array} \\ \begin{bmatrix} 0 & 1 & 0 & 1 & 0 \\ 1 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 1 & 1 \\ 1 & 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 1 & 0 \end{bmatrix} \end{array}$$

3.



4. Graph G

adjacency matrix:

$$\begin{array}{c} \\ V_1 \\ V_2 \\ V_3 \\ V_4 \end{array} \begin{array}{c} \begin{array}{cccc} V_1 & V_2 & V_3 & V_4 \end{array} \\ \begin{bmatrix} 1 & 0 & 2 & 0 \\ 0 & 0 & 1 & 0 \\ 2 & 1 & 0 & 2 \\ 0 & 0 & 2 & 0 \end{bmatrix} \end{array}$$

incidence matrix:

$$\begin{array}{c} \\ V_1 \\ V_2 \\ V_3 \\ V_4 \end{array} \begin{array}{c} \begin{array}{cccccc} e1 & e2 & e3 & e4 & e5 & e6 \end{array} \\ \begin{bmatrix} 2 & 1 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 1 & 1 & 1 & 1 & 1 \\ 0 & 0 & 0 & 0 & 1 & 1 \end{bmatrix} \end{array}$$

Graph H

adjacency matrix:

$$\begin{array}{c} \\ V_1 \\ V_2 \\ V_3 \\ V_4 \end{array} \begin{array}{c} \begin{array}{cccc} V_1 & V_2 & V_3 & V_4 \end{array} \\ \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 1 & 2 \\ 0 & 1 & 1 & 0 \\ 0 & 2 & 0 & 0 \end{bmatrix} \end{array}$$

Incidence matrix:

$$\begin{array}{c} \\ V_1 \\ V_2 \\ V_3 \\ V_4 \end{array} \begin{array}{c} \begin{array}{cccccc} e1 & e2 & e3 & e4 & e5 & e6 \end{array} \\ \begin{bmatrix} 2 & 0 & 0 & 0 & 0 & 0 \\ 0 & 2 & 1 & 1 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 & 2 \\ 0 & 0 & 1 & 1 & 0 & 0 \end{bmatrix} \end{array}$$

NO.: Date:

5. (a) Both have 5 vertices and 5 edges.

Both have 3 vertices with 2 degree and 1 vertices with 3 degree and 1 vertices with 1 degree.

$f(V_1) = u$, $f(V_2) = u_5$, $f(V_3) = u_2$, $f(V_4) = u_4$, $f(V_5) = u_1$

adjucency matrix for G1:

$$\begin{array}{c} \\ V_1 \\ V_2 \\ V_3 \\ V_4 \\ V_5 \end{array} \begin{array}{c} \begin{matrix} V_1 & V_2 & V_3 & V_4 & V_5 \end{matrix} \\ \begin{bmatrix} 0 & 1 & 0 & 1 & 0 \\ 1 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 1 & 1 \\ 1 & 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \end{bmatrix} \end{array}$$

adjacency matrix for G2:

$$\begin{array}{c} \\ U_5 \\ U_4 \\ U_2 \\ U_3 \\ U_1 \end{array} \begin{array}{c} \begin{matrix} U_5 & U_4 & U_2 & U_3 & U_1 \end{matrix} \\ \begin{bmatrix} 0 & 1 & 0 & 1 & 0 \\ 1 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 1 & 1 \\ 1 & 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \end{bmatrix} \end{array}$$

Both adjucency matrix is the same.

G1 and G2 are isomorphic.

(b) Both have 5 vertices and 6 edges.

Graph H_1 has 2 vertices with 1 degree, 1 vertices with 2 degree, 1 vertice 3 degree and 1 vertice with 5 degree.

Graph H_2 has 1 vertices with 1 degree, 1 vertices with 2 degree and 3 vertices with 3 degree.

H_1 and H_2 are not isomorphic since the degree of vertices are different.

6. a) trails

b) walk

c) walk (trivial walk)

d) circuits

e) closed walk

f) path

7. a) $V_1 e_1 V_2 e_3 V_3 e_5 V_4$

$V_1 e_1 V_2 e_2 V_3 e_5 V_4$

$V_1 e_1 V_2 e_4 V_3 e_5 V_4$ Total path = 3

b) $V_1 e_1 V_2 e_2 V_3 e_4 V_2 e_3 V_3 e_5 V_4$ $V_1 e_1 V_2 e_3 V_3 e_5 V_4$ $V_1 e_1 V_2 e_4 V_3 e_3 V_2 e_2 V_3 e_5 V_4$

$V_1 e_1 V_2 e_4 V_3 e_2 V_2 e_3 V_3 e_5 V_4$ $V_1 e_1 V_2 e_2 V_3 e_5 V_4$

$V_1 e_1 V_2 e_3 V_3 e_2 V_2 e_4 V_3 e_5 V_4$ $V_1 e_1 V_2 e_4 V_3 e_5 V_4$ Total trails = 9

 $V_1 e_1 V_2 e_3 V_3 e_4 V_2 e_2 V_3 e_5 V_4$ $V_1 e_1 V_2 e_2 V_3 e_3 V_2 e_4 V_3 e_5 V_4$

c) infinitely number of walk

8\. (a) is a Euler circuit as every vertex has positive even degree, 2 vertices with 2 degree, 3 vertices with 4 degree.
The Euler circuit is $V_2 e_1 V_1 e_8 V_4 e_5 V_3 e_4 V_2 e_2 V_5 e_7 V_4 e_6 V_5 e_3 V_2$.

(b) is not a Euler circuit as it does not consist every vertex with positive even degree.
It consist of 5 vertices with 2 degree, 3 vertices with 3 degree, 1 vertices with 4 degree and 1 vertices 5 degree.

9\. (a) is a Euler path as it consist of 2 vertex with odd degree and all others are positive even degree. The Euler path is $U, V_7, V_0, V_1, u, V_2, V_3, V_4, V_2, V_6, V_4, W, V_6, V_5, W$

(b) is not a Euler path as it consist of 4 vertex with odd degree.

10\. (a) has Hamiltonian circuit which is $V_0, V_6, V_5, V_4, V_3, V_2, V_1, V_7, V_0$. (b) has no Hamiltonian circuit.

11\. $n = 100$

$m = 3$

$$l = \frac{[(3-1)100+1]}{3}$$

$= 67$

$\therefore$ The tree have 67 leaves.

12\. a) root = a, b, d, e, g, h, j, n

b) internal vertices = a, b, d, e, g, h, j, n

c) leaves = c, f, i, k, l, m, o, p, q, r, s

d) children of n = r, s

e) parent of e = b

f) siblings of k = l, m

g) proper ancestors of q = a, d, j, q

h) proper descendants of b = b, e, f, g, k, l, m, n, r, s

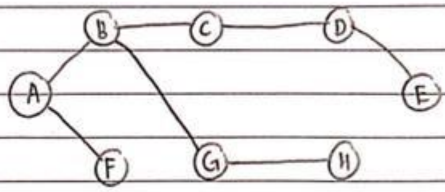
13\. a) preorder = a, b, e, k, l, m, f, g, n, r, s, c, d, h, o, i, j, p, q

inorder = k, e, l, m, b, f, r, n, s, g, a, c, o, h, d, i, p, j, q

postorder = k, l, m, e, f, r, s, n, g, b, c, o, h, i, p, q, j, d, a

14. AB = 1 Choose: AB = 1

AB = 1	AB = 1
GH = 1	GH = 1
BC = 2	BC = 2
AF = 3	AF = 3
BF = 4	CD = 4
CD = 4	BG = 5
BG = 5	DE = 7
CG = 6	
DH = 6	
DG = 7	
DE = 7	
FG = 8	
EH = 8	



Total weight = 1+1+2+3+4+5+7
= 23

15.

iteration	S	N	L(M)	L(N)	L(O)	L(P)	L(Q)	L(R)	L(S)	L(T)
1.	{ }	{M,N,O,P,Q,R,S,T}	0	∞	∞	∞	∞	∞	∞	∞
2.	{M}	{N,O,P,Q,R,S,T}	0	4	∞	2	∞	5	∞	∞
3.	{M,P}	{N,O,Q,R,S,T}	0	4	∞	2	6	5	5	∞
4.	{M,P,N}	{O,Q,R,S,T}	0	4	10	2	6	5	5	∞
5.	{M,P,N,R}	{O,Q,S,T}	0	4	7	2	6	5	5	6
6.	{M,P,N,R,S}	{O,Q,T}	0	4	7	2	6	5	5	6
7.	{M,P,N,R,S,T}	{O,Q}	0	4	7	2	6	5	5	6

shortest path is 6, M-R-T.