SECI 1013

DISCRETE STRUCTURE

TUTORIAL 3

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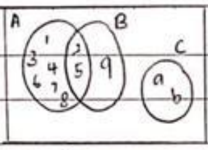
1. a)



i) $A - B = \{1, 3, 4, 6, 7, 8\}$

ii) $(A \cap B) \cup C = \{2, 5\} \cup \{a, b\}$
$= \{2, 5, a, b\}$

iii) $A \cap B \cap C = \phi$

iv) $B \times C = \{2, 5, 9\} \times \{a, b\}$
$= \{(2, a), (2, b), (5, a), (5, b), (9, a), (9, b)\}$

v) $P(C) = \{\phi, \{a\}, \{b\}, \{a, b\}\}$

b) $(P \cap ((P' \cup Q)')) \cup (P \cap Q)$

$= (P \cap ((P')' \cap Q')) \cup (P \cap Q)$ De Morgan's law

$= (P \cap (P \cap Q')) \cup (P \cap Q)$ Double complement law

$= ((P \cap P) \cap Q') \cup (P \cap Q)$ Associative law

$= (P \cap Q') \cup (P \cap Q)$ idempotent law

$= P \cap (Q' \cup Q)$ distributive law

$= P \cap U$ complement law

$= P$ properties of universal set

c)

p	q	$\neg p \vee q$	$q \to p$	A
T	T	T	T	T
T	F	F	T	F
F	T	T	F	F
F	F	T	T	T

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d) For all integer x, if x is odd, then $(x+2)^2$ is odd.

let $P(x)$ = x is odd

$Q(x)$ = $(x+2)^2$ is odd.

$\forall x\ (P(x) \rightarrow Q(x))$

x is odd, $x = 2n+1$

$(x+2)^2$ is odd, $(x+2)^2 = 2m+1$

$(2n+1+2)^2 = (2n+3)(2n+3)$

$= 4n^2 + 6n + 6n + 9$

$= 4n^2 + 12n + 8 + 1$

$= 2(2n^2 + 6n + 4) + 1$

$= 2m+1$

$\therefore (x+2)^2$ is odd is proved.

e) i) $\exists x \exists y\ P(x,y)$

$=$ TRUE

When $x = 2$, $y = 1$, the statement is true.

ii) $\forall x \forall y\ P(x,y)$

$=$ FALSE

when $x = 2$, $y = 1$, the statement is false.

2. a) i) $R = \{(1,1), (1,2), (2,2), (3,1)\}$

Domain $= \{1, 2, 3\}$

Range $= \{1, 2\}$

ii) the relation is not irreflexive, it is not reflexive as it consist of $(1,1)$ and $(2,2) \in R$, but $(3,3) \notin R$

the relation is antisymmetric.

$(1,2) \in R$, $(2,1) \notin R$

$(3,1) \in R$, $(1,3) \notin R$

b) i) $S = \{ (4,5), (5,4), (5,5) \}$

ii)
$$S = \begin{array}{c|cccc} & 2 & 3 & 4 & 5 \\ \hline 2 & 0 & 0 & 0 & 0 \\ 3 & 0 & 0 & 0 & 0 \\ 4 & 0 & 0 & 0 & 1 \\ 5 & 0 & 0 & 1 & 1 \end{array}$$

S is not reflexive since $(2,2)$, $(3,3)$ and $(4,4) \notin S$

$$\begin{array}{c|cccc} & 2 & 3 & 4 & 5 \\ \hline 2 & 0 & 0 & 0 & 0 \\ 3 & 0 & 0 & 0 & 0 \\ 4 & 0 & 0 & 0 & 1 \\ 5 & 0 & 0 & 1 & 1 \end{array} = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 1 \end{bmatrix}$$

S is symmetric, since $M_S = M_S^T$

$$\begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 1 \end{bmatrix} \otimes \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 1 \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 1 & 1 \end{bmatrix}$$

S is not transitive, since $M_S \otimes M_S \neq M_S$

S is not equivalence relation as it is not reflexive and not transitive.

c) i) $f = \{ (1,2), (2,3), (3,4) \}$

ii) $f = \{ (1,1), (1,2), (2,1), (3,2) \}$

iii) $f = \{ (1,1), (1,2), (2,1), (2,2), (3,2) \}$

d) i) $4x + 3 = y$

$4x = y - 3$

$x = \frac{y-3}{4}$

$f^{-1}(y) = \frac{y-3}{4}$

ii) nom $= 2(4x+3) - 4$

$= 8x + 6 - 4$

$= 8x + 2$

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3. a) i) $a_1 = 1$

$a_2 = a_{2-1} + 2(2)$

$= a_1 + 4$

$= 1 + 4$

$= 5$

$a_3 = a_{3-1} + 2(3)$

$= a_2 + 6$

$= 5 + 6$

$= 11$

$\therefore$ first three terms is 1, 5, 11

ii) a(k)

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{ if k=1
    return 1
  return a(k-1)+2k }
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b) $r_1 = 7$ is the initial conditions

$r_k = 2a_{k-1}$ when $k \geq 2$

c) $S(1) = 5$

$S(2) = 5 * S(2-1)$

$= 5 * S(1)$

$= 5 * 5$

$= 25$

$S(3) = 5 * S(3-1)$

$= 5 * S(2)$

$= 5 * 25$

$= 125$

$S(4) = 5 * S(4-1)$

$= 5 * S(3)$

$= 5 * 125$

$= 625$

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4. a) 3 to B = { 3, 4, 5, 6, 7, 8, 9, A, B }

= 9

5 to F = { 5, 6, 7, 8, 9, A, B, C, D, E, F }

= 11

total hexadecimal numbers can form = $9 \times 16 \times 16 \times 11$

= 25344

b) A to Z = 26

0 to 9 = 10

total licence plates = $1 \times 26 \times 26 \times 26 \times 10 \times 10 \times 1$

= 1757600

c) 1 letter = 8

2 letter = 8×7

= 56

3 letter = $8 \times 7 \times 6$

= 336

total arrangements = 8 + 56 + 336

= 400

d) $^7C_4 \times {}^6C_3 = \frac{7!}{4!(7-4)!} \times \frac{6!}{3!(6-3)!}$

$= 35 \times 20$

$= 700$

e) $\frac{11!}{2! \cdot 2!} = 9979200$

f) $\frac{(6+10-1)!}{10!\,5!} = 3003$

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5. a) pigeon, n = 18 people

3 first name and 2 last name = 3x2

= 6

pigeonhole, k = 6

$\left\lceil \frac{18}{6} \right\rceil = 3$

b) odd integer = { 1, 3, 5, 7, 9, 11, 13, 15, 17, 19 }

= 10

even integer = { 2, 4, 6, 8, 10, 12, 14, 16, 18, 20 }

= 10

for worst condition, the person may pick all even number first. hence to get at least 1 odd number the person must pick 11 integer.

c) integers divisible by 5 = { 5, 10, 15, 20, 25, 30, 35, 40, 45, 50, 55, 60, 65, 70, 75, 80, 85, 90, 95, 100 }

= 20

integer not divisible by 5 = 100 - 20

= 80

for worst condition, the person may pick all integer not divisible by 5 first, hence to sure get integer divisible by 5, the person need to pick 81 integer.