SECI 1013

DISCRETE STRUCTURE

TUTORIAL 1

LECTURER:

Dr Razana Alwee

SUBMITTED BY:

Amir Iskandar Bin Norkhairulazaddin (A20EC0011)

Mek Zhi Qing (A20EC0077)

Roshini Thavan Nair (A20EC0139)

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1. $A = \{x \in R \mid 0 < x \leq 2\}$

$B = \{x \in R \mid 1 \leq x < 4\}$

$C = \{x \in R \mid 3 \leq x < 9\}$

a) $A \cup C = \{x \in R \mid 0 < x \leq 2 \text{ or } 3 \leq x < 9\}$

$= \{x \in R \mid 0 < x < 9\}$

b) $(A \cup B)' = \{x \in R \mid \neg(0 < x < 4)\}$

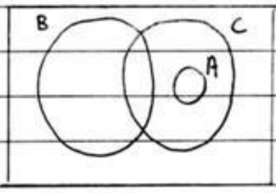
$= \{x \in R \mid x \leq 0 \text{ or } x \geq 4\}$

c) $A' \cup B' = \{x \in R \mid 0 < x \leq 2\}' \cup \{x \in R \mid 1 \leq x < 4\}'$

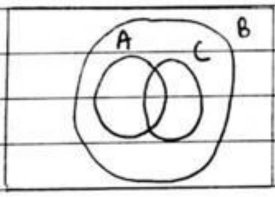
$= \{x \in R \mid x \leq 0 \text{ or } x > 2\} \cup \{x \in R \mid x < 1 \text{ or } x \geq 4\}$

$= \{x \in R \mid x < 1 \text{ or } x > 2\}$

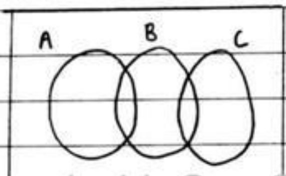
2. (a) $A \cap B = \emptyset$, $A \subsetneq C$, $C \cap B \neq \emptyset$



(b) $A \subseteq B$, $C \subseteq B$, $A \cap C \neq \emptyset$



(c) $A \cap B \neq \emptyset$, $B \cap C \neq \emptyset$, $A \cap C \neq \emptyset$, $A \not\subset B$, $C \not\subset B$



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3. $A = \{-1, 1, 2, 4\}$

$B = \{1, 2\}$

$S = \{(-1,1), (1,1), (2,2)\}$

$T = \{(-1,1), (1,1), (2,2), (4,2)\}$

$A \times B = \{(-1,1), (-1,2), (1,1), (1,2), (2,1), (2,2), (4,1), (4,2)\}$

$S \cap T = \{(-1,1), (1,1), (2,2)\}$

$S \cup T = \{(-1,1), (1,1), (2,2), (4,2)\}$

4. $\neg((\neg p \wedge q) \vee (\neg p \wedge \neg q)) \vee (p \wedge q)$

$= \neg(\neg p \wedge (q \vee \neg q)) \vee (p \wedge q)$ distributive law

$= \neg(\neg p \wedge U) \vee (p \wedge q)$ complement law

$= \neg(\neg p) \vee (p \wedge q)$ identity laws

$= p \vee (p \wedge q)$ double complement law

$= p$ absorption law

5. (a) $R_1 = \{(1,1), (1,2), (1,3), (1,4), (1,5), (2,1), (2,2), (2,3), (2,4), (3,1), (3,2), (3,3), (4,1), (4,2), (5,1)\}$

$$A_1 = \begin{array}{c c} & \begin{array}{ccccc} 1 & 2 & 3 & 4 & 5 \end{array} \\ \begin{array}{c} 1 \\ 2 \\ 3 \\ 4 \\ 5 \end{array} & \begin{bmatrix} 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 0 \\ 1 & 1 & 1 & 0 & 0 \\ 1 & 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 \end{bmatrix} \end{array}$$

(b) $R_2 = \{(2,1), (3,1), (3,2), (4,1), (4,2), (4,3), (5,1), (5,2), (5,3), (5,4)\}$

$$A_2 = \begin{array}{c c} & \begin{array}{cccc} 1 & 2 & 3 & 4 \end{array} \\ \begin{array}{c} 2 \\ 3 \\ 4 \\ 5 \end{array} & \begin{bmatrix} 1 & 0 & 0 & 0 \\ 1 & 1 & 0 & 0 \\ 1 & 1 & 1 & 0 \\ 1 & 1 & 1 & 1 \end{bmatrix} \end{array}$$

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c) R_1 is not reflexive. because (4,4) and (5,5) $\notin R_1$.

R_1 is symmetric.

$$A_1 T = \begin{bmatrix} 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 0 \\ 1 & 1 & 1 & 0 & 0 \\ 1 & 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 \end{bmatrix} = A_1$$

R_1 is not transitive.

$$\begin{bmatrix} 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 0 \\ 1 & 1 & 1 & 0 & 0 \\ 1 & 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 \end{bmatrix} \otimes \begin{bmatrix} 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 0 \\ 1 & 1 & 1 & 0 & 0 \\ 1 & 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 \end{bmatrix}$$

$$A_1 \otimes A_1 \neq A_1$$

$\therefore$ R_1 is not equivalence relation as it is not reflexive and not transitive.

(d) R_2 is irreflexive because all (1,1), (2,2), (3,3), (4,4), (5,5) $\notin R_2$.

R_2 is antisymmetric.

$(2,1) \in R_2$, $(1,2) \notin R_2$

$(3,1) \in R_2$, $(1,3) \notin R_2$

$(3,2) \in R_2$, $(2,3) \notin R_2$

$(4,1) \in R_2$, $(1,4) \notin R_2$

$(4,2) \in R_2$, $(2,4) \notin R_2$

$(4,3) \in R_2$, $(3,4) \notin R_2$

$(5,1) \in R_2$, $(1,5) \notin R_2$

$(5,2) \in R_2$, $(2,5) \notin R_2$

$(5,3) \in R_2$, $(3,5) \notin R_2$

$(5,4) \in R_2$, $(4,5) \notin R_2$

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 1 & 1 & 0 & 0 \\ 1 & 1 & 1 & 0 \\ 1 & 1 & 1 & 1 \end{bmatrix} \otimes \begin{bmatrix} 1 & 0 & 0 & 0 \\ 1 & 1 & 0 & 0 \\ 1 & 1 & 1 & 0 \\ 1 & 1 & 1 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 1 & 1 & 0 & 0 \\ 1 & 1 & 1 & 0 \\ 1 & 1 & 1 & 1 \end{bmatrix}$$

R_2 is transitive since $A_2 \otimes A_2 = A_2$

R_2 is not partial order because it is irreflexive.

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6. $R_1 = \{(1,1), (1,2), (2,3), (3,1), (3,3)\}$
$R_2 = \{(1,2), (2,2), (3,1), (3,3)\}$

a) $R_1 \cup R_2 = \{(1,1), (1,2), (2,2), (2,3), (3,1), (3,3)\}$

$$\begin{array}{c} \\ 1 \\ 2 \\ 3 \end{array}\begin{array}{c} \begin{array}{ccc} 1 & 2 & 3 \end{array} \\ \begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & 1 \\ 1 & 0 & 1 \end{bmatrix} \end{array}$$

b) $R_1 \cap R_2 = \{(2,2), (3,1), (3,3)\}$

$$\begin{array}{c} \\ 2 \\ 3 \end{array}\begin{array}{c} \begin{array}{ccc} 1 & 2 & 3 \end{array} \\ \begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & 1 \end{bmatrix} \end{array}$$

7. if $f(x) = x$ and $g(2) = -x$, then all $(f+g)(x) = 0$
for example $f(1) = 1$ and $g(1) = -1$, $(f+g)(1) = 0$
$f(3) = 3$ and $g(3) = -3$, $(f+g)(3) = 0$
Both $(f+g)(1)$ and $(f+g)(3) = 0$ but $1 \neq 3$, hence $f+g$ is not one to one function.

8. n is number of stair.
$C_1 = 1$ as it only consist of one way to move up the staircase.
$C_2 = 2$ as it can move up by taking one stair or two stair which is two way.
$C_n = C_{n-1} + C_{n-2}$ as C_n is the total number of way to climb the staircase.
Hence the recurrence relation is $C_n = C_{n-1} + C_{n-2}$, $n \geq 3$ where $C_1 = 1$ and $C_2 = 2$ is the initial conditions.

9. a) $t_7 = t_{7-1} + t_{7-2} + t_{7-3}$

$= t_6 + t_5 + t_4$

$t_3 = t_{3-1} + t_{3-2}\, t_{3-3}$

$= t_2 + t_1 + t_0$

$= 1+1+0$

$= 2$

$t_5 = t_{5-1} + t_{5-2} + t_{5-3}$

$= t_4 + t_3 + t_2$

$= 4 + 2 + 1$

$= 7$

$t_4 = t_{4-1} + t_{4-2} + t_{4-3}$

$= t_3 + t_2 + t_1$

$= 2 + 1 + 1$

$= 4$

$t_6 = t_{6-1} + t_{6-2} + t_{6-3}$

$= t_5 + t_4 + t_3$

$= 7 + 4 + 2$

$= 13$

$\therefore\ t_7 = t_6 + t_5 + t_4$

$= 13 + 7 + 4$

$= 24$

b)
```
t(n)
  {
     if (n=0)
        return 0
     if (n=1) or (n=2)
        return 1
     return t(n-1) + t(n-2) + t(n-3)
  }
```