DISCRETE STRUCTURE SECI1013

20202021/1

GROUP/ASSIGNMENT :

GROUP 12/ASSIGNMENT 4

LECTURER'S NAME :

DR NOR AZIZAH ALI

GROUP MEMBERS :

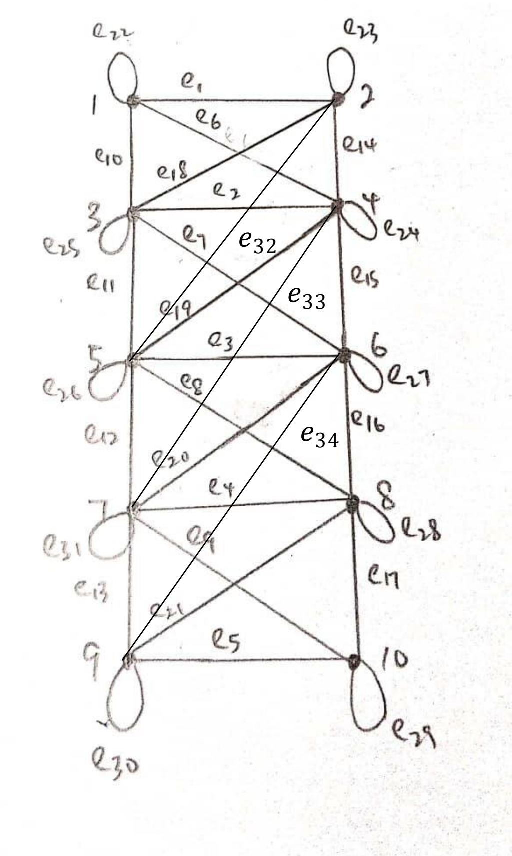
NAME	MATIRC NO.
HONG PEI GEOK	A20EC0044
KONG JIA ROU	A20EC1098
MAIZATUL AFRINA BINTI SAIFUL AZWAN	A20EC0204
RADIN DAFINA BINTI RADIN ZULKARNAIN	A20EC0135

Question 1

1. Let G be a graph with $V(G)$ = {1,2,....,10}, such that two numbers ‘v’ and ‘w’ in $V(G)$ are adjacent if and only if | v – w| ≤ 3. Draw the graph G and determine the numbers of edges, $e(G)$.

   e(G) = { e1, e2, e3, e4, e5, e6, e7, e8, e9, e10, e11, e12, e13, e14, e15, e16, e17, e18, e19, e20, e21, e22, e23, e24, e25, e26, e27, e28, e29, e30, e31, e32, e33, e34 }

   number of edges = 34



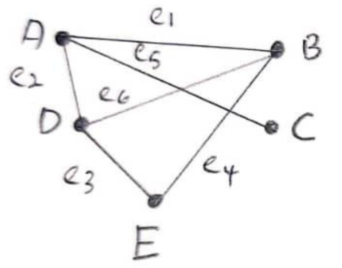
Question 2

Model the following situation as graphs, draw each graphs and gives the corresponding adjacency matrix.

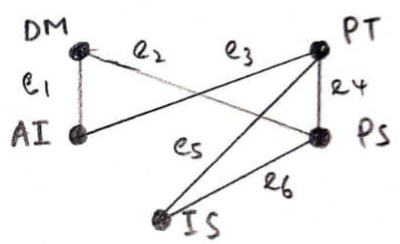
(a) Ahmad and Bakri are friends. Ahmad is also friends with David and Chong. David, Bakri and Ehsan all friends.
(Note that you may use the representation of A= Ahmad; B = Bakri; C = Chong; D = David; E= Ehsan)

(b) There are 5 subjects to be scheduled in the exam week: Discrete Mathematics (DM), Programming Technique (PT), Artificial Intelligence (AI), Probability Statistic (PS) and Information System (IS). The following subjects cannot be scheduled in the same time slot: -

i. DM and IS
ii. DM and PT
iii. AI and PS
iv. IS and AI



(a)



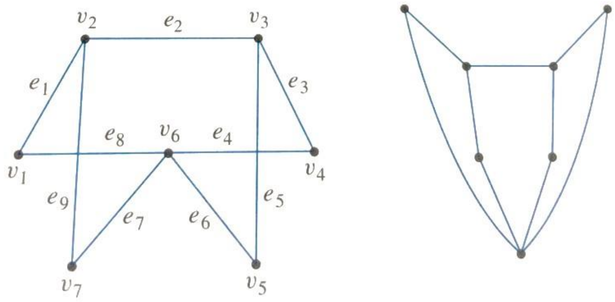
(b)

$$A_G = \begin{array}{c} \\ A \\ B \\ C \\ D \\ E \end{array} \begin{array}{c} \begin{array}{ccccc} A & B & C & D & E \end{array} \\ \begin{bmatrix} 0 & 1 & 1 & 1 & 0 \\ 1 & 0 & 0 & 1 & 1 \\ 1 & 0 & 0 & 0 & 0 \\ 1 & 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 1 & 0 \end{bmatrix} \end{array}$$

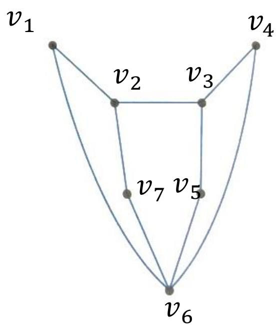
$$B_G = \begin{array}{c} \\ DM \\ PT \\ AI \\ PS \\ IS \end{array} \begin{array}{c} \begin{array}{ccccc} DM & PT & AI & PS & IS \end{array} \\ \begin{bmatrix} 0 & 0 & 1 & 1 & 0 \\ 0 & 0 & 1 & 1 & 1 \\ 1 & 1 & 0 & 0 & 0 \\ 1 & 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 1 & 0 \end{bmatrix} \end{array}$$

Question 3

Show that the two drawing represent the same graph by labeling the vertices and edges of the right-hand drawing to correspond to left-hand drawing.

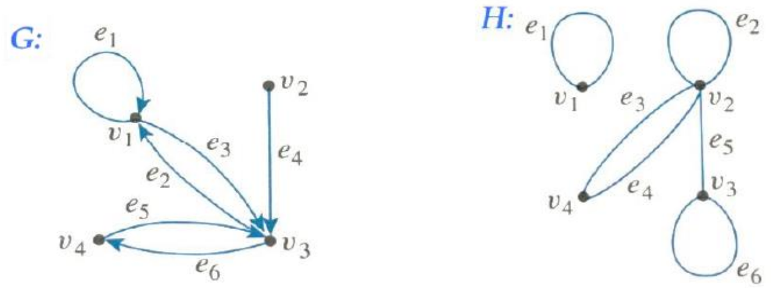


First Graph		
From	To	Degree
v_1	v_2	3
	v_6	4
v_2	v_1	2
	v_3	3
	v_7	2
v_3	v_2	3
	v_4	2
	v_5	2
v_6	v_1	2
	v_4	2
	v_5	2
	v_7	2



Question 4

Find the adjacency and incidence matrices for the following graphs.



<u>Graph G:</u>

$$A_G = \begin{array}{c} \\ \text{V1} \\ \text{V2} \\ \text{V3} \\ \text{V4} \end{array} \begin{array}{c} \begin{array}{cccc} \text{V1} & \text{V2} & \text{V3} & \text{V4} \end{array} \\ \begin{bmatrix} 1 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{bmatrix} \end{array}$$

$$I_G = \begin{array}{c} \\ \text{V1} \\ \text{V2} \\ \text{V3} \\ \text{V4} \end{array} \begin{array}{c} \begin{array}{cccccc} \text{e1} & \text{e2} & \text{e3} & \text{e4} & \text{e5} & \text{e6} \end{array} \\ \begin{bmatrix} 2 & 1 & -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & -1 & 0 & 0 \\ 0 & -1 & 1 & 1 & 1 & -1 \\ 0 & 0 & 0 & 0 & -1 & 1 \end{bmatrix} \end{array}$$

<u>Graph H:</u>

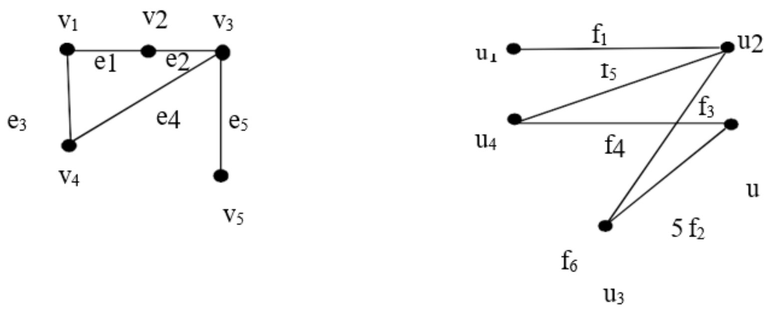
$$A_H = \begin{array}{c} \\ \text{V1} \\ \text{V2} \\ \text{V3} \\ \text{V4} \end{array} \begin{array}{c} \begin{array}{cccc} \text{V1} & \text{V2} & \text{V3} & \text{V4} \end{array} \\ \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 1 & 2 \\ 0 & 1 & 1 & 0 \\ 0 & 2 & 0 & 0 \end{bmatrix} \end{array}$$

$$I_H = \begin{array}{c} \\ \text{V1} \\ \text{V2} \\ \text{V3} \\ \text{V4} \end{array} \begin{array}{c} \begin{array}{cccccc} \text{e1} & \text{e2} & \text{e3} & \text{e4} & \text{e5} & \text{e6} \end{array} \\ \begin{bmatrix} 2 & 0 & 0 & 0 & 0 & 0 \\ 0 & 2 & 1 & 1 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 & 2 \\ 0 & 0 & 1 & 1 & 0 & 0 \end{bmatrix} \end{array}$$

Question 5

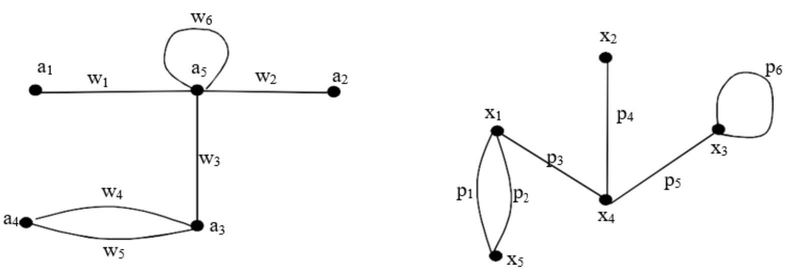
Determine whether the following graphs are isomorphic.

a)



G1 *G2*

b)



H1 *H2*

a)

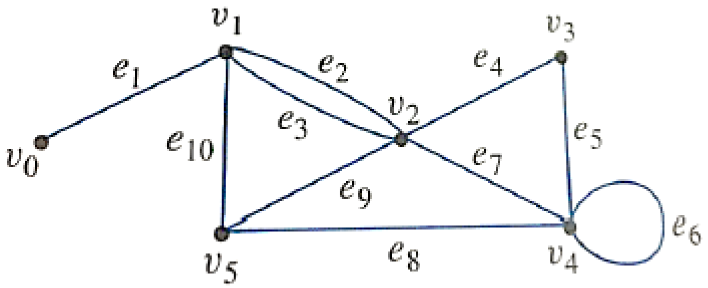
G1 and G2 the same number of vertices and edges;
G1 and G2 the same degrees for corresponding vertices;
G1 and G2 the same number of connected components;
Both graphs are connected or both graph are not connected, pairs of connected vertices must have the corresponding pair of vertices connected.
GI and G2 are isomorphic.

b)

H1 and H2 the same number of vertices and edges;
H1 and H2 has different degrees for corresponding vertices;
H1 and H2 are not isomorphic.

Question 6

In the graph below, determine whether the following walks are trails, paths, closed walks, circuits/cycles, simple circuits or just walks.



a) $v_0 e_1 v_1 e_{10} v_5 e_9 v_2 e_2 v_1$
   - Trails
   - It is because the vertex is repeated and no edge is repeated.

b) $v_4 e_7 v_2 e_9 v_5 e_{10} v_1 e_3 v_2 e_9 v_5$
   - Walk
   - It is because both of vertices and edges are repeated but the vertex at the start and the end are not the same.

c) v_2
   - Not a walk.
   - Because there is no edges.

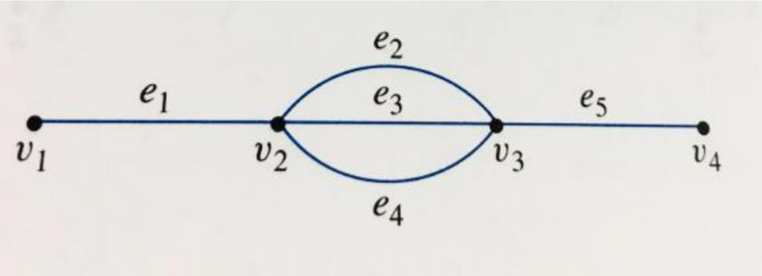
d) $v_5 e_9 v_2 e_4 v_3 e_5 v_4 e_6 v_4 e_8 v_5$
   - Circuits/cycles
   - It is because we can see that the vertex is repeated but the edges are not and also it is a closed walk.

e) $v_2 e_4 v_3 e_5 v_4 e_8 v_5 e_9 v_2 e_7 v_4 e_5 v_3 e_4 v_2$
   - Closed walk
   - It is because we can see here that both vertices and edges are repeated and it start and end at the same vertex.

f) $v_3 e_5 v_4 e_8 v_5 e_{10} v_1 e_3 v_2$
   - Path
   - Because of vertices and edges are not repeated and the start and the end are not at the same vertex.

Question 7

Consider the following graph.



a) How many paths are there from $v1$ to $v4$?

$$v_1 e_1 v_2 e_2 v_3 e_5 v_4$$

$$v_1 e_1 v_2 e_3 v_3 e_5 v_4$$

$$v_1 e_1 v_2 e_4 v_3 e_5 v_4$$

$\therefore$ 3 paths

b) How many trails are there from $v1$ to $v4$?

$$v_1 e_1 v_2 e_2 v_3 e_5 v_4$$

$$v_1 e_1 v_2 e_3 v_3 e_5 v_4$$

$$v_1 e_1 v_2 e_4 v_3 e_5 v_4$$

$$v_1 e_1 v_2 e_2 v_3 e_4 v_2 e_3 v_3 e_5 v_4$$

$$v_1 e_1 v_2 e_2 v_3 e_3 v_2 e_4 v_3 e_5 v_4$$

$$v_1 e_1 v_2 e_3 v_3 e_2 v_2 e_4 v_3 e_5 v_4$$

$$v_1 e_1 v_2 e_3 v_3 e_4 v_2 e_2 v_3 e_5 v_4$$

$$v_1 e_1 v_2 e_4 v_3 e_2 v_2 e_3 v_3 e_5 v_4$$

$$v_1 e_1 v_2 e_4 v_3 e_3 v_2 e_2 v_3 e_5 v_4$$

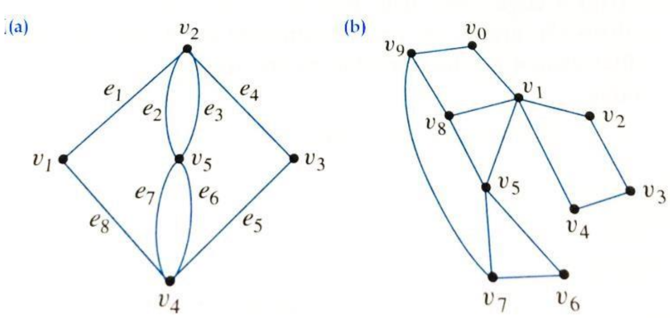
$\therefore$ 9 trails

c) How many walks are there from $v1$ to $v4$?

Infinite, contained an arbitrary large number of repetitions of edges joining a pair of vertices.

Question 8

Determine which of the graphs in (a) – (b) have Euler circuits. If the graph does not have a Euler circuit, explain why not. If it does have a Euler circuit, describe one.



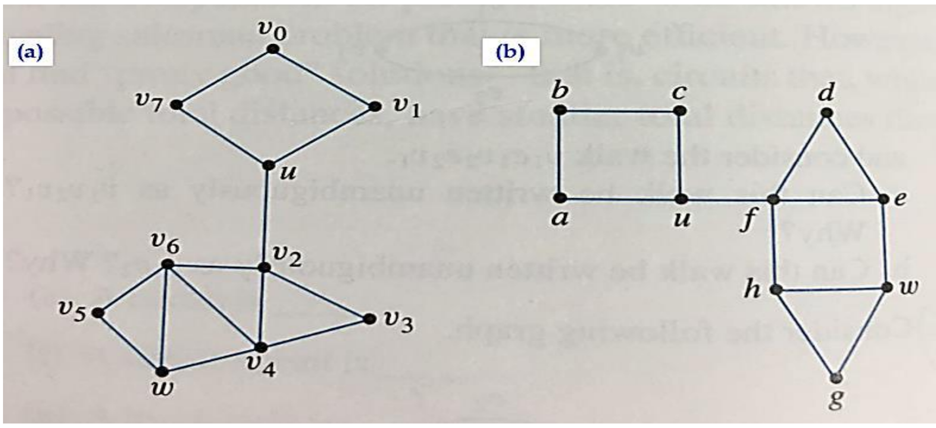
Referring to graph (a), all vertices are even degree, then there is a Euler circuit of graph (a), for example

$$(v_4,\ e_8,\ v_1,\ e_1,\ v_2,\ e_2, v_5,\ e_7,\ v_4,\ e_6, v_5,\ e_3,\ v_2,\ e_4, v_3,\ e_{5,} v_4)$$

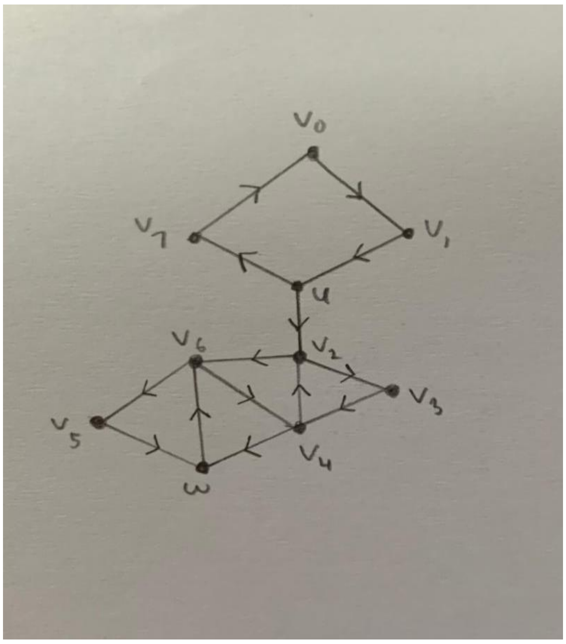
There are three odd vertices in graph (b), which are deg (v_9) = 3, deg (v_7) = 3, deg (v_8) = 3, thus according to theorem of Euler circuit, there is no Euler circuit of graph (b).

Question 9

For each of graph in (a) – (b), determine whether there is an Euler path from u to w. If there is, find such a path.



(a) - Is an Euler path.
- Vertex u and w are the only one that have odd degree.



- (u, v7, v0, v1, u, v2, v3, v4, v2, v6, v4, w, v6, v5, w) is an Euler path.

(b) - Is not an euler path.
- Vertex u, w, e and h have odd degree. So, graph (b) is not an Euler path

Question 10

For each of graph in (a) – (b), determine whether there is Hamiltonian circuit. If there is, exhibit one.

1) Hamiltonian circuit is a path which visits every vertex exactly once. Graph (a) is not a Hamiltonian circuit because the path needs to visit vertex u and vertex v2 more than once.

2) Hamiltonian circuit is a path which visits every vertex exactly once. Same situation with graph (a), graph (b) is not a Hamiltonian circuit because the path needs to visit vertex u and vertex f more than once.

Question 11

How many leaves does a full *3-ary* tree with 100 vertices have?

$$l = \frac{[\,(m-1)\,n+1]}{m}$$

$$l = \frac{[\,(3-1)\;100\;+1]}{3}$$

$$l = \frac{[\,(2)\;101]}{3}$$

$$l = \;67\;leaves$$

Question 12

Find the following vertex/vertices in the rooted tree illustrated below.

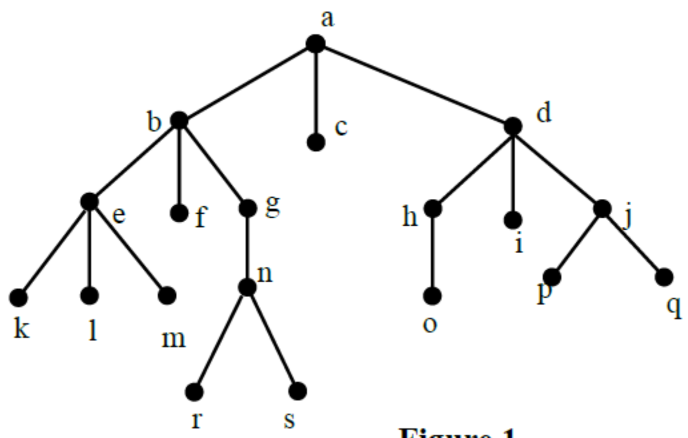


Figure 1

a) Root

= a

b) Internal vertices

= e, b, g, n, h, d, j, a

c) Leaves

= c, f, k, l, m, r, s, o, i, p, q

d) Children of n

= r, s

e) Parent of e

= b

f) Siblings of k

= l, m

g) Proper ancestors of q

= j, d, a

h) Proper descendants of b

= e, k, l, m, f, g, n, r, s

Question 13

In which order are the vertices of ordered rooted tree in **Figure 1** is visited using *preorder*, *inorder* and *postorder*.

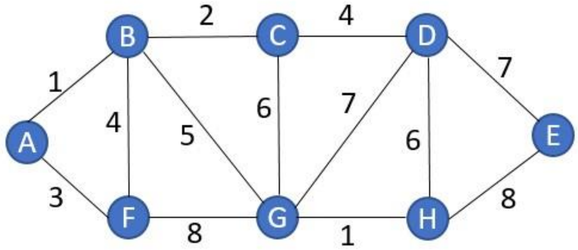
Preorder - a, b, e, k, l, m, f, g, n, r, s, c, d, h, o, i, j, p, q

Inorder - k, e, l, m, b, f, r, n, s, g, a, c, o, h, d, i, p, j, q

Postorder - k, l, m, e, f, r, s, n, g, b, c, o, h, i, p, q, j, d, a

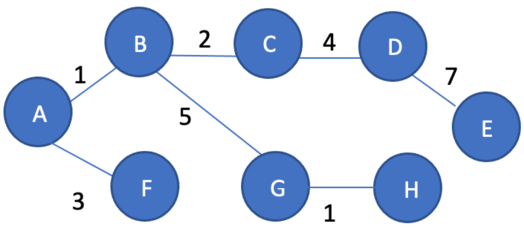
Question 14

Find the minimum spanning tree for the following graph using Kruskal's algorithm.



List of edges in order of size:

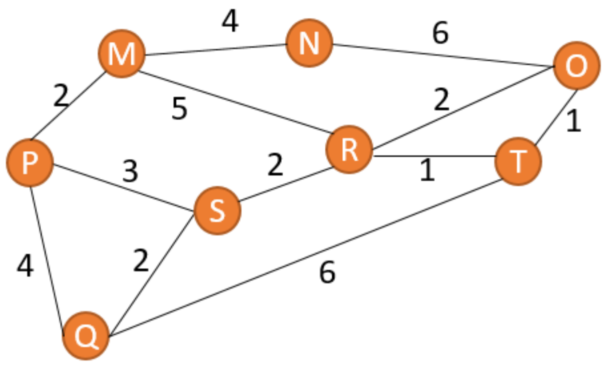
Edge	Weight	Will adding edge make circuit?	Action taken	Cumulative weight of subgraph
(A,B)	1	No	Added	1
(G,H)	1	No	Added	2
(B,C)	2	No	Added	4
(A,F)	3	No	Added	7
(B,F)	4	Yes	Not added	7
(C,D)	4	No	Added	11
(B,G)	5	No	Added	16
(C,G)	6	Yes	Not added	16
(D,H)	6	Yes	Not added	16
(D,E)	7	No	Added	23
(D,G)	7	Yes	Not added	23
(F,G)	8	Yes	Not added	23
(E,H)	8	Yes	Not added	23



The minimum spanning tree is drawn above.

Question 15

Use Dijkstra's algorithm to find the shortest path from **M** to **T** for the following graph.



Shortest length: 6

Shortest path: M → R → T

Iteration	S	N	L(M)	L(N)	L(O)	L(P)	L(Q)	L(R)	L(S)	L(T)
0	{}	{M,N,O,P,Q,R,S,T}	0	∞	∞	∞	∞	∞	∞	∞
1	{M}	{N,O,P,Q,R,S,T}	0	4	∞	2	∞	5	∞	∞
2	{M,P}	{N,O,Q,R,S,T}	0	4	∞	2	6	5	5	∞
3	{M,P,N}	{O,Q,R,S,T}	0	4	10	2	6	5	5	∞
4	{M,P,N,R}	{O,Q,S,T}	0	4	10	2	6	5	5	6
5	{,M,P,N,R,S}	{O,Q,T}	0	4	10	2	6	5	5	6
6	{M,P,N,R,S,O}	{Q,T}	0	4	10	2	6	5	5	6
7	{M,P,N,R,S,O,Q}	{T}	0	4	10	2	6	5	5	6