DISCRETE STRUCTURE SECI1013

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GROUP/ASSIGNMENT :

GROUP 12/ASSIGNMENT 1

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Question 1

Let the universal set be the set R of all real numbers and let A={x ∈ R | 0 < x ≤ 2}, B={x ∈ R | 1 ≤ x < 4} and C={x ∈ R | 3 ≤ x < 9}. Find each of the following :

(a) A ∪ C

$$A \cup C = \{x \in \mathbf{R} \mid 0 < x \leq 2 \text{ or } 3 \leq x < 9\}$$

(b) (A ∪ B)′

$$(A \cup B)' = \{x \in \mathbf{R} \mid \neg(0 < x < 4)\} = \{x \in \mathbf{R} \mid \mathrm{x} \leq 0 \text{ or } \mathrm{x} \geq 4\}$$

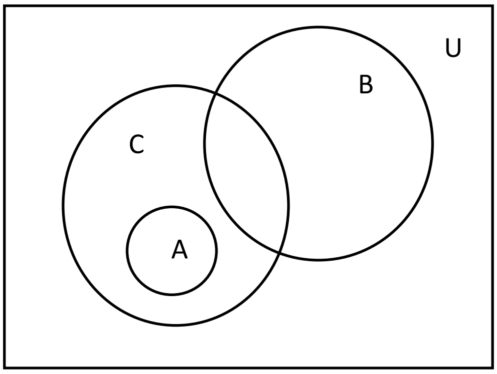
(c) A′ ∪ B′

$$A' \cup B' = \{x \in \mathbf{R} \mid \mathrm{x} < 1 \text{ or } \mathrm{x} > 2\}$$

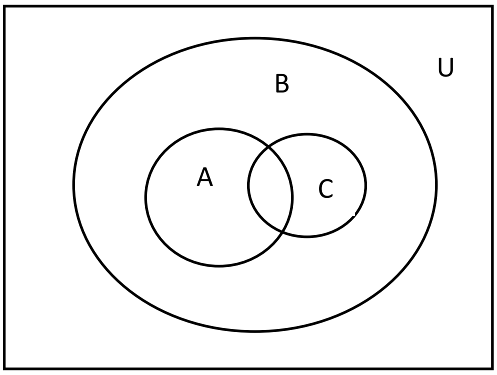
Question 2

Draw Venn diagrams to describe sets A, B, and C that satisfy the given conditions.

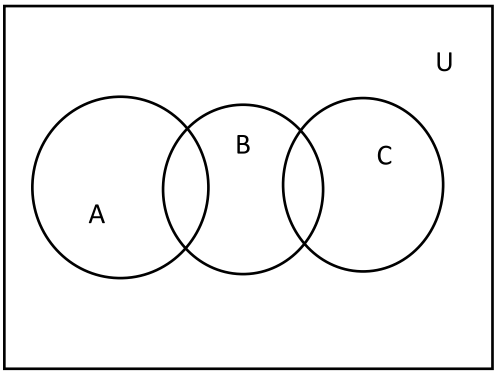
a) $A \cap B = \emptyset$, $A \subseteq C$, $C \cap B \neq \emptyset$



b) $A \subseteq B$, $C \subseteq B$, $A \cap C \neq \emptyset$



c) $A \cap B \neq \emptyset$, $B \cap C \neq \emptyset$, $A \cap C = \emptyset$, $A \not\subset B$, $C \not\subset B$



Question 3

Given two relations S and T from A to B,

S ∩ T = {(x,y) ∈A×B | (x,y) ∈ S and (x,y) ∈ T}

S ∪ T = {(x,y) ∈A×B | (x,y) ∈ S or (x,y) ∈ T}

Let A={−1, 1, 2, 4} and B={1,2} and defined binary relations S and T from A to B as follows:

For all (x,y) ∈A×B, x S y ⟷ |x| = |y|

For all (x,y) ∈A×B, x T y ⟷ x− y is even

State explicitly which ordered pairs are in A× B, S, T, S ∩ T, and S ∪ T.

$$A = \{-1,1,2,4\}\ and\ B = \{1,2\}$$

$$\boldsymbol{A \times B} = \{(\mathbf{-1,1}), (\mathbf{-1,2}), (\mathbf{1,1}), (\mathbf{1,2}), (\mathbf{2,1}), (\mathbf{2,2}), (\mathbf{4,1}), (\mathbf{4,2})\}$$

$$S = \{ (x,y) \in A \times B, x\,S\,y \leftrightarrow |x| = |y|$$

Since,

$$(-1,1) \in A \times B$$

$$|-1| = |1|$$

$$(1,1) \in A \times B$$

$$|1| = |1|$$

$$(2,2) \in A \times B$$

$$|2| = |2|$$

$$\therefore \boldsymbol{S} = \{(\mathbf{-1,1}), (\mathbf{1,1}), (\mathbf{2,2})\}$$

$T = \{\ (x, y) \in A\ \times B, x\ T\ y\ \leftrightarrow\ x - y$ is even

Thus,

$(-1,1);\ -1 - 1 =\ -2\ \ is\ even$

$(-1,2);\ -1 - 2 =\ -3\ is\ odd$

$(1,1);\ 1 - 1 =\ 0\ is\ even$

$(1,2);\ 1 - 2 =\ -1\ is\ odd$

$(2,1);\ 2 - 1 =\ 1\ is\ odd$

$(2,2);\ 2 - 2 =\ 0\ is\ even$

$(4,1);\ 4 - 1 =\ 3\ is\ odd$

$(4,2);\ 4 - 2 =\ 2\ is\ even$

$\therefore \boldsymbol{T} = \{(-\mathbf{1}, \mathbf{1})\ , (\mathbf{1}, \mathbf{1})\ , (\mathbf{2}, \mathbf{2})\ , (\mathbf{4}, \mathbf{2})\}$

$\boldsymbol{S}\ \cap\ \boldsymbol{T} = \{(-\mathbf{1}, \mathbf{1})\ , (\mathbf{1}, \mathbf{1})\ , (\mathbf{2}, \mathbf{2})\}$

$\boldsymbol{S}\ \cup\ \boldsymbol{T} = \{(-\mathbf{1}, \mathbf{1})\ , (\mathbf{1}, \mathbf{1})\ , (\mathbf{2}, \mathbf{2})\ , (\mathbf{4}, \mathbf{2})\}$

Question 4

Show that $\neg((\neg p \wedge q) \vee (\neg p \wedge \neg q)) \vee (p \wedge q) \equiv p$. State carefully which of the laws are used at each stage.

$\neg((\neg p \wedge q) \vee (\neg p \wedge \neg q)) \vee (p \wedge q)$	$= \neg(\neg p \wedge (q \vee \neg q)) \vee (p \wedge q)$	Distributive law
	$= \neg(\neg p \wedge U) \vee (p \wedge q)$	Complement law
	$= \neg(\neg p) \vee (p \wedge q)$	Properties of universal law
	$= p \vee (p \wedge q)$	Double negation law
	$= p$	Absorption law

Question 5

R_1 = {(x,y) | x + y ≤6}; R_1 is from X to Y; R_2 = {(y,z) | y > z}; R_2 is from Y to Z; ordering of X, Y, and Z: 1, 2, 3, 4, 5. Find:

a) The matrix A1 of the relation R1 (relative to the given orderings)

$$M_{A_1} = \begin{array}{c} \\ 1 \\ 2 \\ 3 \\ 4 \\ 5 \end{array} \begin{array}{c} \begin{array}{ccccc} 1 & 2 & 3 & 4 & 5 \end{array} \\ \begin{bmatrix} 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 0 \\ 1 & 1 & 1 & 0 & 0 \\ 1 & 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 \end{bmatrix} \end{array}$$

b) The matrix A2 of the relation R2 (relative to the given orderings)

$$M_{A_2} = \begin{array}{c} \\ 1 \\ 2 \\ 3 \\ 4 \\ 5 \end{array} \begin{array}{c} \begin{array}{ccccc} 1 & 2 & 3 & 4 & 5 \end{array} \\ \begin{bmatrix} 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 \\ 1 & 1 & 0 & 0 & 0 \\ 1 & 1 & 1 & 0 & 0 \\ 1 & 1 & 1 & 1 & 0 \end{bmatrix} \end{array}$$

c) Is R1 reflexive, symmetric, transitive, and/or an equivalence relation?

Reflexive : R_1 is not a reflexive relation because there are value 0 and 1 on its diagonal and there are only some vertex that have loops.

Symmetric : R_1 is a symmetric relation because $m_{R_1} = {m_{R_1}}^T$ and $m_{ij} = m_{ji}$.

Transitive : R_1 is not a transitive relation.

Equivalence : R_1 is not an equivalence relation because R_1 is not a reflexive and a transitive relation.

d) Is R_2 reflexive, antisymmetric, transitive, and/or a partial order relation?

Reflexive : R_2 is not a reflexive relation because there is value 0 on its diagonal.

Antisymmetric : R_2 is an antisymmetric relation because $m_{ij} \neq m_{ji}$.

Transitive : R_2 is not a transitive relation.

Partial order : R_2 is not a partial order relation because R_2 is not a reflexive relation and not a transitive relation.

Question 6

Suppose that the matrix of relation R1 on {1, 2, 3} is

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 1 \\ 1 & 0 & 1 \end{bmatrix}$$

relative to the ordering 1, 2, 3, and that the matrix of relation R2 on {1, 2, 3} is

$$\begin{bmatrix} 0 & 1 & 0 \\ 0 & 1 & 0 \\ 1 & 0 & 1 \end{bmatrix}$$

relative to the ordering 1, 2, 3. Find:

a) The matrix of relation R1 ∪ R2

$$M_{R_1} \cup M_{R_2} = M_{R_1} \vee M_{R_2} = \begin{array}{c c} & \begin{array}{ccc} 1 & 2 & 3 \end{array} \\ \begin{array}{c} 1 \\ 2 \\ 3 \end{array} & \begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & 1 \\ 1 & 0 & 1 \end{bmatrix} \end{array}$$

b) The matrix of relation R1 ∩ R2

$$M_{R_1} \cap M_{R_2} = M_{R_1} \wedge M_{R_2} = \begin{array}{c c} & \begin{array}{ccc} 1 & 2 & 3 \end{array} \\ \begin{array}{c} 1 \\ 2 \\ 3 \end{array} & \begin{bmatrix} 0 & 0 & 0 \\ 0 & 1 & 0 \\ 1 & 0 & 1 \end{bmatrix} \end{array}$$

Question 7

If f :R→ R and g:R→ R are both one-to-one, is f + g also one-to-one? Justify your answer.

A function is said to be one-to one if $f(x_1) = f(x_2)$ and $x_1 = x_2$

Assume that,

$$f(x) = x$$
$$g(x) = -x$$

For all real number for x,

$$(f + g)(x) = f(x) + g(x)$$
$$= (x) + (-x)$$
$$= 0$$

For example,

$$(f + g)(5) = f(5) + g(5)$$
$$= (5) + (-5)$$
$$= 0$$

$$(f + g)(3) = f(3) + g(3)$$
$$= (3) + (-3)$$
$$= 0$$

$$(f + g)(5) = (f + g)(3) = 0, \text{ but } 5 \neq 3$$
$$(f + g)(x_1) = (f + g)(x_2) = 0 \text{ but } x_1 \neq x_2$$

Therefore,

$$\therefore \boldsymbol{f + g} \textbf{ is not one-to-one.}$$

Question 8

With each step you take when climbing a staircase, you can move up either one stair or two stairs. As a result, you can climb the entire staircase taking one stair at a time, taking two at a time, or taking a combination of one- or two-stair increments. For each integer n ≥ 1, if the staircase consists of n stairs, let c_n be the number of different ways to climb the staircase.

Find a recurrence relation for c_1, c_2, …., c_n.

$c_1 = 1,$

$c_2 = 2,$

$\therefore c_n = c_{n-1} + c_{n-2}$, when n ≥ 3

Question 9

The Tribonacci sequence (t_n) is defined by the equations,

$$\boldsymbol{t_0} = \mathbf{0},\ \boldsymbol{t_1} = \boldsymbol{t_2} = \mathbf{1},\ \boldsymbol{t_n} = \boldsymbol{t_{n-1}} + \boldsymbol{t_{n-2}} + \boldsymbol{t_{n-3}} \text{ for all } \mathbf{n \geq 3.}$$

a) Find t_7

$t_0 = 0$
$t_1 = 1$
$t_2 = 1$
$t_3 = 1 + 1 + 0 = 2$
$t_4 = 2 + 1 + 1 = 4$
$t_5 = 4 + 2 + 1 = 7$
$t_6 = 7 + 4 + 2 = 13$
$t_7 = 13 + 7 + 4 = 24$

$\therefore\ t_7 = 24$

b) Write a recursive algorithm to compute t_n, n ≥ 3.

Input: n
Output: t(n)

```
t(n) {
  if (n = 0)
        return 0
  if (n = 1 or n = 2)
        return 1
  return t (n - 1) + t (n - 2) + t (n - 3)
}
```