ASSIGNMENT:- 02

SCHOOL OF COMPUTING
FACULTY OF ENGINEERING
SUBJECT:-DISCRETE STRUCTURE
SUBJECT CODE:-SECI1013-08

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ANS TO THE QUESTION NUMBER:-01

Ans to the question No: 01

ⓐ $A = \{1, 2, 3, 4, 5, 6, 7, 8\}$

$B = \{2, 5, 9\}$

$C = \{a, b\}$

(i) $A - B$

$= \{1, 2, 3, 4, 5, 6, 7, 8\} - \{2, 5, 9\}$

$= \{1, 3, 4, 6, 7, 8\}$

(ii) $(A \cap B) \cup C$

$A \cap B$

$= \{1, 2, 3, 4, 5, 6, 7, 8\} \cap \{2, 5, 9\}$

$= \cancel{\{1, 2, 3, 4, 5, 6, 7, 8, 9\}} = \{2, 5\}$

$(A \cap B) \cup C$

$= \cancel{\{1, 2, 3, 4, 5, 6, 7, 8, 9\}} \therefore \{2, 5\} \cup \{a, b\}$

$= \{a, b, 2, 5\}$.

1(a). (iii) $A \cap B \cap C$

$= \{1,2,3,4,5,6,7,8\} \cap \{2,5,9\} \cap \{a,b\}$

$= \{2,5\}.$

(iv) $B \times C$

$= \{2,5,9\} \times \{a,b\}$

$= \{(2,a),(2,b),(5,a),(5,b),(9,a),(9,b)\}.$

(v) $P(C)$ $\quad C = \{a,b\}$

$= \{\varphi, \{a\}, \{b\}, \{a,b\}\}$

$|C| = |2|$

$P|(C)| = 2^2$

$= 4.$

① (b)

$(P \cap ((P' \cup Q)')) \cup (P \cap Q) = P$

$L.H.S = P \cap (P' \cup Q)' \cup (P \cap Q)$ Distributive law

$= (P \cap P) \cup (P \cap Q') \cup (P \cap Q)$ Idempotent law

$= (P \cap Q') \cup (P \cap Q)$ Absorption Law.

$= P \cap (Q \cup Q')$ - complement law

$= P \cap U$ - Properties of universal set

$= P$

1 c

$A = (\neg P \vee q) \leftrightarrow (q \rightarrow P)$

P	q	¬P		$(\neg P \vee q)$	$(q \rightarrow P)$	$(\neg P \vee q) \leftrightarrow (q \rightarrow P)$
T	T	F		T	T	T
T	F	F		F	T	F
F	T	T		T	F	T
F	F	T		T	T	T

(d) $P(x) = x$ is an odd integer

$\forall x (P(x) \rightarrow Q(x))$

So, a is an ~~odd~~ odd integer

Let,

$a = 2n + 1$

$a^2 = (2n+1)^2$

$\Rightarrow a^2 = 4n^2 + 4n + 1$

$\Rightarrow a^2 = 2(2n^2 + 2n) + 1$

So now, let,

$m = 2n^2 + 2n$

$\Rightarrow a^2 = 2m + 1$

here,

a^2 is an odd integer.

So, Now,

Therefore, for all integers x, if x is odd, then x^2 is odd.

(1) (e)

(i) $\exists x \exists y\, P(x, y)$: True, if any $x \geq y$ or $y \leq x$

(ii) $\forall x \forall y\, P(x, y)$: False, if $x < y$ or $y > x$.

ANS TO THE QUESTION NUMBER :- 02

Ans to the question No: 02

(a) (i)

R = (1, 2, 3)

$$\begin{array}{c|c} & \begin{array}{ccc} 1 & 2 & 3 \end{array} \\ \begin{array}{c} 1 \\ 2 \\ 3 \end{array} & \begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{bmatrix} \end{array}$$

R = { (1,1), (1,2), (2, 2), (3,1) }

domain = {1, 2, 3}

Range = {1, 2}.

(ii) It's obviously a irreflexive because it has 2 elements in diagonal means ~~it has~~ ~~in d~~ (it has (1,1), (2,2).

It's also a antisymmetric, because (1,1) ∈ R, and (2,2) ∈ R and (1,2) ∈ R but (3,1) ∉ R, so, it's anti symmetric.

02 b

(i) let
$S = \{(x,y) \mid x+y \geq 9\}$ is a relation

on $x = \{2,3,4,5\}$.

elements of the set S

as $4+5 \geq 9$ and $5+5 \geq 9$.

So,
S are $= \{4,5\}$.

(ii) $S = \{4,5\}$.

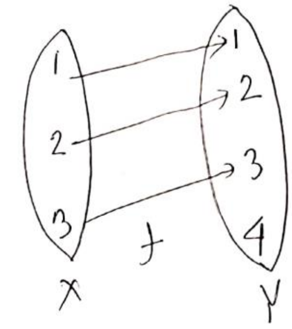
- as $4+4 < 9$, so set S isn't reflexive
- $4+5 \geq 9$, and $5+4 \geq 9$, so s is symmetric
- $4+5 \geq 9$ and $5+5 \geq 9$, so $5+5 \geq 9$, so set s is transitive.

as, set S is n't reflexive, it doesn't contain any equivalance relation.

2. a/

(i). Given,

$X = \{1, 2, 3\}$
$Y = \{1, 2, 3, 4\}$
$Z = \{1, 2\}$



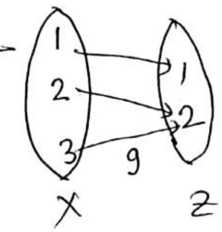
Now,

$f : X \longrightarrow Y$

One-to-one: Yes, because every element of X matched with unique element of Y.

Onto: No, because element 1 of Y didn't match with any element of X.

Ans.



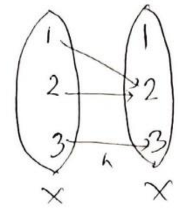
(ii) $g : X \longrightarrow Z$

Onto: Yes, $\forall$ element of Y is having image in X so this is an onto function.

One-to-one: No, because $\forall$ elements of $X \notin Y$.

Ans.

iii/ h: $X \longrightarrow X$



One-to-one : No, because $\forall x \notin Y$.

Onto : No, because all element of ~~x~~ x didn't match with Y uniquely.

So, it is neither onto nor one-to-one.

Ans.

2.d/ (i) Given,

$m(x) = 4x + 3$

$n(x) = 2x - 4$

Now,

inverse of m, =

$y = 4x + 3$

$\Rightarrow 4x = y - 3$

$\Rightarrow x = \frac{y-3}{4}$

$\Rightarrow y = \frac{x-3}{4}$ [$\because$ changing y & x]

$\therefore$ inverse of $f(x)$ is $\frac{x-3}{4}$

Ans.

(ii) composition of nom,

$n(m(x))$

$\Rightarrow 2(4x+3) - 4$

$\Rightarrow 8x + 6 - 4$

$\Rightarrow 8x + 2$

$\Rightarrow 2(4x+1)$

Ans

ANS TO THE QUESTION NUMBER :- 03

Ans. to the Ques. NO. 03.

a/(i)

Given,

$a_k = a_{k-1} + 2k$, For all integers $k \geq 2$

and $a_1 = 1$.

Now, If $k = 2$, $a_2 = a_{2-1} + 2 \times 2$

$= a_1 + 4$

$= 1 + 4$ $[\because a_1 = 1]$

$= 5$

If $k = 3$, $a_3 = a_{3-1} + \cancel{2} \times 3$

$= a_2 + 6$

$= 5 + 6$ $[\because a_2 = 5]$

$= 11$

So therefore, the first three terms are 1, 5 and 11

Ans.

3 a (ii)

Input : k, integer ≥ 1

output , a(k) {

a(k) ← k

if k = 1,

return 1

return a (k-1) + 2k

}.

3.b/

let, r_k = Number of operations when it is run with an input of size k.

When the algorithm is run with an input size 1, then it executes 5 operations.

$\therefore r_1 = 5$

The number of operations with an input of size k is twice the number of operation with an input of size k-1.

$\therefore r_k = 2r_{k-1}$ when $k-1 \geq 1$ or $k \geq 2$

$\Rightarrow r_k = 2r_{k-1}$

$\Rightarrow r_k = 2 \cdot 2 \cdot r_{k-2}$

$\Rightarrow r_k = 2 \cdot 2 \cdot 2 \cdot r_{k-3}$

$\Rightarrow r_k = 2^3 r_{k-3}$

$\Rightarrow r_k = 2^{k-1} r_1$

$\Rightarrow r_k = 5 \cdot 2^{k-1}$ $[r_1 = 5]$

↳ recurrence relation.

Ans.

3.c/

Trace $S(4)$

$\to 5 * S(3)$

$\to 5 * S(2)$

$\to 5 * S(1)$

$\to 5$

$S(4) = 5 \times 5 \times 5 \times 5$

$= 625$

Ans,

ANS TO THE QUESTION NUMBER :- 04

Sub : Time : Date : / /

Answer of the question No.4

a) Let, the numbers are W,X,Y,Z.

these are the digits from 0 to F

W can take 9 possible values from 3 to B

We know,

X and Y can take all possible values.

Z can take 11 values from 5 to F

So, 9×16×16×5 = 11520 numbers

(Ans:)

b) Here, first letter and last digit are fixed

So, we need to fix the 3 letters and 2 digits

Each of the 3 letters can be fixed in 6 ways.

Each of 2 digits can be fixed in 6 ways.

So, there are 6×6×6×10×10 = 21600 number plates.

c) 'COMPUTER' has 8 distinct letters.

we can have one, two, or three letter words.

Single letter words = 8

Two letter words = $P(8,2) = 8 \times 7 = 56$

Three letter words = $P(8,3) = 8 \times 7 \times 6 = 336$

total 336 words.

(Ans.)

d) Way to select 4 out of 7 women = $C(7,4) = (7 \times 6 \times 5)/3!$
$= 35$

way to select 3 out of 6 men = $C(6,3) = (6 \times 5 \times 4) / 3! = 20$

we need to select the women and the men.

so we need to multiply

$\therefore 35 \times 20 = 700$

(Ans.)

e) 'PROBABILITY'

out of 11 letter we have 7 letter that occurs

once → P, R, O, A, L, T, Y

2 letter occur twice → B, I

so, the total possible arrangements are

$11! (2! \times 2!) = 9979200$ (Ans:)

ANS TO THE QUESTION NUMBER :- 04 AND 05

f) pastries of first kind x_1
Pastries of second kind x_2
Pastries of third kind x_3
Pastries of fourth kind x_4
Pastries of fifth kind x_5
Pastries of sixth kind x_6

$$x_1 + x_2 + x_3 + x_4 + x_5 + x_6 = 10$$

we know that.

$$x_1 + x_2 + x_3 + \cdots + x_k = n$$

$$C(n+k-1, k-1)$$

here, its no of solutions are $C(10+6-1, 6-1)$.

$$= C(15, 5) = 3003$$ (Ans:

Answer of the question NO. 5

a) Different 1st and last name combinations are $3 \times 2 = 6$.
We want minimum number of people having 1st and last names. We have to give each name to an entire group of six people at a time.
so, there are 18 person, We need to give the names to 3 groups.
With the most symmetric distribution there must be 3 people having same 1st and last name.

Sub :   Day   Time :   Date :   /   /

b) There are 10 ~~ve~~ even and 10 odd numbers in (1 to 20). We might pick the 10 even numbers upfront in worst case. Then the 11th number will be odd. So, therefore we need to pick 11 numbers to be sure of having picked an odd number.

c) ~~1~~ 20 numbers are divisible by 5 and the rest 80 are not in (1 to 100). The worst case would be picking up 81 numbers that at least one of them is a multiple of 5.

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