



UTM
UNIVERSITI TEKNOLOGI MALAYSIA

ASSIGNMENT:- 02

*SCHOOL OF COMPUTING
FACULTY OF ENGINEERING
SUBJECT:-DISCRETE STRUCTURE
SUBJECT CODE:-SECI1013-08*

Group-08

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Ans to the question No:01

① a

we have numbers : 2, 3, 4, 5, 6 or 7.

Total 6 numbers

first place can be filled in 6 ways ;

2nd place can be filled in 6 ways (numbers will repeat)

3rd place can be filled in 6 ways

Total : $6 \times 6 \times 6 = 216$.

01 b

If all the numbers should be distinct ;

first place can be filled in 6 ways

2nd place can be filled in 5 ways

3rd place can be filled in 4 ways

Total : $6 \times 5 \times 4 = 120$;

01 c

We have to form odd numbers in between 300 and 700.

first place can be filled in 4 ways (only 3, 4, 5, 6 allowed)

2nd place can be filled in 6 ways

3rd place can be filled in 3 ways (only 3, 5, 7 allowed; odd numbers)

Total: $4 \times 6 \times 3 = 72$;

02 a

Men insist to sit next to each other = $(6-1)! (5)! = (5)! (5)!$

In the given data we have total 5 men and 5 women except Anita.

We need all men to sit next to each other, so consider 5 men as a group.

So, we have total 6 person i.e. 1 men group and 5 women.

So, we can arrange 6 person around a round table in $(6-1)!$ way. But the 5 men internally can be arrange in $5!$ ways.

So, we can arrange them in such a way that all men sit together is,

$$(6-1)! (5)! = (5)! (5)! \text{ ways.}$$

02 b

The couple insisted to sit next to each other = $(9-1)! (2)! = (8)! (2)!$

Now, take 1 couple as a group and we have other 8 people. totally 9 people. we can arrange 9 people around a round table in $(9-1)!$ ways.

But the couple can be rearrange in between them in $(2)!$ ways.

So,
Finally, we can ~~re~~arrange people in such a way that couple are insisted to sit to each other

$$(9-1)! (2)! = (8)! (2)! \text{ ways.}$$

02 c

Men and women sit in alternate seats

$$(5-1)! (5)! = (4)! (5)!$$

First let us consider 5 women are seated in alternate seats. It is done in $(5-1)!$ ways. It is same as our main table.

Now the 5 men can be arranged in 5 gaps in $5!$ ways. Now this time it is not $(5-1)!$ because already women are seated.

So, the people are arranged around a table so that men and women sit in alternate seats is

$$(5-1)! (5)! = (4)! (5)! .$$

2d

Now,

In photo shoot, we have 12 people include Anita and her husband.

We need to arrange them in a line so that Anita and her husband stand together.

Consider Anita and her husband as a group. So, we have total 11 persons. We can arrange them in a line in $(11)!$ ways.

But Anita and her husband can be arranged in $2!$ ways between them.

So, we can arrange people, so that, Anita and her husband stand together is, $(11)!(2)!$ ways.

Answer of the question No 3

a) Ways to win if no ties,
there are 5 sprinters
so the number of ways to win is
 $5! = 120$ ways

b) If 2 sprinters tie,
number of ways of winning is
 $4! = 24$ ways of placement

$5C_2 = 10$ ways to choose which 2 sprinters will tie

$\therefore 24 \times 10 = 240$ ways of two sprinters will tie.

c) If 2 Groups of 2 sprinters tie
there are only 3 places 1st, 2nd, 3rd

$3! = 6$ ways of placement

$5C_2 = 10$ ways to choose 2 sprinters in a group

$6C_2 = 15$ ways that 2 groups will tie.

$6 \times 10 \times 3 = 180$ ways

(Ans)

(4a)

Consider that there are six varieties of croissants.

They are plain croissants, cherry croissants, chocolate croissants, almond croissants, apple ~~cro~~ croissants and broccoli croissants.

The objective is to find the number of ways to choose a dozen croissants.

4.b

The objective is to find the numbers of ways to choose two dozen croissants with at least two of each kind.

To pick out the dozen croissants with atleast two of each kind, first pick out two from each kind from the available six kinds which make one dozen croissants.

The remaining dozen croissants can be selected in the unordered selection in,

$$C(6+12-1, 12) = C(17, 12)$$

$$= \frac{(17)!}{(12)! (17-12)!}$$

$$= \frac{17 \times 16 \times 15 \times 14 \times 13}{5 \times 4 \times 3 \times 2 \times 1}$$

$$= 6188 \text{ ways.}$$

4(c)

The objective is to find the number of ways to choose two dozen croissants with at least 5 chocolate croissants and at least 3 almond croissants.

There are 24 croissants in two dozen.

Two dozen croissants with at least five chocolate croissant and at least three almond croissant are picked out.

First,
5 chocolate, three almond

So,
16 more croissants are to be picked out from the six varieties as unordered selection,

Here,

$$n=6, r=16$$

$$C(6+16-1, 16) = C(21, 16)$$

$$= \frac{(21)!}{(5)! (21-5)!}$$

$$= \frac{21 \times 20 \times 19 \times 18 \times 17}{5 \times 4 \times 3 \times 2 \times 1}$$

$$= 20349 \text{ ways.}$$

Ans. to the Ques. NO. 05.

a/ If a team wins 2 games among 4 games,
then, $C(4,2) = {}^4C_2 = 6$

If a team wins 1 game among 3 games
then, $C(3,1) = {}^3C_1 = 3$

Now,

$$\begin{aligned} 2 \text{ wins and } 1 \text{ ties or wins} &= C(4,2) \times C(3,1) \times 2 \\ &= 36 \end{aligned}$$

$$\begin{aligned} 1 \text{ win and } 3 \text{ ties or wins} &= C(3,1) \times C(4,3) \times 2 \times 2 \\ &= 96 \end{aligned}$$

Since there are 2 teams we multiply it by 2,

$$\begin{aligned} \text{Scenarios} &= 2 \times (36 + 96) \\ &= 264 \end{aligned}$$

Ans,

b/ If 10 penalty kicks are executed, we get $= 1024$

$$\begin{aligned}\text{Unsettled games} &= 1024 - 264 \\ &= 760\end{aligned}$$

$$\text{So, 1st game} = 760$$

$$\text{2nd game} = 264$$

$$\text{So, total scenarios} = 760 = 200640$$

Ans.

c/ For sudden death shootout we have 3 options. A wins, B win or a tie.

So, unsettled games scenarios are,

$$\text{Round 1} = 760$$

$$\text{Round 2} = 760$$

For shootout the games was settled so the scenarios are $= 2 = 10$

$$\text{So, final scenario} = 10 = 5776000$$

Ans.

Answer of the question No. 6

The number of different answer sheets that are possible

Every question with on a, b, c and d is 4^{10} as every question has possible answers.

In order to insure that 3 answer sheets are identical we need every possibility filled twice and then one more on $2 \times 4^{10} + 1 = 2097153$ students

(Ans)

Answer of the question No. 7

Students passed in history - $(H_p) = 75\%$

Students passed in mathematics $(M_p) = 65\%$

So, failed in history $(H_F) = (100 - 75) = 25\%$

failed in mathematics $(M_F) = (100 - 65) = 35\%$

50% passed both in history and mathematics
35% failed both in history and mathematics

So, $H_p \cap M_p = 50\%$

$H_F \cap M_F = 35\%$

Let $x =$ total number of students

$$n(H_p) = 0.75x$$

$$n(H_F) = 0.25x$$

$$n(M_p) = 0.65x$$

$$n(M_F) = 0.35x$$

$$n(H_p \cap M_p) = 0.50x$$

$$n(H_F \cap M_F) = 35$$

We know that,

$$n(A \cup B) = n(A) + n(B) - n(A \cap B)$$

$$n(A \cup B)^c = n(A^c \cap B^c)$$

$$\begin{aligned} \therefore n(H_p \cup M_p) &= n(H_p) + n(M_p) - n(H_p \cap M_p) \\ &= (0.75x) + (0.65x) - (0.5x) \\ &= 0.90x \end{aligned}$$

Then, $n(H_p \cup M_p)^c = n(H_p^c \cap M_p^c)$

$$\text{Or, } x - 0.90x = n(H_F \cap M_F)$$

$$\text{Or, } 0.1x = 35$$

$$\text{Or, } x = 350$$

We know,

$$\begin{bmatrix} H_p^c = H_F \\ M_p^c = M_F \end{bmatrix}$$

So, the total number of students will be 350

(Ans)

Answer of the question NO.8

Firstly we need to find the total number of possible outcomes

$$\rightarrow 780 - 299 \\ = 481 \text{ (total)}$$

so we have 481 possible outcome

Now we have to find the successful outcomes.

We need to find 1 in,
3 digit, 2 digit, 1 digit

$\rightarrow 1$ in 3 digits = 0,

because the number range is 300 to 780

$\rightarrow 1$ in 2 digits.

411, 311, 611, 511, 711

total 5

$\rightarrow 1$ in 1 digit

301, 312, 313, 314, 315, 316, 317, 318, 319, 321, 331, 341, 351,

361, 371

total 16

so from 401 to 471 we also get [16]

501 to 571 we get [16]

so, the total successful outcomes = $0 + 5 + (5 \times 16) = 85$

so, the probability is $\frac{85}{481}$

$$= 0.1767$$

(Ans.)

Answer of the question No 9.

a) there are six cars

so,

${}^{10}C_6 = 210$ ways to park the cars in
10 parking lots.

Since cars of the same colour are distinguishable,

$$\frac{6!}{2! 4!} = 15 \text{ ways to arrange}$$

$\therefore 210 \times 15 = 3150$ ways to arrange the cars.

b) We assume the empty lot is an object.

so, then we have 6 cars and group of empty lots

${}^7C_6 = 7$ ways to park the cars so that the empty
lots are together.

$\therefore 15 \times 7 = 105$ ways to park and arrange the car
so the empty lots are together

Probability of the empty lots are together:

$$\frac{105}{3150} = \frac{1}{30} \quad (\text{Ans.})$$

ANS TO THE QUESTION NUMBER :- 10

Ans. to the ques. NO. 10.

Given,

a/ Sending message,

$A_1 = \text{email}, P(A_1) = 0.4$

$A_2 = \text{letter}, P(A_2) = 0.1$

$A_3 = \text{handphone}, P(A_3) = 0.5$

Receiving message,

$P(B|A_1) = 0.6$

$P(B|A_2) = 0.8$

$P(B|A_3) = 1$

$$\begin{aligned} P(B) &= P(B|A_1)P(A_1) + P(B|A_2)P(A_2) + P(B|A_3)P(A_3) \\ &= (0.6 \times 0.4) + (0.8 \times 0.1) + (0.5 \times 1) \\ &= 0.24 + 0.08 + 0.5 \\ &= 0.82 \end{aligned}$$

Ans.

b/ Given,

The trainee receives the messages.

Now,

the conditional probabilities of receiving it via email,

$$P(A|B) = \frac{P(B|A)P(A)}{P(B)}$$

$$= \frac{0.6 \times 0.4}{0.82}$$

$$= 0.29271$$

$$= 0.29$$

Ans.

Ans. to the Ques. NO. 11.

Let,

A = Fatal accident

A' = Not fatal accident

B = Light truck

B' = Cars

Given,

$$P(B) = 0.4$$

$$P(B') = 0.6$$

$$P(A|B) = 0.0002$$

$$P(A'|B') = 0.9998$$

$$P(A|B') = 0.00025$$

$$P(A'|B) = 0.99975$$

Now,

The probability of having light truck (B),

given that accident was fatal (A) = $P(B|A)$

$$\begin{aligned}\therefore P(B|A) &= \frac{P(A|B) P(B)}{P(A|B') P(B') + P(A|B) P(B)} \\ &= \frac{(0.00025)(0.4)}{(0.00025)(0.4) + (0.0002)(0.6)} \\ &= 0.4545 \\ \text{or, } 45.45\%.\end{aligned}$$

Ans.

Ans. to the Ques. NO. 12.

Total number of letters = 9

Total numbers of boxes = 4

As given in the question, all 9 letters having different colors and we have 4 choices when to put them. So, the possible numbers of ways without restriction is,

$$4^9 = 262144$$

This disallowed ways can be counted by inclusion-exclusion. If we put the letters into 2 boxes then we have 4 choices for which boxes the letter get into and two choices per letter.

Leading to $4 \times 3^9 = 78732$ disallowed ways we count $4 \times 1^9 = 4$ ways to put all letters into one box twice. So, we subtract 3.

Therefore, The numbers of allow assignments of letters to boxes is $= 262144 - 78732 + 4$
 $= 183416$ Ans.

