ASSIGNMENT:- 01

SCHOOL OF COMPUTING
FACULTY OF ENGINEERING
SUBJECT:-DISCRETE STRUCTURE
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Group-08

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Section-08

1. Let the universal set be the set R of all real numbers and let A={x∈R | 0 < x ≤ 2}, B={x∈R | 1 ≤ x < 4} and C={x∈R | 3 ≤ x < 9}. Find each of the following:

a) A ∪ C

b) (A ∪ B)′

c) A′ ∪ B′

ANSWER: 01

Given that,

U = R = {…-2,-1,0,+1,+2…} (All real numbers)

A={x∈R | 0 < x ≤ 2}, A={1,2}

B={x∈R | 1 ≤ x < 4}, B={1,2,3,4}

C={x∈R | 3 ≤ x < 9}, C={3,4,5,6,7,8}

a). A ∪ C = {1,2} U {3,4,5,6,7,8}

= {1,2,3,4,5,6,7,8,}

= {x∈R | 0 < x < 9}

b). (A U B) = {1,2} U {1,2,3}

= {1,2,3}

(A ∪ B)′ = R – (A U B)

= R – {1,2,3}

= {x∈R | 1 > x > 3 }

c). $A' \cup B' = (A \cap B)'$ (DeMorgan's Law)

$(A \cap B) = \{1,2\}$

$(A \cap B)' = R - (A \cap B)$

$= R - \{1,2\}$

$A' \cup B' = R - \{1,2\}$

$= \{x \in R \mid 1 > x > 2\}$

2. Draw Venn diagrams to describe sets A, B, and C that satisfy the given conditions.

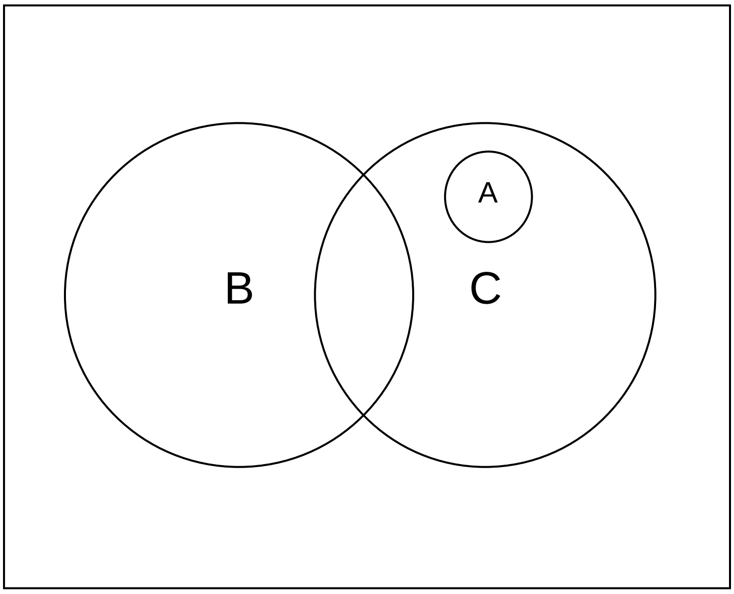
a) $A \cap B = \emptyset$, $A \subseteq C$, $C \cap B \neq \emptyset$

b) $A \subseteq B$, $C \subseteq B$, $A \cap C \neq \emptyset$

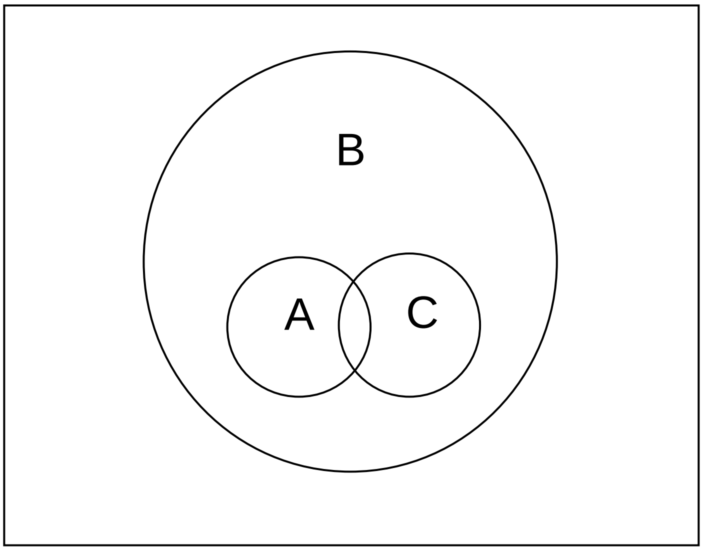
c) $A \cap B \neq \emptyset$, $B \cap C \neq \emptyset$, $A \cap C = \emptyset$, $A \not\subset B$, $C \not\subset B$

ANSWER : 02

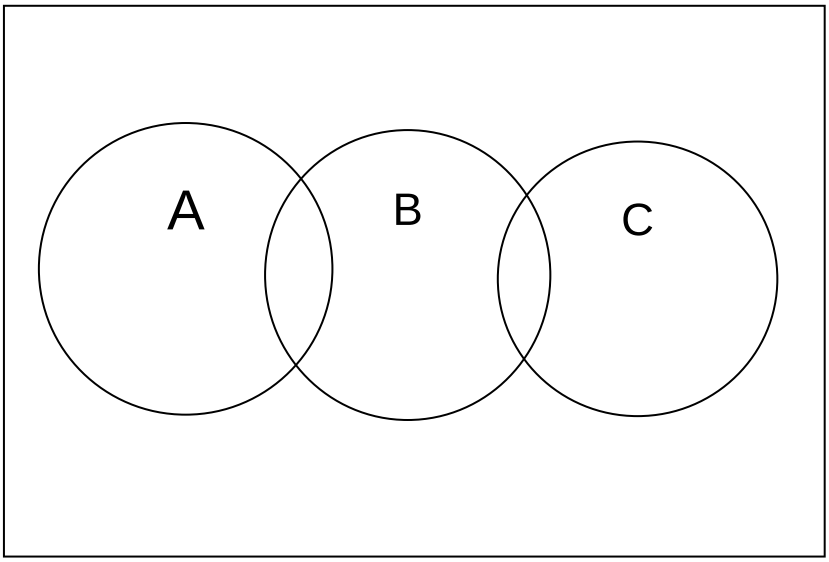
a)



b)



c)



3. Given two relations S and T from A to B, S ∩ T = {(x,y) ∈A×B | (x,y) ∈ S and (x,y) ∈ T} S ∪ T = {(x,y) ∈A×B | (x,y) ∈ S or (x,y) ∈ T} Let A={−1, 1, 2, 4} and B={1,2} and defined binary relations S and T from A to B as follows: For all (x,y) ∈A×B, x S y ↔ |x| = |y| For all (x,y) ∈A×B, x T y ↔ x− y is even State explicitly which ordered pairs are in A×B, S, T, S ∩ T, and S ∪ T

ANSWER NO: 03

Part -1

Given that

A={-1,1,2,4} and B={1,2}

A X B = {(-1, 1), (-1, 2), (1, 1), (1, 2), (2, 1), (2, 2), (4, 1), (4, 2)}

Part-2

For all (x,y) ∈A×B, x S y ↔ |x| = |y|

So, S = {(-1, 1), (1, 1), (2, 2)}

For all (x,y) ∈A×B, x T y ↔ x− y is even

So, T = {(-1, 1), (1, 1), (2, 2), (4, 2)}

Part-3

S ∩ T = {(-1, 1), (1, 1), (2, 2)}

AND

S ∪ T = {(-1, 1), (1, 1), (2, 2), (4, 2)}

4. Show that $\neg ((\neg p \wedge q) \vee (\neg p \wedge \neg q)) \vee (p \wedge q) \equiv p$. State carefully which of the laws are used at each stage.

ANSWER NO: 04

$\neg ((\neg p \wedge q) \vee (\neg p \wedge \neg q)) \vee (p \wedge q)$

$= ((p \wedge \neg q) \wedge (p \wedge q)) \vee (p \wedge q)$ **[De Morgan's Law]**

$= ((p \vee \neg q) \wedge (p \vee q)) \vee (p \wedge q)$ **[De Morgan's Law, Double negation Law]**

$= p \vee (\neg q \wedge q) \vee (p \wedge q)$ **[Associative Law]**

$= p \vee (p \wedge q)$

$= p$ **[Absorption Law]**

(showed)

5. R1={(x,y)| x+y ≤6}; R1 is from X to Y; R2={(y,z)| y>z}; R2 is from Y to Z; ordering of X, Y, and Z: 1, 2, 3, 4, 5.

Find:

a) The matrix A1 of the relation R1 (relative to the given orderings)

b) The matrix A2 of the relation R2 (relative to the given orderings)

c) Is R1 reflexive, symmetric, transitive, and/or an equivalence relation?

d) Is R Answed by :2 reflexive, antisymmetric, transitive, and/or a partial order relation?

ANSWER NO: 05

a)

R1={(x,y)| x+y ≤6}; [R1 is from X to Y]

Therefore,

R1= (1,1), (1,2), (1,3), (1,4), (1,5), (2,1), (2,2), (2,3), (2,4), (3,1), (3,2), (3,3), (4,1), (4,2), (5,1)

The matrix of A1=

	1	2	3	4	5
1	1	1	1	1	1
2	1	1	1	1	0
3	1	1	1	0	0
4	1	1	0	0	0
5	1	0	0	0	0

b)

R2={(y,z)| y>z}; [R2 is from Y to Z;]

Therefore,

R2= (2,1), (3,1), (3,2), (4,1), (4,2), (4,3), (5,1), (5,2), (5,3), (5,4)

The matrix of A2**;**

$$
\begin{array}{c|ccccc}
 & 1 & 2 & 3 & 4 & 5 \\
\hline
1 & 0 & 0 & 0 & 0 & 0 \\
2 & 1 & 0 & 0 & 0 & 0 \\
3 & 1 & 1 & 0 & 0 & 0 \\
4 & 1 & 1 & 1 & 0 & 0 \\
5 & 1 & 1 & 1 & 1 & 0
\end{array}
$$

c) R1= (1,1), (1,2), (1,3), (1,4), (1,5), (2,1), (2,2), (2,3), (2,4), (3,1), (3,2), (3,3), (4,1), (4,2), (5,1)

R1 is not reflexive. The matrix has (1,1), (2,2), (3,3). But it doesn't have (4,4) and (5,5)

This matrix is symmetric. Because all the (a,b) ∈ **R.** (1,2) and (2,1) both ∈ to **R**

(1,3) and (3,1) ∈ to **R**

(1,4) and (4,1) ∈ to **R**

So, (a,b) ∈ **R.**

The matrix is not transitive. Because we cannot find any (a,b) ∈ **R** ^ (b,c) ∈ **R** (a,c) ∈ **R.**
That is why we cannot relate to this relation to R1.

So we call, it cannot be an equivalent relation.

d)

R2= (2,1), (3,1), (3,2), (4,1), (4,2), (4,3), (5,1), (5,2), (5,3), (5,4)

R2 is an irreflexive relation, because in relation there is no (a,a) ∉**R.**

R2 is antisymmetric because there is

(2,1) ∈ R but (1,2) ∉ **R**

(3,1) ∈ R but (1,3) ∉ **R**

(3,2) ∈ R but (2,3) ∉ **R**

R2 is not transitive.

Because there is no (a,b) ∈**R** ^ (b,c) ∈**R** (a,c) ∈ **R** there.

That is why we cannot relate to this relation to R1.

So we call, It is a partial order relation.

6. Suppose that the matrix of relation R1 on {1, 2, 3} is � 1 0 0 0 1 1 1 0 1 � relative to the ordering 1, 2, 3, and that the matrix of relation R2 on {1, 2, 3} is � 0 1 0 0 1 0 1 0 1 � relative to the ordering 1, 2, 3. Find: a) The matrix of relation R1∪ R2 b) The matrix of relation R1∩ R2

ANSWER NO: 06

So, We get

R_1= {(1, 1), (2, 2), (2, 3), (3, 1), (3, 3)}
R_2= {(1, 2), (2, 2), (3, 1), (3, 3)}

(a)
$R_1 \cup R_2$ = {(1, 1), (2, 2), (2, 3), (3, 1), (3, 3)} ∪ {(1, 2), (2, 2), (3, 1), (3, 3)}

={(1, 1), (1, 2), (2, 2), (2, 3), (3, 1), (3, 3)}

The matrix of relation $R_1 \cup R_2$ is=

$$\begin{pmatrix} 1 & 1 & 0 \\ 0 & 1 & 1 \\ 1 & 0 & 1 \end{pmatrix}$$

(b)$R_1 \cap R_2$ = {(1, 1), (2, 2), (2, 3), (3, 1), (3, 3)} ∩ {(1, 2), (2, 2), (3, 1), (3, 3)

={(2, 2), (3, 1), (3, 3)}

The matrix of relation $R_1 \cap R_2$ is=

$$\begin{pmatrix} 0 & 0 & 0 \\ 0 & 1 & 0 \\ 1 & 0 & 1 \end{pmatrix}$$

7. If f :R→ R and g:R→ R are both one-to-one, is f + g also one-to-one? Justify your answer.

ANSWER NO: 07

f :**R→ R =** f (R) = R …….(i)
g:**R→ R =** g (R) = R …….(ii)
From the question f + g also one to one relaation
We know that, for all a1,a2 if f(a1)=f(a2) then a1=a2 after that this will be one to one relation.
So we get from (i+ ii)

(f + g) (R) = R + R

= 2R …….. (iii)

From we know,

Let (f + g) (R_1) = (f + g) (R_2)

2 R_1 = 2 R_2 [from iii]

$R_1 = R_2$

So, we can show that (f + g) (R) is one-to-one.

[Showed]

8. With each step you take when climbing a staircase, you can move up either one stair or two stairs. As a result, you can climb the entire staircase taking one stair at a time, taking two at a time, or taking a combination of one- or two-stair increments. For each integer n≥1, if the staircase consists of n stairs, let cn be the number of different ways to climb the staircase. Find a recurrence relation for c1, c2, …., cn.

ANSWER NO: 08

Here

So,
Let n= number of total stairs;
c_n=the number of ways to climb "n"

Now,
When n=1
Only one way to climb. So c_1=1

When, n=2
Only two ways to climb.So.c_2 =2

When , n>= 3
We need combination of one or two stair increments for 1 steps : can climb (n-1)steps
So, it can be done in c_{n-1} ways.

Or by taking 2 steps: can climb (n-2) steps.
So it can be done in c_{n-1} ways.

So now, the required recurrence relation can be formed as:

$$c_n = c_{n-1} + c_{n-2} \text{ when } n>=3$$

9. The Tribonacci sequence (tn) is defined by the equations, t1 = t2 = t3 = 1, tn = tn-1 + tn-2 + tn-3 for all n≥4. a) Find t7. b) Write a recursive algorithm to compute tn, n≥1.9

ANSWER NO: 09

a)
t1 = t2 = t3 = 1,[t1 = 1, t2 = 1, t3 = 1] and
tn = tn-1 + tn-2 + tn-3 for all n≥4

t1 = t2 = t3 = 1 -1st equation
tn = tn-1 + tn-2 + tn-3 - 2nd equation

So now,

t4 = t(4-1) + t(4-2) + t(4-3)
t4 = t3 + t2 + t1 = 1 + 1 + 1 = 3
t4 =3
t5 = t(5-1) + t(5-2) + t(5-3) = t4 + t3 + t2 = 3 + 1 + 1 = 5
t5 = 5
t6 = t(6-1) + t(6-2) + t(6-3) = t5 + t4 + t3 = 5 + 3 + 1 = 9
t6 = 9
t7 = t(7-1) + t(7-2) +t(7-3) = t6 + t5 + t4 = 9 + 5 + 3 = 17
t7 = 17

SO, t7=17

b)
```
input : n
0utput : tribonacci(n)
tribonacci(n) {
        if ( n =1 )
            Return 1
        else if ( n = 2 )
            Return 1
        else if ( n = 3 )
            Return 1
        else
            Return tribonacci(n-1) + tribonacci(n-2) + tribonacci(n-3)
        }
end if
end
```