



UTM
UNIVERSITI TEKNOLOGI MALAYSIA

Assignment 4

SECI 1013 SECTION 01
DISCRETE STRUCTURE

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SUBMITTED BY:

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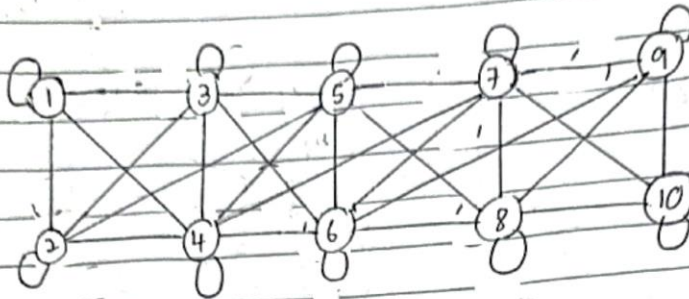
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Semester 1 2020/2021

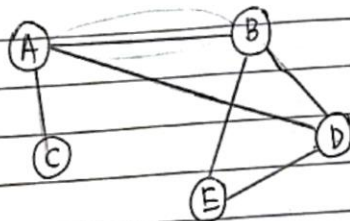
No.:

1. $V(G) = \{1, 2, 3, \dots, 10\}$



$\therefore e(G) = 34$

2. A



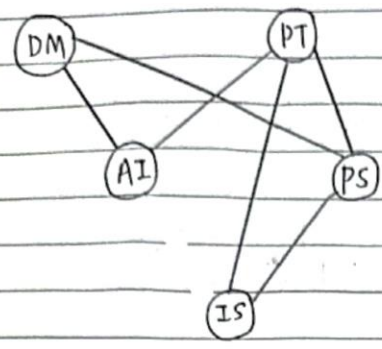
$A_G =$

	A	B	C	D	E
A	0	1	1	1	0
B	1	0	0	1	1
C	1	0	0	0	0
D	1	1	0	0	1
E	0	1	0	1	0

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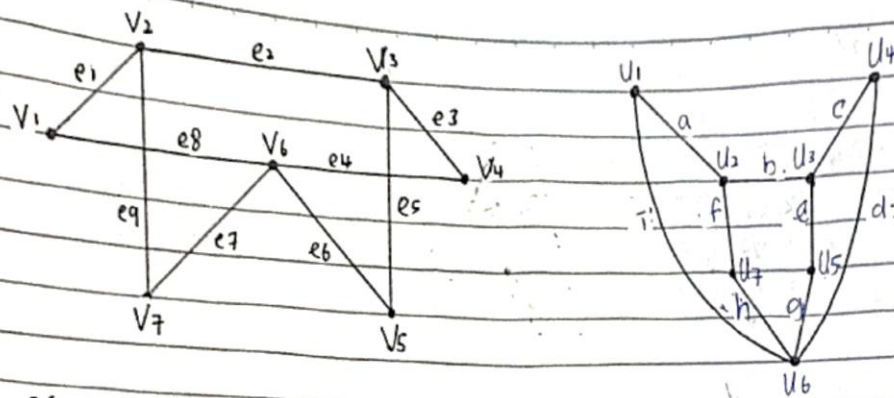
b



Ag.

	DM	PT	AI	PS	IS
DM	0	0	1	1	0
PT	0	0	1	1	1
AI	1	1	0	0	0
PS	1	1	0	0	1
IS	0	1	0	1	0

3.



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4. i)

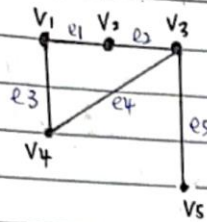
$$A_G = \begin{matrix} & \begin{matrix} V_1 & V_2 & V_3 & V_4 \end{matrix} \\ \begin{matrix} V_1 \\ V_2 \\ V_3 \\ V_4 \end{matrix} & \begin{bmatrix} 2 & 0 & 2 & 0 \\ 0 & 0 & 1 & 0 \\ 2 & 1 & 0 & 2 \\ 0 & 0 & 2 & 0 \end{bmatrix} \end{matrix}$$

$$I_G = \begin{matrix} & \begin{matrix} e_1 & e_2 & e_3 & e_4 & e_5 & e_6 \end{matrix} \\ \begin{matrix} V_1 \\ V_2 \\ V_3 \\ V_4 \end{matrix} & \begin{bmatrix} 2 & 1 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 1 & 1 & 1 & 1 & 1 \\ 0 & 0 & 0 & 0 & 1 & 1 \end{bmatrix} \end{matrix}$$

$$A_H = \begin{matrix} & \begin{matrix} V_1 & V_2 & V_3 & V_4 \end{matrix} \\ \begin{matrix} V_1 \\ V_2 \\ V_3 \\ V_4 \end{matrix} & \begin{bmatrix} 2 & 0 & 0 & 0 \\ 0 & 2 & 1 & 2 \\ 0 & 1 & 2 & 0 \\ 0 & 2 & 0 & 0 \end{bmatrix} \end{matrix}$$

$$I_G = \begin{matrix} & \begin{matrix} e_1 & e_2 & e_3 & e_4 & e_5 & e_6 \end{matrix} \\ \begin{matrix} V_1 \\ V_2 \\ V_3 \\ V_4 \end{matrix} & \begin{bmatrix} 2 & 0 & 0 & 0 & 0 & 0 \\ 0 & 2 & 1 & 1 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 & 2 \\ 0 & 0 & 1 & 1 & 0 & 0 \end{bmatrix} \end{matrix}$$

5.a)



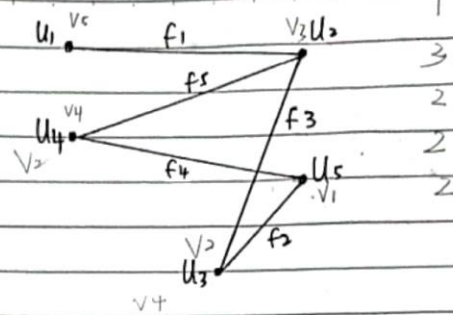
2

2

2

3

1



1

3

2

2

2

 G_1 G_2

Both graph have five vertices and five edges.

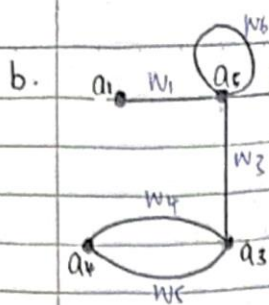
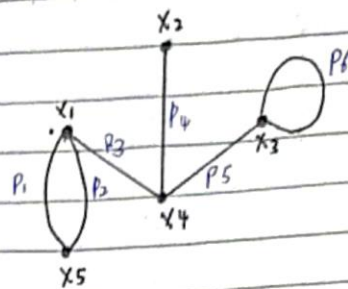
Both graph are connected and simple graph.

Both have one vertex with 1 degree, three vertex with 2 degree and one vertex with 3 degree.

	v_1	v_2	v_3	v_4	v_5		u_5	u_3	u_2	u_4	u_1		
$A_{G_1} =$	v_1	0	1	0	1	0	$A_{G_2} =$	u_5	0	1	0	1	0
	v_2	1	0	1	0	0		u_3	1	0	1	0	0
	v_3	0	1	0	1	1		u_2	0	1	0	1	1
	v_4	1	0	1	0	0		u_4	1	0	1	0	0
	v_5	0	0	1	0	0		u_1	0	0	1	0	0

Since A_{G_1} and A_{G_2} are the same.

G_1 and G_2 are isomorphic.

 H_1  H_2

Both graph have five vertices and six edges.

Both graph are connected graph.

Both graph have different degree for corresponding vertices.

Graph H_1 have 2 vertices with 1 degree, 1 vertex with 2 degree, 1 vertex with 3 degree and 1 vertex with 5 degree.

Graph H_2 have 1 vertex with 1 degree, 1 vertex with 2 degree and 3 vertices with 3 degree.

$f: H_1 \rightarrow H_2$ cannot be defined

$\therefore H_1$ and H_2 are not isomorphic.

6a) It is a trail. This is because it is a walk from v_0 to v_1 that does not contain repeated edge. It contains repeated vertex, v_1 .

b) It is a walk. This is because it contains a repeated edge, e_9 and repeated vertex, v_5 .

c) It is a closed walk. This is because it is a walk that start and ends at the same vertex, v_2 .

d) It is a circuit. This is because it is a closed walk that have at least one edge and does not contain repeated edges. It contains a repeated vertex, v_4 .

e) It is a closed walk. This is because it is a walk that start and end at the same vertex, v_2 . It contains repeated edges, e_4 and e_5 and repeated vertices, v_3 and v_4 besides the first and the last vertices.

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f. It is a path. This is because it is a trail from V_3 to V_2 that does not contain a repeated vertex.

7a) $V_1, e_1, V_2, e_2, V_3, e_5, V_4$

$V_1, e_1, V_2, e_3, V_3, e_5, V_4$

$V_1, e_1, V_2, e_4, V_3, e_5, V_4$

$\therefore$ number of path from V_1 to $V_4 = 3$

b) number of trails from V_1 to $V_4 = 3 + 3!$

$= 9$

c) number of walks from V_1 to V_4 is infinite. This is because a walk from one vertex v to a vertex w is a finite alternating sequence of adjacent vertices and edges of G . From vertex V_1 to V_4 , there are infinite possible of walks.

8. a) Graph in (a)

vertex	v_1	v_2	v_3	v_4	v_5
Degree	2	4	2	4	4

Graph in (a) has an Euler circuit since every vertex has even degree. One of the Euler circuit is $(v_1, e_1, v_2, e_2, v_5, e_7, v_4, e_6, v_5, e_3, v_2, e_4, v_3, e_5, v_4, e_8, v_1)$.

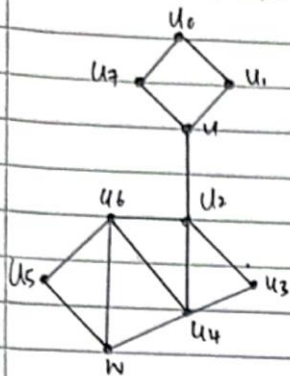
Graph in (b)

Vertex	v_0	v_1	v_2	v_3	v_4	v_5	v_6	v_7	v_8	v_9
Degree	2	5	2	2	2	4	2	3	3	3

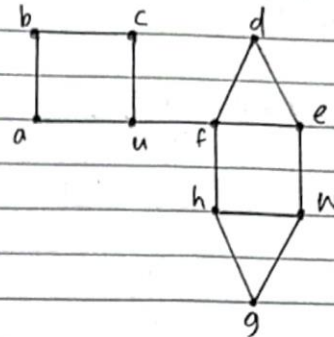
Graph in (b) does not have an Euler circuit. This is because not all vertices have even degree. Vertices v_1 , v_7 , v_8 , and v_9 have odd degree. $\deg(v_1)=5$, $\deg(v_7)=3$, $\deg(v_8)=3$ and $\deg(v_9)=3$.

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(a)



(b)

9) Graph a

Vertex	u_0	u_1	u	u_2	u_3	u_4	w	u_5	u_6	u_7
Degree	2	2	3	4	2	4	3	2	4	2

There is a euler path from u to w since it is connected and u and w are the only vertices having odd degree.

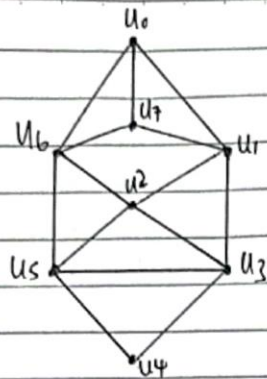
The euler path is $u, u_1, u_0, u_7, u, u_2, u_3, u_4, u_2, u_6, u_4, w, u_6, u_5, w$.

Graph b

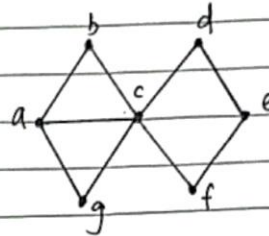
Vertex	a	b	c	u	f	d	e	w	h	g
Degree	2	2	2	3	4	2	3	3	3	2

There is no euler path from u to w . This is because u and w are not the only vertices have odd degree. Vertices e and h also have odd degree. $\deg(e) = 3$, $\deg(h) = 3$.

10.



(a)



(b)

Graph a has a Hamiltonian circuit.

$u_0 - u_6 - u_2 - u_5 - u_4 - u_3 - u_1 - u_7 - u_0$

Graph b does not have a Hamiltonian circuit.

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11. $m=3$, $n=100$

$$l = \frac{(m-1)n+1}{m}$$

$$= \frac{(3-1)100+1}{3}$$

$$= \frac{201}{3}$$

$$= 67$$

∴ A full-3-ary tree with 100 vertices will have 67 leaves.

12a) Root = a

b) Internal vertices = b, e, g, n, d, h, j

c) leaves = c, f, k, l, m, r, s, o, i, p, q

d) children of n = r, s

e) Parent of e = b

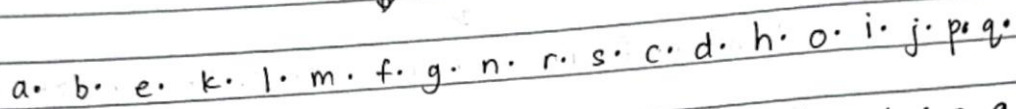
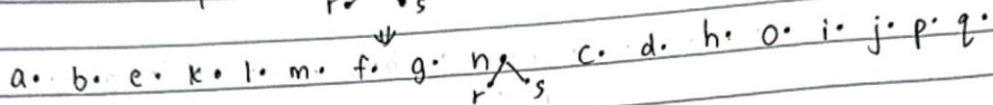
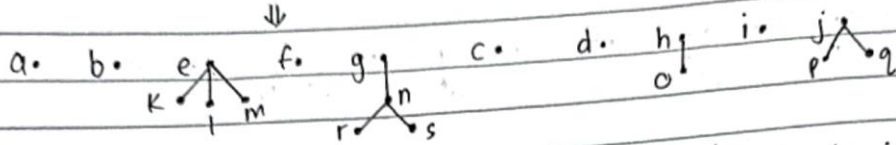
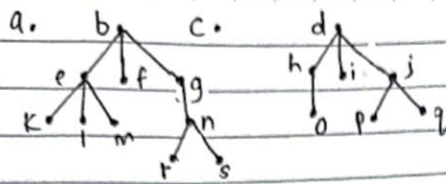
f) Siblings of k = l, m

g) proper ancestors of q = a, d, j

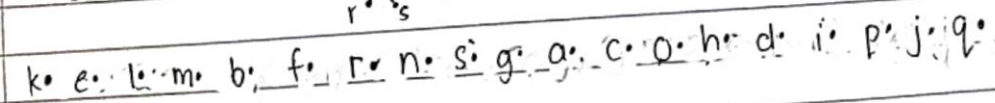
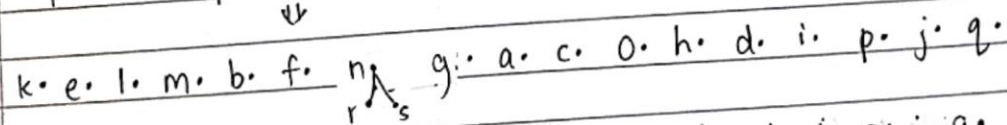
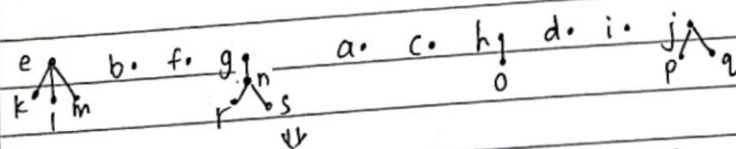
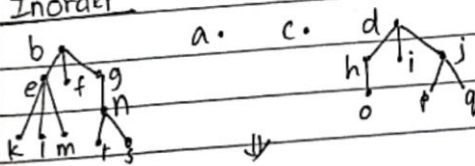
h) proper descendants of b = e, f, g, k, l, m, n, r, s

Preorder

13.

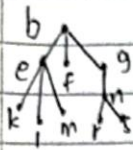


$\therefore$ preorder = a, b, e, k, l, m, f, g, n, r, s, c, d, h, o, i, j, p, q

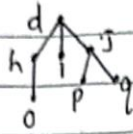
Inorder

Inorder = k, e, l, m, b, f, n, r, s, g, a, c, o, h, d, i, p, j, q

Postorder



c.



a.

↓



f.



b.

c.

h.

i.



d.

a.

↓

k. l. m. e. f. n. g. b. c. o. h. i. p. q. j. d. a.



↓

k. l. m. e. f. r. s. n. g. b. c. o. h. i. p. q. j. d. a.

∴ Postorder = k, l, m, e, f, r, s, n, g, b, c, o, h, i, p, q, j, d, a

14.

$AB = 1 \checkmark$

$GH = 1 \checkmark$

$BC = 2 \checkmark$

$AF = 3 \checkmark$

$BF = 4$

$CD = 4 \checkmark$

$BG = 5 \checkmark$

$CG = 6$

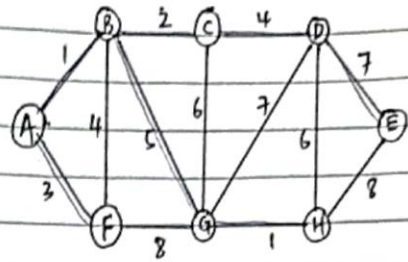
$DH = 6$

$DE = 7 \checkmark$

$DG = 7$

$FG = 8$

$EH = 8$



$\therefore$ Minimum spanning tree = $1+1+2+3+4+5+7$
 $= 23$

15.

i	S	N	$L(M)$	$L(N)$	$L(O)$	$L(P)$	$L(Q)$	$L(R)$	$L(S)$	$L(T)$
0	{ }	{M, N, O, P, Q, R, S, T}	0	∞	∞	∞	∞	∞	∞	∞
1	{M}	{N, O, P, Q, R, S, T}	0	4	∞	(2)	∞	5	∞	∞
2	{M, P}	{N, O, Q, R, S, T}	0	(4)	∞	(2)	6	5	5	∞
3	{M, P, N}	{O, Q, R, S, T}	0	(4)	10	(2)	6	(5)	5	∞
4	{M, P, N, R}	{O, Q, S, T}	0	(4)	7	(2)	6	(5)	(5)	6
5	{M, P, N, R, S}	{O, Q, T}	0	(4)	7	(2)	6	(5)	(5)	(6)
6	{M, P, N, R, S, T}	{O, Q}	0	(4)	7	(2)	6	(5)	(5)	(6)

$\therefore$ Shortest path from m to T is 6, m-R-T.