**Assignment 3**

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**SECI 1013 SECTION 01**

**DISCRETE STRUCTURE**

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# Question 1

a) $A = \{1,2,3,4,5,6,7,8\}$
$B = \{2,5,9\}$
$C = \{a,b\}$

i) $A - B = \{1, \cancel{2}, 3, 4, \cancel{5}, 6, 7, 8\} - \{\cancel{2}, \cancel{5}, 9\}$
$= \{1, 3, 4, 6, 7, 8\}$

ii) $(A \cap B) = \{2, 5\}$
$\therefore (A \cap B) \cup C = \{2, 5, a, b\}$

iii) $A \cap B \cap C = \emptyset$

iv) $B \times C = \{(2,a), (2,b), (5,a), (5,b), (9,a), (9,b)\}$

v) $P(C) = \{\emptyset, \{a\}, \{b\}, \{a,b\}\}$

b) $[P \cap ((P' \cup Q)')] \cup (P \cap Q)$

| | |
|---|---|
| $= [P \cap ((P')' \cap Q')] \cup (P \cap Q)$ | De Morgan's Laws |
| $= [P \cap (P \cap Q')] \cup (P \cap Q)$ | Double complement Laws |
| $= [(P \cap P) \cap Q'] \cup (P \cap Q)$ | Associative laws |
| $= (P \cap Q') \cup (P \cap Q)$ | Idempotent Laws |
| $= P \cap (Q' \cup Q)$ | Distributive laws |
| $= P \cap U$ | complement laws |
| $= P$ – shown | Identity Laws |

c)

| P | q | ¬P | ¬P ∨ q | q → P | A |
|---|---|---|---|---|---|
| T | T | F | T | T | T |
| T | F | F | F | T | F |
| F | T | T | T | F | F |
| F | F | T | T | T | T |

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d Let $x = 2n + 1$

$(x+2)^2 = (2n+1+2)^2$

$= (2n+3)^2$

$= 4n^2 + 12n + 9$

$= 4n^2 + 12n + 8 + 1$

$= 2(2n^2 + 6n + 4) + 1$

$= 2k + 1$, where $k = 2n^2 + 6n + 4$ is an integer

$\therefore (x+2)^2$ is an odd. - shown

e.i) $\exists x \exists y\, P(x, y)$

The statement is true.

Because there exist a value of x and y that satisfy $x \geqslant y$.

When $x = 5$, $y = 1$, $x$ is greater or equal to y.

ii) $\forall x \forall y\, P(x, y)$

The statement is false.

Because not all values of x and y satisfy $x \geqslant y$. When $x = 1$, $y = 5$, $x$ is not greater or equal to y. $P(1,5)$ is a counterexample.

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Question 2

a)

$$M_R = \begin{array}{c|ccc} & 1 & 2 & 3 \\ \hline 1 & \multicolumn{3}{c}{} \\ \end{array}$$

$$M_R = \begin{array}{cc} & \begin{array}{ccc} 1 & 2 & 3 \end{array} \\ \begin{array}{c} 1 \\ 2 \\ 3 \end{array} & \begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{bmatrix} \end{array}$$

$R = \{ (1,1), (1,2), (2,2), (3,1) \}$

i) Domain $= \{ 1, 2, 3 \}$

Range $= \{ 1, 2 \}$

ii) irreflexive ?

R is not irreflexive because the main diagonal of the matrix is not all 0's.

$$M_R = \begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{bmatrix}$$

antisymmetric ?

R is antisymmetric because

$(1,2) \in R$ but $(2,1) \notin R$

$(3,1) \in R$ but $(1,3) \notin R$

b $S = \{ (x,y) \mid x+y \geq 9 \}$

$X = \{2, 3, 4, 5\}$

i) when $x = 4$, $y = 5$, $x+y = 9$, $\geq 9$

$x = 5$, $y = 4$, $x+y = 9$, $\geq 9$

$x = 5$, $y = 5$, $x+y = 10$, $\geq 9$

$\therefore$ $S = \{ (4,5), (5,4), (5,5) \}$

ii) Reflexive ?

$$M_S = \begin{array}{c} \\ 2 \\ 3 \\ 4 \\ 5 \end{array} \begin{array}{c} \begin{array}{cccc} 2 & 3 & 4 & 5 \end{array} \\ \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 1 \end{bmatrix} \end{array}$$

S is not reflexive since the main diagonal of the matrix relation is not all 1's.

Symmetric ?

S is symmetric since

$(4,5) \in S$ and $(5,4) \in S$.

transitive ?

S is not transitive because

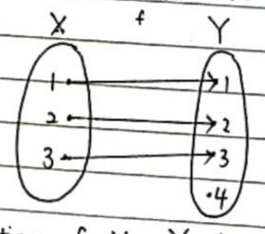
$(4,5) \in S$ , $(5,4) \in S$ but $(4,4) \notin S$.

Equivalent relation ?

S is not equivalent relation since it is symmetric but not reflexive and transitive.

c) $X = \{1, 2, 3\}$ , $Y = \{1, 2, 3, 4\}$ , $Z = \{1, 2\}$

i) $f = \{(1,1), (2,2), (3,3)\}$




function $f: X \to Y$ is one-to-one since it have at most one arrow point to element in Y.

function $f: X \to Y$ is not onto because there is no arrow point to element 4 in Y.

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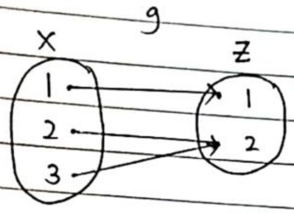
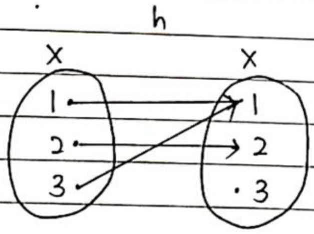
ii) $g = \{(1,1), (2,2), (3,2)\}$.




function g is onto because there is at least one arrow point to each element in Z.
function g is not one-to-one because there is two arrow point to element 2 in Z.

iii) $h = \{(1,1), (2,2), (3,1)\}$




function h is not one-to-one because there is two arrow point to element 1.
function h is not onto because there is no arrow point to element 3.

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d i) Let $y = 4x + 3$

$4x = y - 3$

$x = \frac{y-3}{4}$

$\therefore m^{-1}(x) = \frac{x-3}{4}$

ii) $n \circ m = n[m(x)]$

$= n[4x+3]$

$= 2(4x+3) - 4$

$= 8x + 6 - 4$

$= 8x + 2$

$\therefore n \circ m = 8x + 2$

Question 3

a $a_k = a_{k-1} + 2k \quad , \quad k \geq 2 \quad , \quad a_1 = 1$

i) $a_2 = a_1 + 2(2)$

$= 1 + 4$

$= 5$

$a_3 = a_2 + 2(3)$

$= 5 + 6$

$= 11$

$\therefore$ The first three term is 1, 5, 11.

ii) Input = k

output = ak

```
a(k)
{
    if  k = 1
        return 1
    else
        return a(k-1) + 2(k)
}
```

b Let $r_k$ = no. of executes with an input size k

$k > 1$

$r_1 = 7$

$r_2 = 2(r_1) = 2(7) = 14$

$r_3 = 2(r_2) = 2(14) = 28$

$r_4 = 2(r_3) = 2(28) = 56$

$\vdots$

$r_k = 2r_{k-1} \quad , \quad k > 1 \quad , \quad r_1 = 7.$

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C. $S(4) = 5 \times S(3) = 5 \times 125 = 625$

$S(3) = 5 \times S(2) = 5 \times 25 = 125$

$S(2) = 5 \times S(1) = 5 \times 5 = 25$

$S(1) = 5$

$\therefore S(4) = 625$

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Question 4

a) first digit = { 3, 4, 5, 6, 7, 8, 9, A, B }
= 9 ways.

Second digit = 16 ways

Third digit = 16 ways

Fourth digit = { 5, 6, 7, 8, 9, A, B, C, D, E, F }
= 11 ways.

$\underline{9}\ \underline{16}\ \underline{16}\ \underline{11}$

$\therefore$ n(ways) = 9 × 16 × 16 × 11
= 25 344

b)

| letter | digit |
|---|---|
| $\underset{A}{\underline{1}}\ \underline{26}\ \underline{26}\ \underline{26}$ | $\underline{10}\ \underline{10}\ \underset{0}{\underline{1}}$ |

letter = 26 (A to Z)
digit = 10 ( 0 to 9)

$\therefore$ n(ways) = 1 × 26 × 26 × 26 × 10 × 10 × 1
= 1757600

c) one letter used = 8
two letter used = $\underline{8}\ \underline{7}$ = ${}^{8}P_{2}$
three letter used = $\underline{8}\ \underline{7}\ \underline{6}$ = ${}^{8}P_{3}$

$\therefore$ total n(ways) = $8 + {}^{8}P_{2} + {}^{8}P_{3}$
= 8 + 56 + 336
= 400

d) 7W , 6M
n(ways) = ${}^{7}C_{4} \times {}^{6}C_{3}$
= 35 × 20
= 700

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e) PROBABILITY

2B, 2I

$$n(\text{ways}) = \frac{11!}{2! \times 2!}$$

$$= 997\ 9200$$

f) $n = 6$

$r = 10$

$$n(\text{ways}) = C(6+10-1,\ 10)$$

$$= \frac{(6+10-1)!}{10!\ (6-1)!}$$

$$= \frac{15!}{10!\ 5!}$$

$$= 3003$$

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# Question 5

a) Pigeon = no of person

= 18

Pigeonhole = no. of combination of first name and last name

= 3 x 2

= 6

By generalized pigeonhole principle

$$k = \left\lceil \frac{n}{m} \right\rceil$$

$$= \left\lceil \frac{18}{6} \right\rceil$$

$$= \lceil 3 \rceil = 3$$

∴ There is at least three person have the same first and last names. — shown.

b) odd number = {1, 3, 5, 7, 9, 11, 13, 17, 19}

|odd number| = 9

even number = {2, 4, 6, 8, 10, 12, 14, 16, 18, 20}

|even number| = 10

The first ten integers picked may be all ten even number.
Then, one more pick is needed to be sure getting at least one is odd integer. 10+1 = 11

∴ 11 integer must be pick to get sure at least one is odd integer.

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C Let A = integers from 1 through 100 which is divisible by 5.
B = integer from 1 through 100 which is not divisble by 5.

$A = \{5, 10, 15, 20, 25, 30, 35, 40, 45, 50, 55, 60, 65, 70, 75, 80, 85, 90, 95, 100\}$

$|A| = 20$

$\therefore |B| = 100 - 20$
$= 80$

The 80 integer may all from set B. Thus, one more pick is needed. from set A to be sure one integer is divisible by 5.

$\therefore 80 + 1 = 81$

81 integers must be picked to get sure one is divisible by 5.