Assignment 2

SECI 1013 SECTION 01

DISCRETE STRUCTURE

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Semester 1 2020/2021

Subject :

Date:

1.a) $\underline{6}\ \underline{6}\ \underline{6}$

$n(ways) = 6^3$

$= 216$

b) $\underline{6}\ \underline{5}\ \underline{4}$

$n(ways) = 6 \times 5 \times 4$

$= 120$

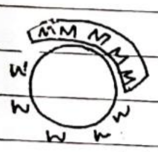
c) odd digits $= \{3, 5, 7\}$

$301 \rightarrow 699$ $\quad \underset{\times 7}{\underline{2}}\ \underline{3}\ \underline{3}$

$n(ways) = 2 \times 3 \times 3$

$= 18$

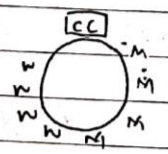
2 a)



$n(ways) = (6-1)! \times 5!$

$= 14400$

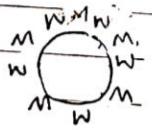
b)



$n(ways) = (9-1)! \times 2!$

$= 80\,640$

c)



n(ways) men arrange first $= (5-1)!$

$= 4!$

$= 24$

n(ways) women arrange later $= 5!$

$= 120$

Total n(ways) men and women sit alternate $= 24 \times 120 = 2880$

d. $\boxed{A\ H}$ F F F F F F P F P P

$n(\text{ways}) = 11! \times 2!$

$= 79833600$

3.a) $n(\text{ways}) = 5!$

$= 120$

b) no. of ways choose 2 sprinters tie $= {}^5C_2$

$= 10$

no. of ways to finish if two sprinters tie $= 10 \times 4!$

$= 240$

c) no. of ways choose two group of two sprinters tie $= \dfrac{5!}{2! \times 2!}$

$= 30$

no. of ways to finish if two group of two sprinters tie $= 30 \times 3!$

$= 180$

Subject :

Date:

4. a) $n = 6$, $r = 12$

$$n(ways) = C(6+12-1, 12)$$
$$= \frac{(6+12-1)!}{12!(6-1)!}$$
$$= \frac{17!}{12!\,5!}$$
$$= 6188$$

b) There are 6 different types of croissants. and there are at least 2 item of each types need to choose.

Thus, we choose 2 croissants from each types first (12 croissants chosen)

$$n(ways) = 1$$

Next, we choose 12 additional to get 2 dozen.

$$n(ways) = C(6+12-1, 12)$$
$$= \frac{17!}{12!\,5!}$$
$$= 6188$$

$$\text{Total } n(ways) = 1 \times 6188$$
$$= 6188$$

c) We choose 5 chocolate croissants and 3 almond croissants,

$$n(ways) = 1 \times 1$$
$$= 1$$

no of croissant need to choose left $= 24 - 8$
$= 16.$

Next, we choose 16 out of 6 types to get 2 dozen,

$n = 6$, $r = 16$,

$$n(ways) = C(6+16-1, 16)$$

Subject :

$$n(\text{ways}) = \frac{(6+16-1)!}{16!\,(6-1)!}$$

$$= \frac{21!}{16!\,5!}$$

$$= 20349$$

$\therefore$ Total $n(\text{ways}) = 1 \times 20349$

$$= 20349$$ #

5.a) Condition for the round ends once it is impossible for a team to equal the number of goals scored by the other team:

- 2 wins for one of the team with 1 additional tie or wins for that teams

OR

1 wins for one of the team with 3 additional tie or wins for that teams.

$\therefore$ 2 wins among 4 games $= C(4,2)$

$$= \frac{4!}{2!(4-2)!}$$

$$= \frac{4!}{2!2!}$$

$$= 6$$

1 wins among 3 games $= C(3,1)$

$$= \frac{3!}{1!(3-1)!}$$

$$= \frac{3!}{1!2!}$$

$$= 3$$

2 option for each of the tie/win games and this tie/win need to occur in the remaining games (5 games excluding the known games)

2 wins and 1 tie/wins $= C(4,2) \times C(3,1) \times 2$

$$= 6 \times 3 \times 2$$

$$= 36$$

1 wins and 3 tie/wins $= C(3,1) \times C(4,3) \times 2^3$

$$= 96$$

$\therefore$ Total no of ways $= (36 + 96) \times 2$

$$= 264$$

Date:

Subject :

b) 10 penalty kicks are played, the no. of ways for first round $= 2^{10}$
$= 1024$

The result in the game not being settled in the first round of 10 penalty kicks $= 1024 - 264$
$= 760$

n(ways) the game no settle in first round $= 760$
n(ways) the game settle in second round $= 264$

$\therefore$ Total n(ways) of scenarios $= 760 \times 264$
$= 200\,640$

c) n(ways) the game no settle in first round $= 760$
n(ways) the game no settle in second round $= 760$
n(ways) for sudden death $= 2+2+2+2 = 10$

$\therefore$ Total no of scenarios $= 760 \times 760 \times 10$
$= 5\,776\,000$

Subject : Date:

6. Pigeonhole = no. of possible answer sheet

$= 4^{10}$

Let Pigeon = : n

By generalized pigeonhole principle,

$$\left\lceil \frac{n}{4^{10}} \right\rceil = 3$$

$$n = 4^{10}(3-1)+1$$
$$= 4^{10}(2)+1$$
$$= 2097153$$

∴ The min no. of students that must be in the professor's class in order to guarantee that at least three answer sheets must be identical is 2097153.

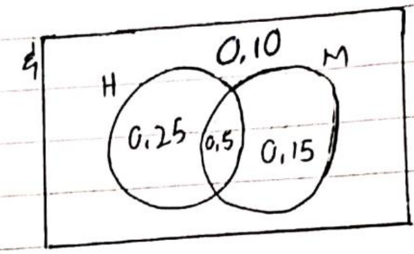
Date:

Subject :

7 Let

H : students have passed in history

M : Students have passed in Mathematics



$\therefore P(H) = 0.75$

$P(M) = 0.65$

$P(H \cap M) = 0.50$

$n(H' \cap M') = 35$

$P(H' \cap M') = 1 - (0.25 + 0.5 + 0.15)$

$= 0.10$

Let total no. of candidates $= x$

$$x \times 0.10 = 35$$

$$x = 350$$

$\therefore$ no. of candidates sits for the exam $= 350$

Subject :

Date:

8. Total no. of integer from 300 through 780 (inclusive) = 780 - 300 + 1

= 481

no. of digit = {0, 1, 2, 3, 4, 5, 6, 7, 8, 9}

= 10 digit

n(ways) '1' in one digit :

$3 \_ \underset{1}{\_}$ = $1 \times 9 \times 1 \times 2 = 18$

$4 \_ \underset{1}{\_}$ = $1 \times 9 \times 1 \times 2 = 18$

$5 \_ \underset{1}{\_}$ = $1 \times 9 \times 1 \times 2 = 18$

$6 \_ \underset{1}{\_}$ = $1 \times 9 \times 1 \times 2 = 18$

$7 \_ \underset{1}{\_}$ or $7 \underset{1}{\_} \_$ = $1 \times 7 \times 1 + 1 \times 1 \times 9 = 16$

7 digit can used ('9','8' x '1' x)

9 digit can used

∴ Total n(ways) '1' in one digit = 4(18) + 16

= 88

n(ways) '1' in two digit :

$3 \underset{1}{\_} \underset{1}{\_}$ = $1 \times 1 \times 1 = 1$

∴ 4 _ _, 5 _ _, 6 _ _, 7 _ _ = 1×4

= 4

∴ Total n(ways) '1' in two digit = 1×5

= 5

∴ total n(ways) the no. choosen will have at least '1' in the digit

= 88 + 5

= 93

Subject :

∴ Probability that the number is choosen will have 1 as at least one digit = $\frac{93}{481}$

9. 2B, 4Y

a) n(ways) the cars be parked in the parking lots

$$= \frac{{}^{10}P_6}{4! \times 2!}$$

$= 3150$

b) $\underbrace{\boxed{- - - -}}_{\text{Empty lots (identical)}}$ B B Y Y Y Y

n(ways) the cars parked so that empty lots are next to each other $= \frac{7!}{2! \times 4!} \times \frac{4!}{4!}$

$= 105$

Probability that the empty slot next to each other $= \frac{105}{3150}$

$= \frac{1}{30}$

Subject :

Date:

10\. Let E : The coach uses email

L = The coach uses letter

H = The coach uses handphone

R : The trainee receive message

$P(E) = 0.4$ $\quad$ $P(R|E) = 0.6$

$P(L) = 0.1$ $\quad$ $P(R|L) = 0.8$

$P(H) = 0.5$ $\quad$ $P(R|H) = 1$

a) $P(R) = P(R|E)P(E) + P(R|L)P(L) + P(R|H)P(H)$

$= 0.6(0.4) + 0.8(0.1) + 1(0.5)$

$= 0.82$

b) $P(E|R) = \dfrac{P(R|E) \times P(E)}{P(R)}$

$= \dfrac{0.6 \times 0.4}{0.82}$

$= \dfrac{0.24}{0.82}$

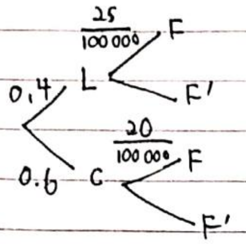
$= \dfrac{12}{41}$

Subject : Date:

11. Let L = light trucks

C = cars

F = fatal accidents



$\therefore$ $P(L) = 0.4$

$P(C) = 0.6$

$P(F|L) = \frac{25}{100\,000}$

$= 0.00025$

$P(F|C) = \frac{20}{100\,000}$

$= 0.0002$

$$P(L|F) = \frac{P(F|L) \times P(L)}{P(F)}$$

$$= \frac{0.00025 \times 0.4}{0.00025(0.4) + 0.0002(0.6)}$$

$$= \frac{5}{11}$$

Subject: Date:

12. Let X : total no of ways 9 letters can be distribute into 4 boxes.

$\therefore |X| = 4^9$

Let T : the no of ways such that the tetrahedon box has no letters

C : the no of ways such that cube box has no letters.

P : the no of ways such that polyhedron box has no letters.

D : the no of ways such that dodecahedron box has no letters.

Formula using : $\sum_{i=1}^{n} |A_i| - \sum_{1\le i<j\le n} |A_i \cap A_j| + \sum_{1\le i<j<k\le n} |A_i \cap A_j \cap A_k| - \dots$

$$+ (-1)^{n-1} |A_1 \cap \dots \cap A_n|$$

$|T| = |C| = |P| = |D| = {}^4C_1 \times 3^9$

$|T \cap C| = |T \cap P| = |T \cap D| \dots = |P \cap D| = {}^4C_2 \times 2^9$

$|T \cap C \cap P| = |T \cap C \cap D| = |T \cap P \cap D| = \dots = |C \cap P \cap D| = {}^4C_3 \times 1^9$

$|T \cap C \cap P \cap D| = {}^4C_4 \times 0^9 = 0$

Total no. of ways such that each box contain at least 1 letter

$= |X| - |T \cup C \cup P \cup D|$

$= 4^9 - [{}^4C_1 \times 3^9 - {}^4C_2 \times 2^9 + {}^4C_3 \times 1^9 - {}^4C_4 \times 0^9]$

$= 4^9 - (78732 - 3072 + 4 - 0)$

$= 186480$