



UTM
UNIVERSITI TEKNOLOGI MALAYSIA

SECI 1013 SECTION 01
DISCRETE STRUCTURE

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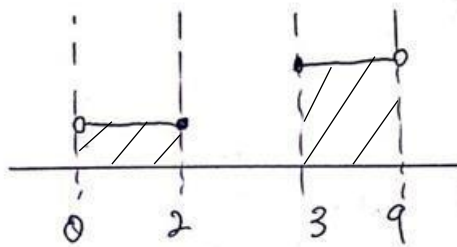
Semester 1 2020/2021

$$1) a) A = \{x \in \mathbb{R} \mid 0 < x \leq 2\}$$

$$B = \{x \in \mathbb{R} \mid 1 \leq x < 4\}$$

$$C = \{x \in \mathbb{R} \mid 3 \leq x < 9\}$$

$$A \cup C = \{x \in \mathbb{R} \mid 0 < x \leq 2 \text{ or } 3 \leq x < 9\}$$



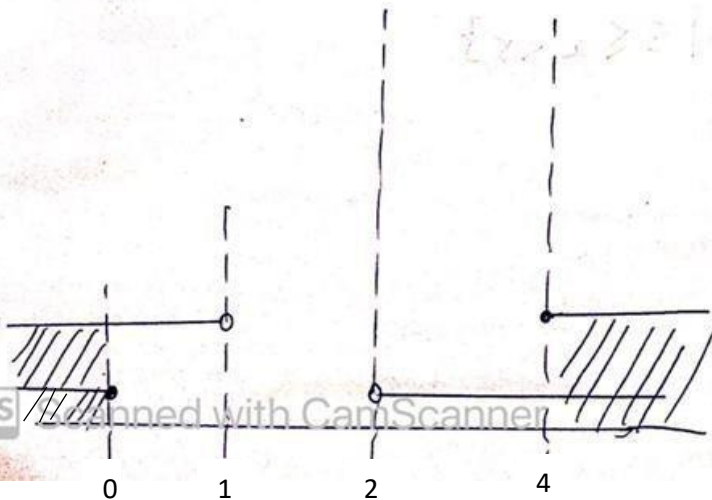
$$b) A' = \{x \in \mathbb{R} \mid x \leq 0 \text{ or } x > 2\}$$

$$B' = \{x \in \mathbb{R} \mid x < 1 \text{ or } x \geq 4\}$$

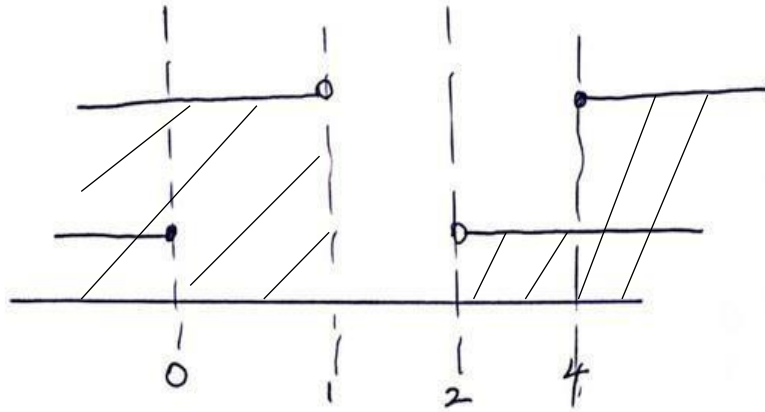
$$(A \cup B)' = A' \cap B' \quad \text{De Morgan's Law}$$

$$= \{x \in \mathbb{R} \mid (x \leq 0 \text{ or } x > 2) \text{ and } (x < 1 \text{ or } x \geq 4)\}$$

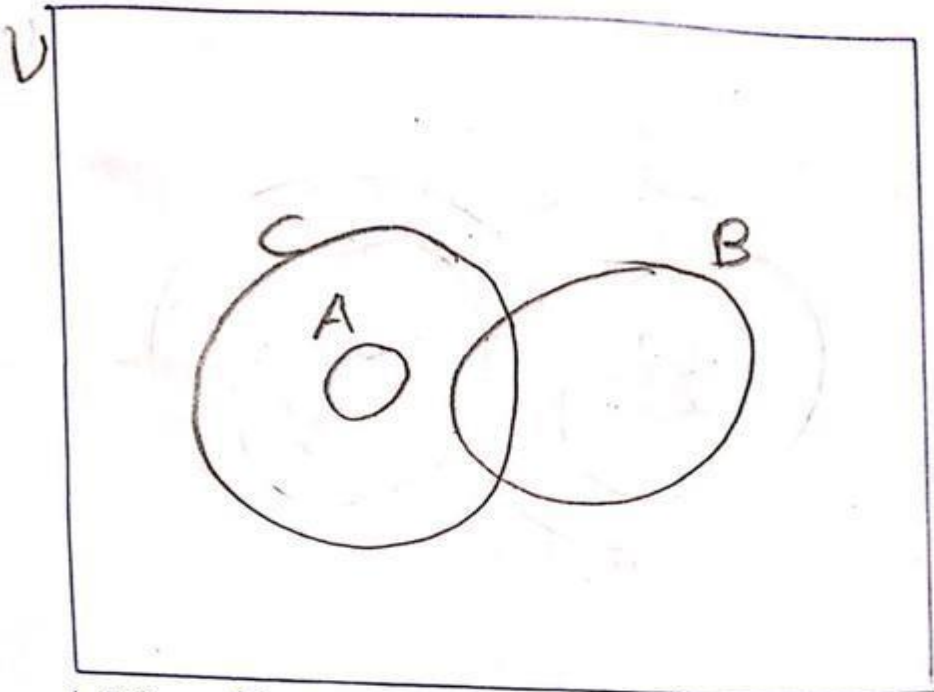
$$= \{x \in \mathbb{R} \mid x \leq 0 \text{ or } x \geq 4\}$$



$$\begin{aligned} \text{d) } A' \cup B' &= \{x \in \mathbb{R} \mid (x \leq 0 \text{ or } x \geq 2) \text{ or } (x < 1 \text{ or } x \geq 4)\} \\ &= \{x \in \mathbb{R} \mid x < 1 \text{ or } x \geq 2\} \end{aligned}$$



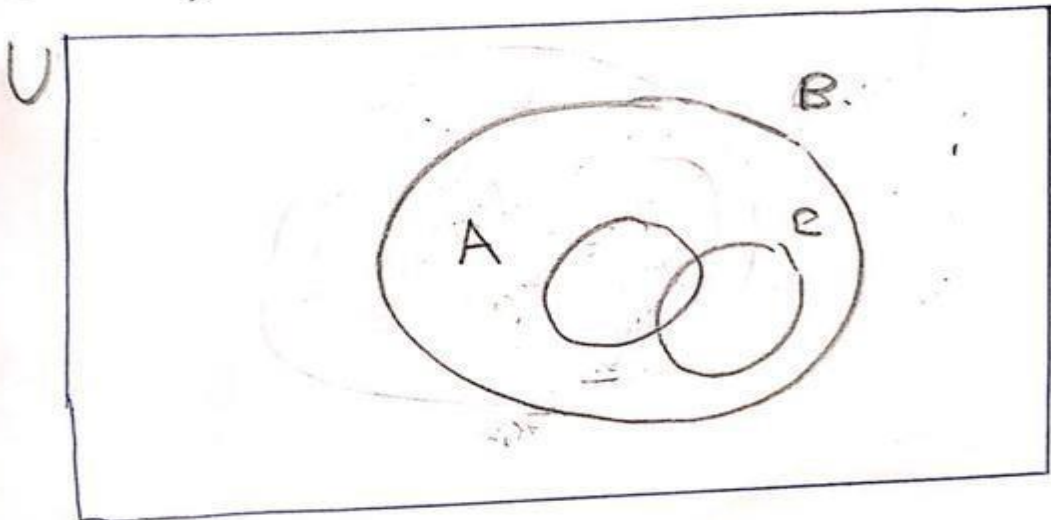
2) a)



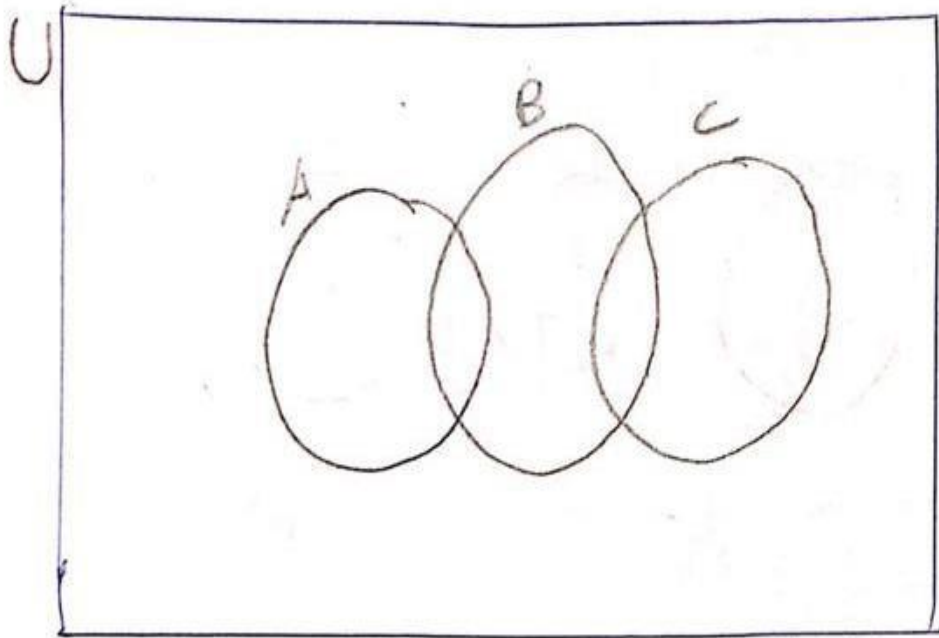
$$A \cap B = \emptyset, A \subseteq C, C \cap B \neq \emptyset$$

b)

$$A \subseteq B, C \subseteq B, A \cap C \neq \emptyset$$



(c) $A \cap B \neq \emptyset$, $B \cap C \neq \emptyset$, $A \cap C = \emptyset$, $A \not\subset B$, $C \not\subset B$



$$3) A \times B = \{(-1,1), (-1,2), (1,1), (1,2), (2,1), (2,2), (4,1), (4,2)\}$$

$$S = \{(-1,1), (1,1), (2,2)\}$$

$$T = \{(-1,1), (1,1), (2,2), (4,2)\}$$

$$S \cap T = \{(-1,1), (1,1), (2,2)\}$$

$$S \cup T = \{(-1,1), (1,1), (2,2), (4,2)\}$$

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$$4. \neg[(\neg p \wedge q) \vee (\neg p \wedge \neg q)] \vee (p \wedge q)$$

$$\equiv [\neg(\neg p \wedge q) \wedge \neg(\neg p \wedge \neg q)] \vee (p \wedge q)$$

De Morgan's Law

$$\equiv [(\neg(\neg p) \vee \neg q) \wedge (\neg(\neg p) \vee \neg(\neg q))] \vee (p \wedge q)$$

De Morgan's Law

$$\equiv [(p \vee \neg q) \wedge (p \vee q)] \vee (p \wedge q)$$

Double negation Law

$$\equiv [p \vee (\neg q \wedge q)] \vee (p \wedge q)$$

Distribution Law

$$\equiv (p \vee F) \vee (p \wedge q)$$

 $\neg q \wedge q \equiv \text{False}$

$$\equiv p \vee (p \wedge q)$$

Identity Law for False

$$\equiv p \text{ — shown}$$

Absorption Law

$$5a) R_1 = \{(x, y) \mid x + y \leq 6\}$$

$$x, y = 1, 2, 3, 4, 5$$

$$R_1 = \{(1, 1), (1, 2), (1, 3), (1, 4), (1, 5), (2, 1), (2, 2), (2, 3), (2, 4), (3, 1), (3, 2), (3, 3), (4, 1), (4, 2), (5, 1)\}$$

$$M_{A1} =$$

	1	2	3	4	5
1	1	1	1	1	1
2	1	1	1	1	0
3	1	1	1	0	0
4	1	1	0	0	0
5	1	0	0	0	0

$$b) R_2 = \{(y, z) \mid y > z\}$$

$$y, z = 1, 2, 3, 4, 5$$

$$R_2 = \{(2, 1), (3, 1), (3, 2), (4, 1), (4, 2), (4, 3), (5, 1), (5, 2), (5, 3), (5, 4)\}$$

$$M_{A2} =$$

	1	2	3	4	5
1	0	0	0	0	0
2	1	0	0	0	0
3	1	1	0	0	0
4	1	1	1	0	0
5	1	1	1	1	0

c) Is R_1 reflexive?

R_1 is not reflexive because the main diagonal of matrix relation is 1's and 0's and not all 1's.

$$M_{A_1} = \begin{bmatrix} 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 0 \\ 1 & 1 & 1 & 0 & 0 \\ 1 & 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 \end{bmatrix}$$

Is R_1 symmetric?

$$M_{A_1}^T = \begin{bmatrix} 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 0 \\ 1 & 1 & 1 & 0 & 0 \\ 1 & 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 \end{bmatrix}$$

R_1 is symmetric because $M_{A_1} = M_{R_1}^T$

Is R_1 transitive?

$$\begin{bmatrix} 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 0 \\ 1 & 1 & 1 & 0 & 0 \\ 1 & 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 \end{bmatrix} \otimes \begin{bmatrix} 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 0 \\ 1 & 1 & 1 & 0 & 0 \\ 1 & 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 \end{bmatrix}$$

R_1 is not transitive since $M_{A_1} \otimes M_{A_1} \neq M_{A_1}$

Is R_1 an equivalence relation?

R_1 is not an equivalence relation since it is symmetric but not reflexive and transitive.

d) Is R_2 reflexive?

R_2 is not reflexive because the main diagonal of the matrix of relation is all 0's and not all 1's.

$$M_{A_2} = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 \\ 1 & 1 & 0 & 0 & 0 \\ 1 & 1 & 1 & 0 & 0 \\ 1 & 1 & 1 & 1 & 0 \end{bmatrix}$$

Is R_2 antisymmetric?

R_2 is antisymmetric because,

$$(2,1) \in R_2 \text{ and } (1,2) \notin R_2$$

$$(3,1) \in R_2 \text{ and } (1,3) \notin R_2$$

$$(3,2) \in R_2 \text{ and } (2,3) \notin R_2$$

$$(4,1) \in R_2 \text{ and } (1,4) \notin R_2$$

$$(4,3) \in R_2 \text{ and } (3,4) \notin R_2$$

$$(5,1) \in R_2 \text{ and } (1,5) \notin R_2$$

$$(5,2) \in R_2 \text{ and } (2,5) \notin R_2$$

$$(5,3) \in R_2 \text{ and } (3,5) \notin R_2$$

$$(5,4) \in R_2 \text{ and } (4,5) \notin R_2$$

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6. .

$$M_{R_1} = \begin{matrix} & \begin{matrix} 1 & 2 & 3 \end{matrix} \\ \begin{matrix} 1 \\ 2 \\ 3 \end{matrix} & \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 1 \\ 1 & 0 & 1 \end{bmatrix} \end{matrix}$$

$$\therefore R_1 = \{(1,1), (2,2), (2,3), (3,1), (3,3)\}$$

$$M_{R_2} = \begin{matrix} & \begin{matrix} 1 & 2 & 3 \end{matrix} \\ \begin{matrix} 1 \\ 2 \\ 3 \end{matrix} & \begin{bmatrix} 0 & 1 & 0 \\ 0 & 1 & 0 \\ 1 & 0 & 1 \end{bmatrix} \end{matrix}$$

$$\therefore R_2 = \{(1,2), (2,2), (3,1), (3,3)\}$$

$$a) R_1 \cup R_2 = \{(1,1), (1,2), (2,2), (2,3), (3,1), (3,3)\}$$

$$\therefore M_{R_1 \cup R_2} = \begin{matrix} & \begin{matrix} 1 & 2 & 3 \end{matrix} \\ \begin{matrix} 1 \\ 2 \\ 3 \end{matrix} & \begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & 1 \\ 1 & 0 & 1 \end{bmatrix} \end{matrix}$$

$$b) R_1 \cap R_2 = \{(2,2), (3,1), (3,3)\}$$

$$\therefore M_{R_1 \cap R_2} = \begin{matrix} & \begin{matrix} 1 & 2 & 3 \end{matrix} \\ \begin{matrix} 1 \\ 2 \\ 3 \end{matrix} & \begin{bmatrix} 0 & 0 & 0 \\ 0 & 1 & 0 \\ 1 & 0 & 1 \end{bmatrix} \end{matrix}$$

7. If $f: \mathbb{R} \rightarrow \mathbb{R}$ and $g: \mathbb{R} \rightarrow \mathbb{R}$ are both one-to-one, $f+g$ is not one-to-one.

Let $f(x) = x$, $\forall x \in \mathbb{R}$

$f(x)$ is one-to-one since

$$f(a) = f(b)$$

$$a = b$$

Let $g(x) = -x$, $\forall x \in \mathbb{R}$

$g(x)$ is one-to-one since

$$g(a) = g(b)$$

$$-a = -b$$

$$a = b$$

$$(f+g)(x) = f(x) + g(x)$$

$$= x + (-x)$$

$$= 0$$

$$\therefore (f+g)(x) = 0, \forall x \in \mathbb{R}$$

Let $x = 1$,

$$(f+g)(1) = 0$$

Let $x = 2$,

$$(f+g)(2) = 0$$

$$\therefore (f+g)(1) = (f+g)(2)$$

$$1 \neq 2$$

$\therefore f+g$ is not one-to-one.

Subject :

8. For n staircase, $n \geq 1$

C_n : number of different ways to climb staircase

$$C_1 = 1$$

$$C_2 = 2$$

$$C_3 = 2 + 1 = 3 \Rightarrow C_2 + C_1$$

$$C_4 = 3 + 2 = 5 \Rightarrow C_3 + C_2$$

$$C_5 = 5 + 3 = 8 \Rightarrow C_4 + C_3$$

$\vdots$

$$C_n = C_{n-1} + C_{n-2}, \quad C_1 = 1, C_2 = 2, n > 2$$

q. $t_0=0, t_1=1, t_2=1, t_n = t_{n-1} + t_{n-2} + t_{n-3}$ for all $n \geq 3$

a) $t_3 = t_2 + t_1 + t_0$ $t_5 = t_4 + t_3 + t_2$ $t_7 = t_6 + t_5 + t_4$
 $= 1 + 1 + 0$ $= 4 + 2 + 1$ $= 13 + 7 + 4$
 $= 2$ $= 7$ $= 24$

$t_4 = t_3 + t_2 + t_1$ $t_6 = t_5 + t_4 + t_3$ $\therefore t_7 = 24$
 $= 2 + 1 + 1$ $= 7 + 4 + 2$
 $= 4$ $= 13$

b) $t(n)$

{ if $(n=0)$

return 0

else if $(n=1$ or $n=2)$

return 1

else

return $t(n-1) + t(n-2) + t(n-3)$

}