SECI 1013 (Discrete Structure)

SEMESTER 1, 2020/2021

GROUP ASSIGNMENT

SECTION: 3

GROUP MEMBER:

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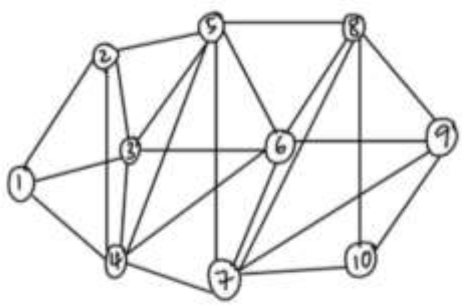
1. Let G be a graph with $V(G) = \{1, 2, \ldots, 10\}$, such that two numbers ‘v’ and ‘w’ in $V(G)$ are adjacent if and only if | v – w| ≤ 3. Draw the graph G and determine the numbers of edges,
   $e(G)$.

   1 = {(1,2), (1,3), (1,4)}
   2 = {(2,1}, (2,3), (2,4), (2,5)}
   3 = {(3,1), (3,2), (3,4), (3,5), (3,6)}
   4 = {(4,1), (4,2), (4,3), (4,5), (4,6), (4,7)}
   5 = {(5,2), (5,3), (5,4), (5,6), (5,7), (5,8)}
   6 = {(6,3), (6,4), (6,5), (6,7), (6,8), (6,9)}
   7 = {(7,4), (7,5), (7,6), (7,8), (7,9), (7,10)}
   8 = {(8,5), (8,6), (8,7), (8,9), (8,10)}
   9 = {(9,6), (9,7), (9,8), (9,10)}
   10 = {(10,7), (10,8), (10,9)}

$$A = \begin{bmatrix} 0 & 1 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 1 & 1 & 1 & 0 & 0 & 0 & 0 & 0 \\ 1 & 1 & 0 & 1 & 1 & 1 & 0 & 0 & 0 & 0 \\ 1 & 1 & 1 & 0 & 1 & 1 & 1 & 0 & 0 & 0 \\ 0 & 1 & 1 & 1 & 0 & 1 & 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 & 1 & 0 & 1 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 & 1 & 1 & 0 & 1 & 1 & 1 \\ 0 & 0 & 0 & 0 & 1 & 1 & 1 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 & 0 & 1 & 1 & 1 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 & 1 & 0 \end{bmatrix}$$

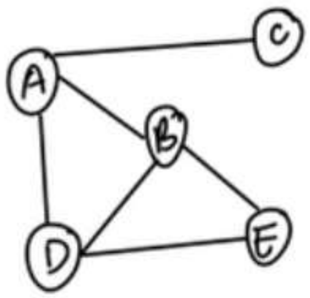


There are total 24 edges

e(G) = 24

2. Model the following situation as graphs, draw each graphs and gives the corresponding adjacency matrix.

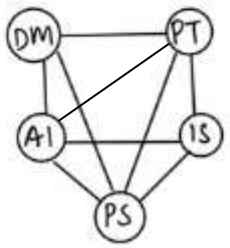
(a) Ahmad and Bakri are friends. Ahmad is also friends with David and Chong. David, Bakri and Ehsan all friends.
(Note that you may use the representation of A= Ahmad; B = Bakri; C = Chong; D = David; E= Ehsan)



$$A = \begin{bmatrix} 0 & 1 & 1 & 1 & 0 \\ 1 & 0 & 0 & 1 & 1 \\ 1 & 0 & 0 & 0 & 0 \\ 1 & 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 1 & 0 \end{bmatrix}$$

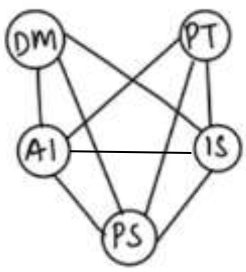
(b) There are 5 subjects to be scheduled in the exam week: Discrete Mathematics (DM), Programming Technique (PT), Artificial Intelligence (AI), Probability Statistic (PS) and Information System (IS). The following subjects cannot be scheduled in the same time slot:-

i. DM and IS



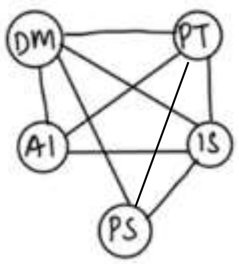
	DM	PT	AI	PS	IS
DM	0	1	1	1	0
PT	1	0	1	1	1
AI	1	1	0	1	1
PS	1	1	1	0	1
IS	0	1	1	1	0

ii. DM and PT



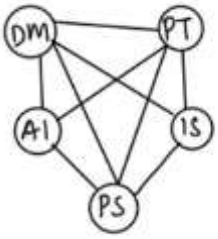
	DM	PT	AI	PS	IS
DM	0	0	1	1	1
PT	0	0	1	1	1
AI	1	1	0	1	1
PS	1	1	1	0	1
IS	1	1	1	1	0

iii. AI and PS



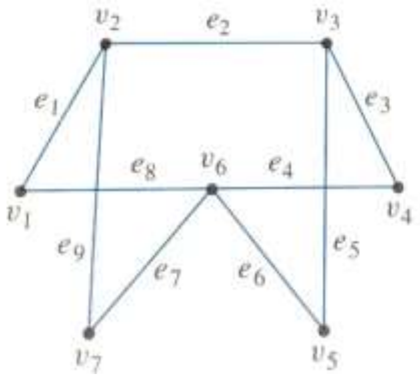
	DM	PT	AI	PS	IS
DM	0	1	1	1	1
PT	1	0	1	1	1
AI	1	1	0	0	1
PS	1	1	0	0	1
IS	1	1	1	1	0

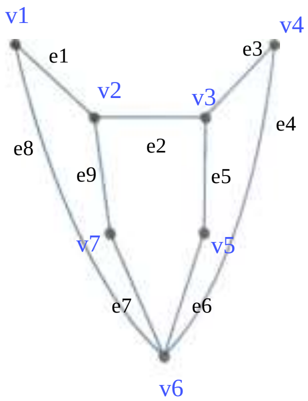
iv. IS and AI



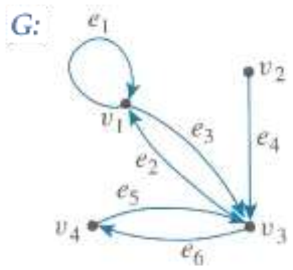
	DM	PT	AI	PS	IS
DM	0	1	1	1	1
PT	1	0	1	1	1
AI	1	1	0	1	0
PS	1	1	1	0	1
IS	1	1	0	1	0

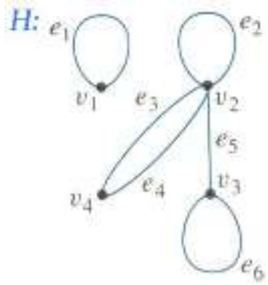
3. Show that the two drawing represent the same graph by labeling the vertices and edges of the right-hand drawing to correspond to left-hand drawing.





4. Find the adjacency and incidence matrices for the following graphs.



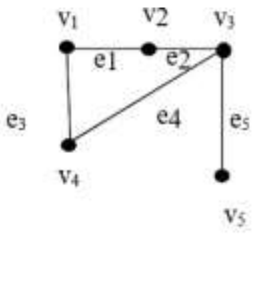


$$A_G = \begin{bmatrix} 1 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{bmatrix} \qquad I_G = \begin{bmatrix} 2 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

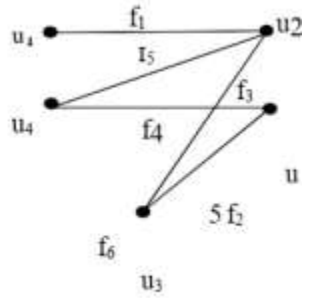
$$A_H = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 1 & 2 \\ 0 & 1 & 1 & 0 \\ 0 & 2 & 0 & 0 \end{bmatrix} \qquad I_H = \begin{bmatrix} 2 & 0 & 0 & 0 & 0 & 0 \\ 0 & 2 & 1 & 1 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 & 2 \\ 0 & 0 & 1 & 1 & 0 & 0 \end{bmatrix}$$

5. Determine whether the following graphs are isomorphic.

a)



G1



G2

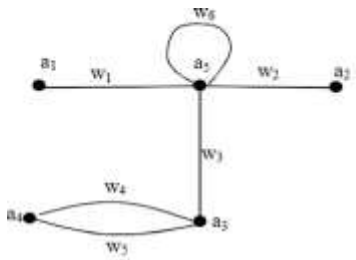
- Both graphs have the same number of 5 vertices and 5 edges
- G1 and G2 have 3 vertices with deg(2), 1 vertices with deg(1) and 1 vertices with deg(3)

$$A_{G1} = \begin{bmatrix} 0 & 1 & 0 & 1 & 0 \\ 1 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 1 & 1 \\ 1 & 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \end{bmatrix} \qquad A_{G2} = \begin{bmatrix} 0 & 1 & 0 & 0 & 0 \\ 1 & 0 & 1 & 1 & 0 \\ 0 & 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 & 1 \\ 0 & 0 & 1 & 1 & 0 \end{bmatrix}$$

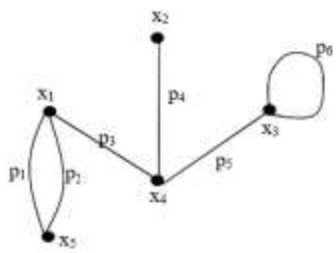
$$A_{G1} \neq A_{G2}$$

- However, $A_{G1} \neq A_{G2}$ hence G1 and G2 are not isomorphic.

b)



H1



H2

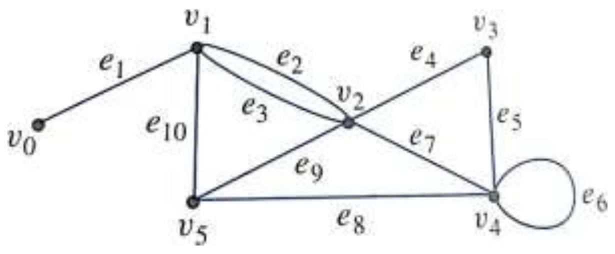
- Both graphs have the same number of 5 vertices and 6 edges
- There are 4 vertex with deg(1), 1 vertex with deg(2) and 2 vertex with deg(2) in both graphs

$$A_{\mathrm{H1}} = \begin{bmatrix} 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 2 & 1 \\ 0 & 0 & 2 & 0 & 0 \\ 1 & 1 & 1 & 0 & 1 \end{bmatrix} \qquad A_{\mathrm{H2}} = \begin{bmatrix} 0 & 0 & 0 & 1 & 2 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & 1 & 0 \\ 1 & 1 & 1 & 0 & 0 \\ 2 & 0 & 0 & 0 & 0 \end{bmatrix}$$

$$A_{\mathrm{H1}} \neq A_{\mathrm{H2}}$$

- However, $A_{\mathrm{H1}} \neq A_{\mathrm{H2}}$ hence H1 and H2 are not isomorphic.

6. In the graph below, determine whether the following walks are trails, paths, closed walks, circuits/cycles, simple circuits or just walks.



a) $v_0 e_1 v_1 e_{10} v_5 e_9 v_2 e_2 v_1$ = trail , walk

b) $v_4 e_7 v_2 e_9 v_5 e_{10} v_1 e_3 v_2 e_9 v_5$ = walk

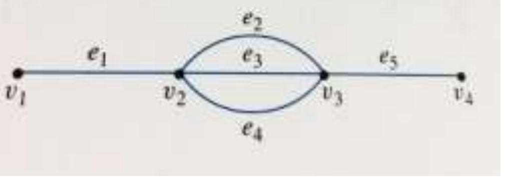
c) v_2 = walk

d) $v_5 e_9 v_2 e_4 v_3 e_5 v_4 e_6 v_4 e_8 v_5$ = closed walk, simple circuit, trail

e) $v_2 e_4 v_3 e_5 v_4 e_8 v_5 e_9 v_2 e_7 v_4 e_5 v_3 e_4 v_2$ = closed walk, cycle

f) $v_3 e_5 v_4 e_8 v_5 e_{10} v_1 e_3 v_2$ = path , trail , wall

7. Consider the following graph.



a) How many paths are there from v_1 to v_4?

= 3

b) How many trails are there from v_1 to v_4?

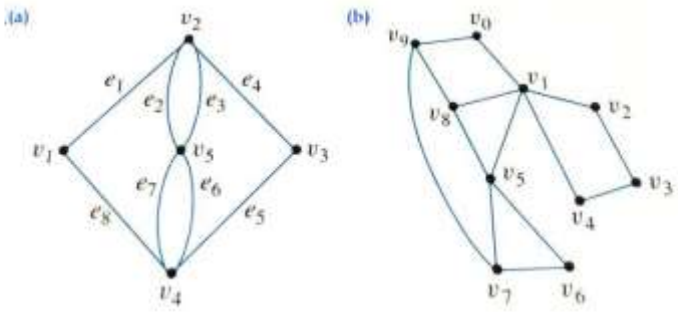
= 3! + 3

= 9

c) How many walks are there from v_1 to v_4?

= infinite

8. Determine which of the graphs in (a) – (b) have Euler circuits. If the graph does not have a Euler circuit, explain why not. If it does have a Euler circuit, describe one.



(a) Yes, because it is connected graph as all degree of vertex are even.

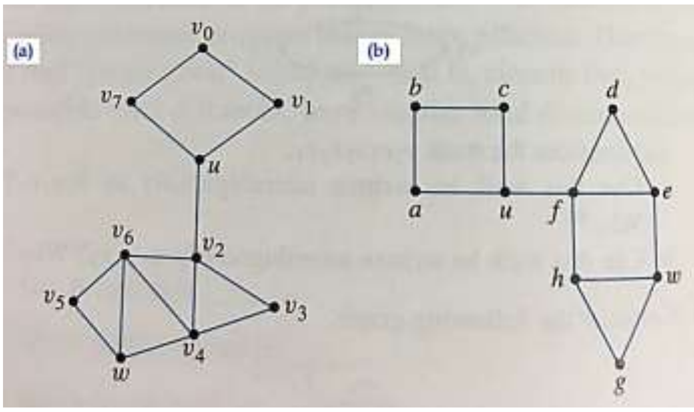
$(V_1, e_1, V_2, e_2, V_5, e_3, V_2, e_4, V_3, e_5, V_4, e_6, V_5, e_7, V_4, e_8, V_1)$

Vertex	1	2	3	4	5
Degree	2	4	2	4	4

(b) No, because it is connected graph having degree of vertex v1, v7, v8 and v9 are odd.

Vertex	0	1	2	3	4	5	6	7	8	9
Degree	2	5	2	2	2	4	2	3	3	3

9. For each of graph in (a) – (b), determine whether there is an Euler path from u to w. If there is, find such a path.



(a) Yes, because there are at most two vertices with odd degree (deg(w) = 3 and deg (v2) = 3). The path (u, v1, v0, v7, u, v2, v3, v4, v6, v5, w)

Vertex	U	V0	V1	V2	V3	V4	V5	V6	V7	W
Degree	3	2	2	4	2	2	2	4	2	3

(b) No, because the edges must only be visited once.

10. For each of graph in (a) – (b), determine whether there is Hamiltonian circuit. If there is, exhibit one.

(a) No, because vertex (2) and vertex (u) have to be visited more than once. That is (6, 5, w, 4, 3, 2, u, 1, 0, 7, u, 2, 6)

(b) No, because vertex (f) and vertex (u) have to be visited more than once. That is (g, w, e, d, f, u, a, b, c, u, f, h, g)

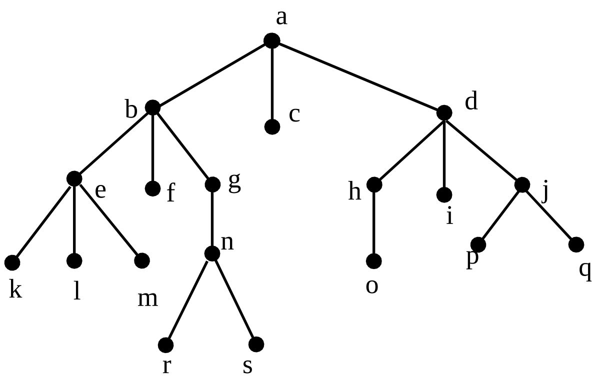
11. How many leaves does a full *3-ary* tree with 100 vertices have?

m = 3, n = 100

$$n - \frac{n-1}{m}$$

$$= 100 - \frac{100-1}{3}$$

12. Find the following vertex/vertices in the rooted tree illustrated below.



a) Root
= a

b) Internal vertices
= a, *b, e, g, n, d, h, j*

c) Leaves
= *r, s, k, l, m, o, p, q, f, c, i*

d) Children of *n*
= *r, s*

e) Parent of *e*
= *b*

f) Siblings of *k*
= *l, m*

g) Proper ancestors of *q*
= *j, d,* a

h) Proper descendants of *b*
= *e, k, l, m, f, g, n, r, s*

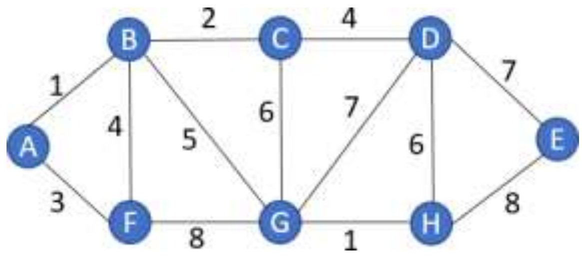
13. In which order are the vertices of ordered rooted tree in **Figure 1** is visited using *preorder*, *inorder* and
postorder.

Preorder = a, b, e, k, l, m, f, g, n, r, s, c, d, h, o, i, j, p, q

Inorder = k, e, l, m, b, f, r, n, s, g, a, c, o, h, d, i, p, j, q

Postorder = k, l, m, e, f, r, s, n, g, b, c, o, h, i, p, q, j, d, a

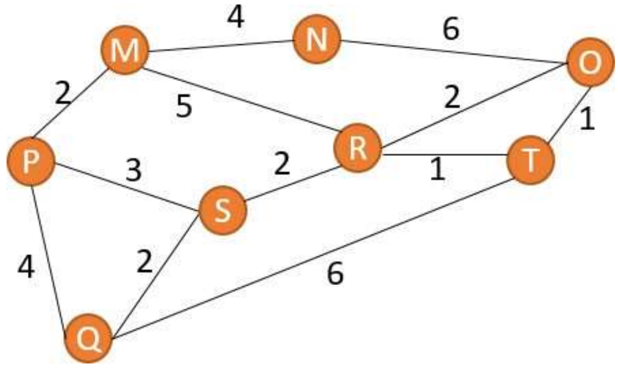
14. Find the minimum spanning tree for the following graph using Kruskal's algorithm.



AB	GH	BC	AF	BF	CD	BG	CG	DH	DG	DE	FG	EH
1	1	2	3	4	4	5	6	6	7	7	8	8

Total weight = 1 + 1 + 2 + 3 + 4 + 5 + 7 = 23

15. Use Dijsktra's algorithm to find the shortest path from **M** to **T** for the following graph.



iteration	S	N	L(M)	L(N)	L(O)	L(P)	L(Q)	L(R)	L(S)	L(T)
0	{}	{M, N, O, P, Q, R, S, T}	0	∞	∞	∞	∞	∞	∞	∞
1	{M}	{N, O, P, Q, R, S, T}	0	4	∞	2	∞	5	∞	∞
2	{M, P}	{N, O, Q, R, S, T}	0	4	∞	2	6	5	5	∞
3	{M, P, N}	{O, Q, R, S, T}	0	4	10	2	6	5	5	∞
4	{M, P, N, R}	{O, Q, S, T}	0	4	7	2	6	5	5	6
5	{M, P, N, R, S}	{O, Q, T}	0	4	7	2	6	5	5	6
6	{M, P, N, R, S, T}	{O, Q}	0	4	7	2	6	5	5	6
7	{M, P, N, R, S, T, Q}	{O}	0	4	7	2	6	5	5	6
8	{M, P, N, R, S, T, Q, O}	{ }	0	4	7	2	6	5	5	6

Shortest path = {M, R, T}

Shortest length = 6