SECI 1013 (Discrete Structure)

SEMESTER 1, 2020/2021

GROUP ASSIGNMENT

SECTION: 3

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1. Let the universal set be the set R of all real numbers and let A={x∈R | 0 < x ⩽ 2},

B={x∈R | 1 ⩽ x < 4} and C={x∈R | 3 ⩽ x < 9}. Find each of the following:

a) A ∪ C

b) (A ∪ B)′

c) A′ ∪ B′

Solutions:

a) A ∪ C

A = (0, 2]

C = [3, 9)

Since, A ∪ C = {x ∈ R | x ∈ A or x ∈ C}

Thus, A ∪ C = {x ∈ R | 0 < x ⩽ 2 or 3 ⩽ x < 9}

b) (A ∪ B)′

A = (0, 2]

B = [1, 4)

Since, A′ = {x ∈ R | x ∉ A} and A ∪ B = {x ∈ R | x ∈ A or x ∈ B}

Thus, (A ∪ B)′ = {x ∈ R | x ∉ (0, 4)}

(A ∪ B)′ = {x ∈ R | x ≤ 0 or x ≥ 4)}

c) A′ ∪ B′

A = (0, 2]

B = [1, 4)

Since, A′ = {x ∈ R | x ∉ A} and A ∪ B = {x ∈ R | x ∈ A or x ∈ B}

Thus, A′ ∪ B′ = {x ∈ R | x ∈ (-∞, 0] or (2, ∞) or (-∞, 1) or [4, ∞)}

A′ ∪ B′ = {x ∈ R | x ∈ (-∞, 1) or (2, ∞)}

A′ ∪ B′ = {x ∈ R | x < 1 or x > 2}

2. Draw Venn diagrams to describe sets A, B, and C that satisfy the given conditions.

a) $A \cap B = \emptyset$, $A \subseteq C$, $C \cap B \neq \emptyset$

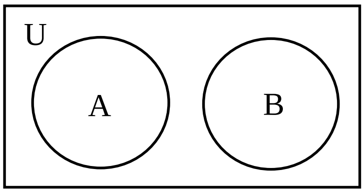
b) $A \subseteq B$, $C \subseteq B$, $A \cap C \neq \emptyset$

c) $A \cap B \neq \emptyset$, $B \cap C \neq \emptyset$, $A \cap C = \emptyset$, $A \not\subset B$, $C \not\subset B$

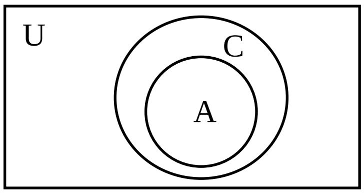
Solutions:

a) $A \cap B = \emptyset$, $A \subseteq C$, $C \cap B \neq \emptyset$

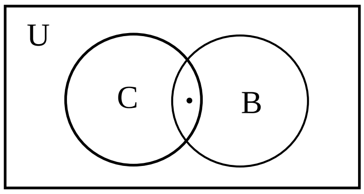
$A \cap B = \emptyset$



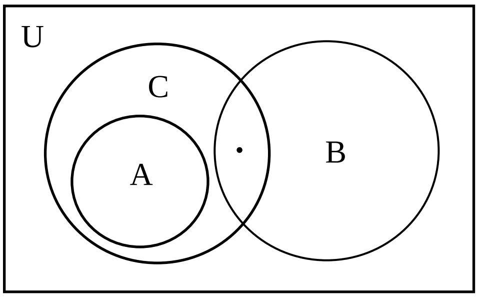
$A \subseteq C$



$C \cap B \neq \emptyset$

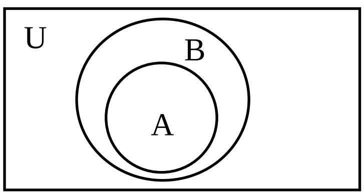


Thus, $A \cap B = \emptyset$, $A \subseteq C$, $C \cap B \neq \emptyset$

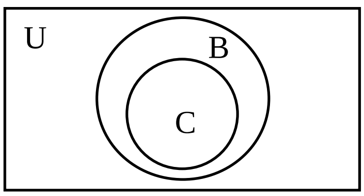


b) $A \subseteq B$, $C \subseteq B$, $A \cap C \neq \emptyset$

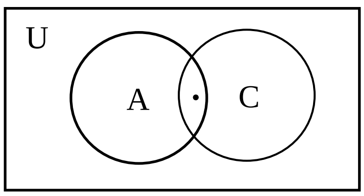
$A \subseteq B$



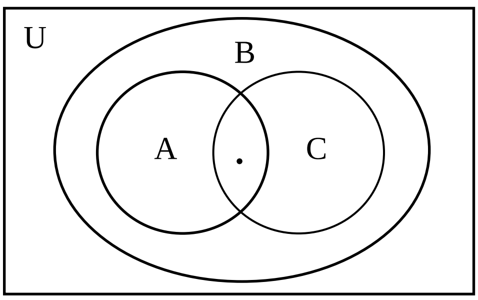
$C \subseteq B$



$A \cap C \neq \emptyset$

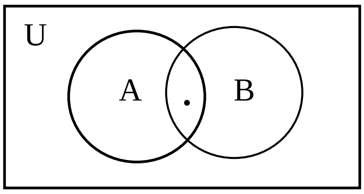


Thus, $A \subseteq B$, $C \subseteq B$, $A \cap C \neq \emptyset$

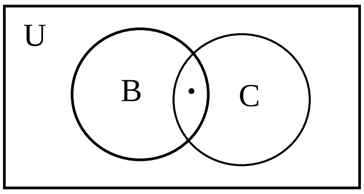


c) A ∩ B ≠ ∅, B ∩ C ≠ ∅, A ∩ C = ∅, A ⊄ B, C ⊄ B

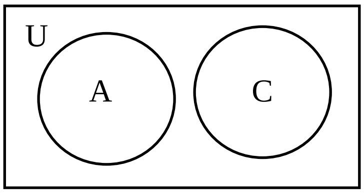
A ∩ B ≠ ∅



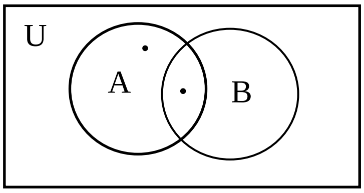
B ∩ C ≠ ∅



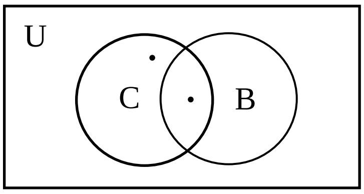
A ∩ C = ∅



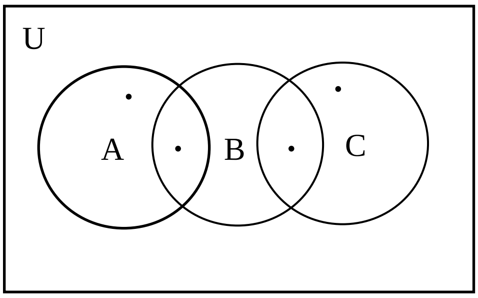
A ⊄ B



C ⊄ B



Thus, A ∩ B ≠ ∅, B ∩ C ≠ ∅, A ∩ C = ∅, A ⊄ B, C ⊄ B



3. Given two relations S and T from A to B,

$S \cap T = \{(x,y) \in A \times B \mid (x,y) \in S \text{ and } (x,y) \in T\}$

$S \cup T = \{(x,y) \in A \times B \mid (x,y) \in S \text{ or } (x,y) \in T\}$

Let $A=\{-1, 1, 2, 4\}$ and $B=\{1,2\}$ and defined binary relations S and T from A to B as follows:

For all $(x,y) \in A \times B$, $x \, S \, y \leftrightarrow |x| = |y|$

For all $(x,y) \in A \times B$, $x \, T \, y \leftrightarrow x - y$ is even

State explicitly which ordered pairs are in $A \times B$, S, T, $S \cap T$, and $S \cup T$.

Solutions:

$A \times B = \{(-1, 1), (-1, 2), (1, 1), (1, 2), (2, 1), (2, 2), (4, 1), (4, 2)\}$

$S = \{(-1, 1), (1, 1), (2,2)\}$

$T = \{(-1, 1), (1, 1), (2,2), (4,2)\}$

$S \cap T = \{(-1, 1), (1, 1), (2,2)\}$

$S \cup T = \{(-1, 1), (1, 1), (2,2), (4,2)\}$

4. Show that $\neg ((\neg p \wedge q) \vee (\neg p \wedge \neg q)) \vee (p \wedge q) \equiv p$. State carefully which of the laws are used at each stage.

$\neg ((\neg p \wedge q) \vee (\neg p \wedge \neg q)) \vee (p \wedge q)$

$= \neg \mathbf{(\neg p \wedge (q \vee \neg q))} \vee (p \wedge q)$ (Distributive law)

$= \mathbf{\neg (\neg p)} \vee (p \wedge q)$ (Double Negation law)

$= \mathbf{p \vee (p \wedge q)}$ (Absorption law)

$= p$

5. R1={(x,y)| x+y $\leqslant$6}; R1 is from X to Y; R2={(y,z)| y>z}; R2 is from Y to Z; ordering of X, Y, and Z: 1, 2, 3, 4, 5.

Find:

a) The matrix A1 of the relation R1 (relative to the given orderings)

b) The matrix A2 of the relation R2 (relative to the given orderings)

c) Is R1 reflexive, symmetric, transitive, and/or an equivalence relation?

d) Is R2 reflexive, antisymmetric, transitive, and/or a partial order relation?

Solutions:

a) The matrix A1 of the relation R1 (relative to the given orderings)

R1 = {(1, 1), (1, 2), (1, 3), (1, 4), (1, 5), (2, 1), (2, 2), (2, 3), (2, 4), (3, 1), (3, 2), (3, 3), (4, 1), (4, 2), (5, 1)}

$$\begin{bmatrix} 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 0 \\ 1 & 1 & 1 & 0 & 0 \\ 1 & 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 \end{bmatrix}$$

b) The matrix A2 of the relation R2 (relative to the given orderings)

R2 = {(2, 1), (3, 1), (3, 2), (4, 1), (4, 2), (4, 3), (5, 1), (5, 2), (5, 3), (5, 4)}

$$\begin{bmatrix} 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 \\ 1 & 1 & 0 & 0 & 0 \\ 1 & 1 & 1 & 0 & 0 \\ 1 & 1 & 1 & 1 & 0 \end{bmatrix}$$

c) Is R1 reflexive, symmetric, transitive, and/or an equivalence relation?

Reflexive: not all (a, a) $\in$ R1; (4, 4) and (5, 5) $\notin$ R1, thus R1 is not reflexive relation.

Symmetric: (a, b) $\in$ R1 and (b, a) $\in$ R1; (1, 2) $\in$ R1 and (2, 1) $\in$ R1

Thus, R1 is symmetric relation.

$$\text{Transitive: } \begin{bmatrix} 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 0 \\ 1 & 1 & 1 & 0 & 0 \\ 1 & 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 \end{bmatrix} \otimes \begin{bmatrix} 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 0 \\ 1 & 1 & 1 & 0 & 0 \\ 1 & 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 \end{bmatrix}$$

Thus, R1 is not transitive relation.

Equivalence: R1 is symmetric but not reflexive and not transitive,

Thus, R1 is not equivalence relation.

d) Is R2 reflexive, antisymmetric, transitive, and/or a partial order relation?

Reflexive: since for each a $\in$ Y, (a, a) $\notin$ R2, thus R2 is irreflexive relation.

Antisymmetric: (2, 1) $\in$ R2 but (1, 2) $\notin$ R2, thus R2 is antisymmetric relation.

Transitive: $\begin{bmatrix} 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 \\ 1 & 1 & 0 & 0 & 0 \\ 1 & 1 & 1 & 0 & 0 \\ 1 & 1 & 1 & 1 & 0 \end{bmatrix} \otimes \begin{bmatrix} 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 \\ 1 & 1 & 0 & 0 & 0 \\ 1 & 1 & 1 & 0 & 0 \\ 1 & 1 & 1 & 1 & 0 \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 \\ 1 & 1 & 0 & 0 & 0 \\ 1 & 1 & 1 & 0 & 0 \end{bmatrix}$

Thus, R2 is not transitive relation.

Partial order relation: R2 is antisymmetric but not reflexive and not transitive,

so R2 is not partial order relation.

6. Suppose that the matrix of relation R1 on {1, 2, 3} is

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 1 \\ 1 & 0 & 1 \end{bmatrix}$$

relative to the ordering 1, 2, 3, and that the matrix of relation R2 on {1, 2, 3} is

$$\begin{bmatrix} 0 & 1 & 0 \\ 0 & 1 & 0 \\ 1 & 0 & 1 \end{bmatrix}$$

relative to the ordering 1, 2, 3. Find:

a) The matrix of relation $R1 \cup R2$

b) The matrix of relation $R1 \cap R2$

Solutions:

a) The matrix of relation $R1 \cup R2$

$$\begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & 1 \\ 1 & 0 & 1 \end{bmatrix}$$

b) The matrix of relation $R1 \cap R2$

$$\begin{bmatrix} 0 & 0 & 0 \\ 0 & 1 & 0 \\ 1 & 0 & 1 \end{bmatrix}$$

7. If $f:R\rightarrow R$ and $g:R\rightarrow R$ are both one-to-one, is f + g also one-to-one? Justify your answer.

Solutions:

Assume: $f(x) = x$, $g(x) = -x$ (x is for all real numbers)

$f + g = (f + g)(x)$

$= f(x) + g(x)$

$= (x) + (-x)$

$= 0$ (it is a constant function)

For example, $(f + g)(1) = 0$ and $(f + g)(2) = 0$, but $1 \neq 2$

Thus, f + g is not one-to-one.

8. With each step you take when climbing a staircase, you can move up either one stair or two stairs. As a result, you can climb the entire staircase taking one stair at a time, taking two at a time, or taking a combination of one- or two-stair increments. For each integer n⩾1, if the staircase consists of n stairs, let C_n be the number of different ways to climb the staircase. Find a recurrence relation for c1, c2, …., C_n.

Solutions:

When one is climbing a staircase consisting of *n* stairs, the last step taken is either a single stair or two stairs together.

Numbers of ways to climb the staircase = C_n

The number of ways to climb the staircase and have the final step be a single stair is C_{n-1} .
The number of ways to climb the staircase and have the final step be two stairs is C_{n-2} .

When n ≥ 3, the staircase contains more than 2 stairs and so will need to use a combination of single stair and two stairs.

Therefore, $C_n = C_{n-1} + C_{n-2}$, n ≥ 3. Mention that C_1 = 1 and C_2 = 2.

9. The Tribonacci sequence (t_n) is defined by the equations,

t0 = 0, t1 = t2 =1, t_n = tn-1 + tn-2 + tn-3 for all n ⩾ 3.

a) Find t7.

b) Write a recursive algorithm to compute tn, n ⩾3.

Solutions:

a) Find t7.

$$t_n = t_{n-1} + t_{n-2} + t_{n-3}$$

$$\begin{aligned} t_3 &= t_{3-1} + t_{3-2} + t_{3-3} \\ &= t_2 + t_1 + t_0 \\ &= 1 + 1 + 0 \\ &= 2 \end{aligned}$$

$$\begin{aligned} t_4 &= t_{4-1} + t_{4-2} + t_{4-3} \\ &= t_3 + t_2 + t_1 \\ &= 2 + 1 + 1 \\ &= 4 \end{aligned}$$

$$\begin{aligned} t_5 &= t_{5-1} + t_{5-2} + t_{5-3} \\ &= t_4 + t_3 + t_2 \\ &= 4 + 2 + 1 \\ &= 7 \end{aligned}$$

$$\begin{aligned} t_6 &= t_{6-1} + t_{6-2} + t_{6-3} \\ &= t_5 + t_4 + t_3 \\ &= 7 + 4 + 2 \\ &= 13 \end{aligned}$$

$$\begin{aligned} t_7 &= t_{7-1} + t_{7-2} + t_{7-3} \\ &= t_6 + t_5 + t_4 \\ &= 13 + 7 + 4 \\ &= 24 \end{aligned}$$

b) Write a recursive algorithm to compute t_n, n ⩾3.

Input: n

Output: t (n)

t (n) {

if (n=1 or n=2)

return 1

return $t_{n-1} + t_{n-2} + t_{n-3}$

}