SECI 1013 (Discrete Structure)

SEMESTER 1, 2020/2021

GROUP ASSIGNMENT 3

SECTION: 3

GROUP MEMBER:

NAME	MATRIC NUMBER
EDDIE WONG CHUNG PHENG	A20EC0031
CHLOE RACQUELMAE KENNEDY	A20EC0026
NUR SYAMALIA FAIQAH BINTI MOHD KAMAL	A20EC0118

QUESTION 1 **[25 marks]**

a) Let $A=\{1, 2, 3, 4, 5, 6, 7, 8\}$, $B=\{2, 5, 9\}$, and $C=\{a, b\}$. Find each of the following:

i. $A-B$

$= \{1, 3, 4, 6, 7, 8\}$

ii. $(A\cap B) \cup C$

$= \{2, 5\} \cup \{a, b\}$

$= \{2, 5, a, b\}$

iii. $A\cap B\cap C$

$= \emptyset$

iv. $B\times C$

$= \{(2,a), (2,b), (5,a), (5,b), (9,a), (9,b)\}$

v. $P(C)$

$= \{\emptyset, \{a\}, \{b\}, \{a,b\}\}$

b) By referring to the properties of set operations, show that: (4 marks)

$(P\cap ((P'\cup Q)')) \cup (P\cap Q) = P$

$= ((P \cap P') \cup (P \cap Q')) \cup (P\cap Q)$ distributive law

$= (P \cap Q') \cup (P\cap Q)$ complement law

$= P \cap (Q' \cup Q)$ distributive law

$= P$ complement law

c) Construct the truth table for, $\mathbf{A} = (\neg p \vee q) \leftrightarrow (q \rightarrow p)$.

p	q	$\neg p$	$(\neg p \vee q)$	$(q \rightarrow p)$	$(\neg p \vee q) \leftrightarrow (q \rightarrow p)$
T	T	F	T	T	T
T	F	F	F	T	F
F	T	T	T	F	F
F	F	T	T	T	T

d) Proof the following statement using direct proof

"For all integer x, if x is odd, then $(x+2)^2$ is odd" (4 marks)

Let $P(x) = x$ be an odd integer
Let $Q(x) = (x+2)^2$ be an odd integer
Let a is an odd integer

$a = 2n + 1$ for some integer n
$a + 2 = 2n + 1 + 2$
$a + 2 = 2n + 3$
$(a+2)^2 = (2n+3)^2$
$(a+2)^2 = 4n^2 + 12n + 9$
$(a+2)^2 = k + 9$ $k = 4n^2 + 12n$

Because k is an even integer which $k = 4n^2 + 12n$, so k + 9 is an odd integer.

e) Let $P(x,y)$ be the propositional function $x \geq y$. The domain of discourse for x and y is the set of all positive integers. Determine the truth value of the following statements. Give the value of x and y that make the statement TRUE or FALSE. (4 marks)

i. $\exists x \, \exists y \, P(x, y)$
$=$ True as long as $x \geq y$, $x \neq 0$ and $y \neq 0$

ii. $\forall x \, \forall y \, P(x, y)$
$=$ False if x=1, y=2

QUESTION 2 **[25 marks]**

a) Suppose that the matrix of relation R on {1, 2, 3} is $\begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{bmatrix}$ relative to the ordering 1, 2, 3. (7 marks)

i. Find the domain and the range of R.

R = {(1,1), (1,2), (2,2), (3,1)}

Answer:
Domain = {1, 2, 3}
Range = {1, 2}

ii. Determine whether the relation is irreflexive and/or antisymmetric. Justify your answer.

Irreflexive = No, because $\exists x$ $(x, x) \in$ R and the main diagonal contains 0 and 1

Antisymmetric = Yes, because $(1, 2) \in R\ but\ (2,1) \notin$ R

b) Let $S=\{(x,y)|\ x+y \geq 9\}$ is a relation on $X=\{2, 3, 4, 5\}$. Find: (6 marks)

i. The elements of the set S.

S = {(4,5), (5,4), (5,5)}

ii. Is S reflexive, symmetric, transitive, and/or an equivalence relation? Justify your answer.

Reflexive = No, because $\forall x$ $(x, x) \notin$ R and the main diagonal contains 0 and 1

$$M_S = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 1 \end{bmatrix}$$

QUESTION 2 **[25 marks]**

Symmetric = Yes, because (4,5) ∈ R and (5,4) ∈ R. Also, M_S = M_S^T

$$M_S = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 1 \end{bmatrix} \qquad M_S^T = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 1 \end{bmatrix}$$

Transitive = No, because product of boolean is not the same.

$$\begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 1 \end{bmatrix} \otimes \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 1 \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 1 & 1 \end{bmatrix}$$

Equivalence = No, because it needs to be a reflexive, symmetric and transitive.

QUESTION 2 **[25 marks]**

c) Let $X=\{1, 2, 3\}$, $Y=\{1, 2, 3, 4\}$, and $Z=\{1, 2\}$. (6 marks)

i. Define a function $f: X \to Y$ that is one-to-one but not onto.

$f = \{(1,1), (2,2), (3,3)\}$

ii. Define a function $g: X \to Z$ that is onto but not one-to-one.

$f = \{(1,1), (2,2), (3,1)\}$

iii. Define a function $h: X \to X$ that is neither one-to-one nor onto.

$f = \{(1,1), (2,1), (3,2)\}$

d) Let m and n be functions from the positive integers to the positive integers defined by the equations:

$m(x) = 4x+3$, $n(x) = 2x-4$ (6 marks)

i. Find the inverse of m.

y = 4x + 3

$$4x = y - 3$$

$$x = \frac{y - 3}{4}$$

$$m^{-1}(x) = \frac{x - 3}{4}$$

ii. Find the compositions of n o m.

$$n \circ m = n(m(x))$$

$$= n\,(4x + 3)$$

$$= 2(4x + 3) - 4$$

$$= 8x + 6 - 4$$

$$= 8x + 2$$

QUESTION 3 **[15 marks]**

a) Given the recursively defined sequence.

$$a_k = a_{k-1} + 2k, \text{ for all integers } k \geq 2, \quad a_1 = 1$$

i) Find the first three terms. (2 marks)

$a_{1=}$ 1

$a_{2=}$ a_1+ 2(2)

= 1 + 4

= 5

$a_{3=}$ a_2+ 2(3)

= 5 + 6

= 11

$a_{1=}$ 1 , $a_{2=}$ 5, $a_{3=}$ 11

ii) Write the recursive algorithm. (5 marks)

```
Input : n, integer ≥ 2
Output: f(n)
f(n) {
        if (n = 2)
        return 5
        return f(n-1) + 2n
}
```

b) A certain computer algorithm executes twice as many operations when it is run with an input of size k as it is run with an input of size k–1 (where k is an integer that is greater than 1). When the algorithm is run with an input of size 1, it executes seven operations. Let r_k = the number of executes with an input size k. Find a recurrence relation for r_1, r_2, …. r_k. (4 marks)

when k =1, $r_k = 7 \times 2^{k-1}$
$= 7 \times 2^{1-1}$
$= 7 \times 1$
$= 7$

Since k > 1, k =2
$r_1 = 7 \times 2^{2-1}$
$= 7 \times 2^1$
$= 7 \times 2$
$= 14$

$r_2 = 7 \times 2^{3-1}$
$= 7 \times 2^2$
$= 7 \times 4$
$= 28$

QUESTION 3 **[15 marks]**

c) Given the recursive algorithm:

Input: *n*

Output: *S* (*n*)

S(*n*) {

if (*n*=1)

return 5

return 5**S*(*n*–1)

}

Trace *S*(4). (4 marks)

When n=1, S(1) = 5

When n=2, S(2) = 5 × S(1)

= 5 × 5

=25

When n=3, S(3) = 5 × S(2)

= 5 × 25

= 125

When n=4, S(4)= 5 × S(3)

=5 × 125

=625

QUESTION 4 **[25 marks]**

a) Hexadecimal numbers are made using the sixteen digits 0, 1, 2, 3, 4, 5, 6, 7, 8, 9, A, B, C, D, E, F. They are denoted by the subscript 16. How many hexadecimal numbers begin with one of the digits 3 through B, end with one of the digits 5 through F and are 4 digits long?

(4 marks)

Case 1: first digit: 3, 4, 5, 6, 7, 8, 9, A, B (9 ways)
Case 2: second digit: all 16 digits (16 ways)
Case 3: third digit: all 16 digits (16 ways)
Case 4: fourth digit: 5, 6, 7, 8, 9, A, B, C, D, E, F (11 ways)

9x 16 x 16 x 11= 25344

b) Suppose that in a certain state, all automobile license plates have four letters followed by three digits. How many license plates could begin with A and end in 0?

(4 marks)

Case 1: first letter is A (1 way)
Case 2: second letter: all 26 letters (26 ways)
Case 3: third letter: all 26 letters (26 ways)
Case 4: fourth letter: all 26 letters (26 ways)
Case 5: first digit: 9 digits, exclude 0 (9 ways)
Case 6: second digit: 10 digits (10 ways)
Case 7: third digit is 0 (1 way)

1 x 26 x 26 x 26 x 9 x 10 x 1 = 1581840

c) How many arrangements in a row of no more than three letters can be formed using the letters of word *COMPUTER* (with no repetitions allowed)?

(5 marks)

Total number of letters: 8
No repeated letters

Case 1: first letter 8 ways
Case 2: second letter 7 ways
Case 3: third letter 6 ways

Total arrangement of 2 letters:
= 8 x 7
= 56

Total arrangement of 3 letters:
= 8 x 7 x 6
= 336

Total arrangement no more than 3 letters:
= 8 + 56 + 336
= 400

QUESTION 4 **[25 marks]**

d) A computer programming team has 13 members. Suppose seven team members are women and six are men. How many groups of seven can be chosen that contain four women and three men?

(4 marks)

Ways to select 4 women (7,4)

$$= \frac{7!}{4!\,(7-4)!}$$

$$= \frac{7!}{4!\,(3)!}$$

$$= 35$$

Ways to select 3 men is (6,3)

$$= \frac{6!}{3!\,(6-3)!}$$

$$= \frac{6!}{2!\,(3)!}$$

$$= 20$$

Total groups of seven that can be chosen:
= 35 x 20
= 700

e) How many distinguishable ways can the letters of the word *PROBABILITY* be arranged?

(4 marks)

Total number of letters: 11
Repeated letters: B, I (2 letters)

Total ways:

$$= \frac{11!}{(2!\ \times\ 2!)}$$

$$= 9979200$$

f) A bakery produces six different kinds of pastry. How many different selections of ten pastries are there?

(4 marks)

Total number of pastry to select: 10
Type of pastry: 6

Total selection:

C (10 + 6 – 1, 10)

= C (15, 10)

$$= \frac{15!}{10!\ (15-10)!}$$

= 3003

QUESTION 5 **[10 marks]**

a) Eighteen persons have first names Ali, Bahar, and Carlie and last names Daud and Elyas. Show that at least three persons have the same first and last names.

(4 marks)

Total people with same first name and same last name

= 3 x 2

= 6

$$m = \left\lceil \frac{18}{6} \right\rceil$$

= 3 (at least 3 has same first and last names)

b) How many integers from 1 through 20 must you pick in order to be sure of getting at least one that is odd?

(3 marks)

Odd integers: {1, 3, 5, 7, 9, 11, 13, 15, 17, 19}

Total integers to pick: 10 + 1 = 11 (at least one integer is odd)

c) How many integers from 1 through 100 must you pick in order to be sure of getting one that is divisible by 5?

(3 marks)

Integers that is divisible by 5: (20 integers)

{5, 10, 15, 20, 25, 30, 35, 40, 45, 50, 55, 60, 65, 70, 75, 80, 85, 90, 95, 100}

integers that is not divisible by 5: 100 – 20 = 80

total integers to pick: 80 + 1 = 81 (at least one integer is divisible by 5)