ASSIGNMENT 3 TUTORIAL

SCHOOL OF COMPUTING, FACULTY OF ENGINEERING

SECI 1013 | DISCRETE STRUCTURE

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SECTION 9

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Tutorial 3

Question (1)

a) $A = \{1, 2, 3, 4, 5, 6, 7, 8\}$ $B = \{2, 5, 9\}$ and $C = \{a, b\}$

i) $A - B = \{1, 3, 4, 6, 7, 8\}$ //

ii) $(A \cap B) \cup C \Rightarrow A \cap B = \{2, 5\}$ $= \{2, 5, a, b\}$ //

$\cup$

$C = \{a, b\}$

iii) $A \cap B \cap C = \emptyset$

iv) $B \times C = \{2, 5, 9\} \times \{a, b\}$

$= \{2a, 2b, 5a, 5b, 9a, 9b\}$ //

v) $P(C) = \{a\}, \{b\}, \{a, b\}$ //

b) $(P \cap ((P' \cup Q)') \cup (P \cap Q)$

$P \cap (P \cap Q') \cup (P \cap Q)$ - demorgan's

$(P \cap P) \cap Q' \cup (P \cap Q)$ - Assoc

$(P \cap Q') \cup (P \cap Q)$ - Idea

$P \cap (Q' \cup Q)$ - Dist

$P \cap U = P$ //

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c)

| $p$ | $q$ | $\neg p \vee q$ | $q \to p$ | $(\neg p \vee q) \leftrightarrow (q \to p)$ |
|---|---|---|---|---|
| T | T | T | T | T |
| T | F | T | T | T |
| F | T | T | F | F |
| F | F | F | T | F |
| | | | | |
| | | | | |

d) $P(x)$ = $x$ is odd
$Q(x)$ = $(x+2)^2$ is odd

~~Let a be an odd integer~~

$a = 2n+1$
$a+2 = 2n+1+2$
$(a+2)^2 = (2n+3)^2$

$(a+2)^2 = 4n^2 + 6n + 6n + 9$
$(a+2)^2 = 4n^2 + 12n + 9$
$(a+2)^2 = 2(2n^2 + 6n) + 9$
$(a+2)^2 = 2k + 9$ $\quad k = 2n^2 + 6n$ is an integer

$(a+2)^2$ = odd ~~int~~ integer

Therefore for all integer $x$, if $x$ is odd the $(x+2)^2$ is odd //

e) i) $\exists x \exists y \; P(x, y)$ = True $(4, 3)$ or $(2, 2)$

ii) $\forall x \forall y \; P(x, y)$ = false $(3, 4)$ or $(2, 6)$

Question 2

a) $m_R$ :
$$\begin{array}{c|ccc} & 1 & 2 & 3 \\ 1 & \\ 2 & \\ 3 & \end{array}\left[\begin{array}{ccc} 1 & 1 & 0 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{array}\right]$$

i. domain → {1, 3}

range → {1, 2}

ii. - the relation is not irreflexive in fact it is not reflexive because the main diagonal have value '0' and '1'.

- The relation is antisymmetric because
  - $(1,2) \in R$ but $(2,1) \notin R$
  - $(3,1) \in R$ but $(1,3) \notin R$
  - $(1,1) \in R$ and $(1,1) \in R$
  - $(2,2) \in R$ and $(2,2) \in R$ $\}$ implies $a = b$

b) $X = \{2, 3, 4, 5\}$ $S = \{(x,y) \mid x + y \geq 9\}$

i. $S = \{(4,5), (5,4), (5,5)\}$

$m_S$ =
$$\begin{array}{c|cccc} & 2 & 3 & 4 & 5 \\ 2 \\ 3 \\ 4 \\ 5 \end{array}\left[\begin{array}{cccc} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 1 \end{array}\right]$$

ii. - relation S is not reflexive because the main diagonal consist value '0' and '1'

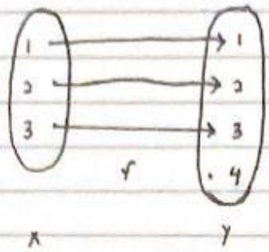
- relation S is symmetric because $\forall a, b \in X, (a,b) \in S \rightarrow (b,a) \in S$

- relation S is not transitive because $m_S \otimes m_S \neq m_S$

# relation S is not equivalence relation because it is symmetric but not reflexive and not transitive.

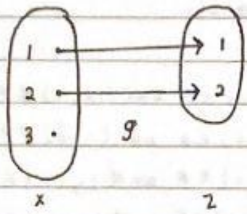
c) $X = \{1,2,3\}$ , $Y = \{1,2,3,4\}$ , $Z = \{1,2\}$

i.



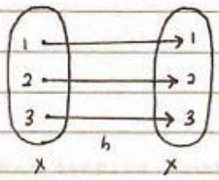

$f : X \to Y$

ii.




$g : X \to Z$

iii.




$h : X \to X$

d) $m(x) = 4x + 3$ , $n(x) = 2x - 4$

i. $y = 4x + 3$

$y - 3 = 4x$

$x = \dfrac{y-3}{4}$

$m^{-1}(x) = \dfrac{x-3}{4}$

ii. $n \circ m = n(m(x))$

$= 2(4x+3) - 4$

$= 8x + 6 - 4$

$n \circ m = 8x + 2$

Question (3)

a) $a_n = a_{n-1} + 2n, \quad n \geq 2$

$a_1 = 1$

i) $a_2 = a_1 + 2(2)$

$= 5$

$a_3 = a_2 + 2(3)$

$= 11$

$a_4 = a_3 + 2(4)$

$= 19$

ii) Input = n

Output : a(n)

```
a(n) {
    if(n = 1)
        return 1
    return a(n-1) + (2 * n)
}
```

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b) ~~let~~ $r_1 = 7$

given the number of operations is twice that of input $k-1$ when input is $k$

$$r_k = 2r_{k-1} \quad \text{for } k \geq 2$$

c) Input: n
output: S(n)

```
S(n){
    if (n=1)
        return 5
    return 5 * S(n-1)
}
```

$S(1) = 5$

$S_n = 5 \times S_{n-1}$

$S(2) = 5 \times 5 = 25$

$S(3) = 5 \times 25 = 125$

$S(4) = 5 \times 125 = 625$

# QUESTION (4)

a) 0, 1, 2, 3, 4, 5, 6, 7, 8, 9, A, B, C, D, E, F

total num = 16

FIRST DIGIT : 9 way (3 through B)
SECOND DIGIT : 16 ways
THIRD DIGIT : 16 ways
FOURTH DIGIT : 11 ways (5 through F)

$= 9 \cdot 16 \cdot 16 \cdot 11 = 25344$ ways #

b) Alphabet = 26
DIGITS = 10 (0, 1, 2, 3, 4, 5, 6, 7, 8, 9)

FIRST LETTER : 1 way (only A)
SECOND LETTER : 26 ways
THIRD LETTER : 26 ways
FOURTH LETTER : 26 ways
FIRST DIGIT : 10 ways
SECOND DIGIT : 10 ways
THIRD DIGIT : 1 ways (only 0)

$= 1 \cdot 26 \cdot 26 \cdot 26 \cdot 10 \cdot 10 \cdot 1$
$= 1757600$ ways #

c) C O M P U T E R = 8 letters

1st letter : 8 ways
2nd letter : 7 ways
3rd letter : 6 ways

$= 8 \cdot 7 \cdot 6$
$= 336$ ways #

d) Group of seven = 4 women 3 men
13 m = 7 w + 6 m

4 women $= 7C4 \quad \frac{7!}{4!(7-4)!} = 35$
$= 35$

3 men $= 6C3 \quad \frac{6!}{3!(6-3)!} = 20$
$= 20$

$= 35 \cdot 20 = 700$ #

e) $\frac{n!}{(n_1!)(n_2!)}$

P R O B A B I L I T Y

Total = 11 letters

$\frac{11!}{2!\,2!} = \frac{39\,916\,800}{4}$

$= 9\,979\,200$ #

f) $n = 6$ pastries
$r = 10$ pastries

* order is not important
* allow repetition

$C(6+10-1, 10) = \binom{15}{10}$

$C(n+r-1, r)$

$= \frac{(n+r-1)!}{r!(n-1)!}$

$= \frac{15!}{10!(15-10)!}$

$= \frac{15!}{10!(5!)}$

$= 3003$ #

Typo

Question 5

a) Pigeonhole principle (Third form)

→ $n$ = pigeon

$k$ = pigeonhole

$m = \left[\frac{n}{k}\right]$, some pigeonhole must contain at least $m$ pigeons.

$n$ = 18 persons

$k$ = number of first names and last names combination

$= 2 \times 3 = 6$

$m = \left[\frac{18}{6}\right] = 3$

∴ hence there are at least 3 persons have the same first and last names.

b) odd integer from 1 - 20 = {1, 3, 5, 7, 9, 11, 13, 15, 17, 19} = 10 numbers

even integer from 1 - 20 = {2, 4, 6, 8, 10, 12, 14, 16, 18, 20} = 10 numbers

∴ we must pick at least 11 numbers to get at least 1 odd number.

c) integer from 1 - 100 that divisible by 5

= {5, 10, 15, 20, 25, 30, 35, 40, 45, 50, 55, 60, 65, 70, 75, 80, 85, 90, 95, 100}

= 20 numbers

Integer from 1 - 100 that cannot divisible by 5

= 100 - 20 = 80 numbers

∴ we must pick at least 81 numbers to get at least 1 number that divisible by 5.