UTM

UNIVERSITI TEKNOLOGI MALAYSIA

ASSIGNMENT 2 TUTORIAL

SCHOOL OF COMPUTING, FACULTY OF ENGINEERING

SECI 1013 | DISCRETE STRUCTURE

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SECTION 9

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DISCRETE STRUCTURE
SEC] 1013
TUTORIAL 2 [CHAPTER 3]

[QUESTION 1]

① Consider 3-digit number whose digit are either 2, 3, 4, 5, 6 or 7

a) How many numbers are there ?

b) How many numbers are there if the digits are instinct ?

c) How many numbers between 300 to 700 is only odd digit

a) ⊔⊔⊔ * assume number will repeat

First place 6 ways
2nd place 6 ways
3rd place 6 ways

Total ways : $6 \times 6 \times 6 = 216$

b) ↙6 ↙5 ↙4
⊔⊔⊔ * number is distinct

First place : 6 ways
2nd place : 5 ways
3rd place : 4 ways

Total ways : $6 \times 5 \times 4 = 120$

c) ⊔⊔⊔

First place : 4 ways (3, 4, 5, 6)
2nd place : 6 ways
3rd place : 3 ways (3, 5, 7) (odd)

Total ways : $4 \times 6 \times 3 = 72$

[QUESTION 2]

② Anita invited 10 of her friends ... couple, four men and four women to her anniversary dinner, how many ways Anita can arrange them around a round dinner table

a) couple = 1 men, 1 women
Men = 4 men
women = 4 women

Total ways = $5! \times 5!$
$= 14\,400$

Men sit next to each other = $5!$
women sit next to each other = $5!$

b) couple as 1 unit
circular permutation

$(8+1) = 9$ unit of men and women

$(9-1)! = 8!$

* couple can rearrange themselves in two ways

Total num of ways = $8! \times 2! = 80640$

c) men can rearrange around the table = $(5-1)! = 4!$
women can arrange in between = $(5)!$

Total number of ways = $4! \times 5!$
$= 2880$

d) Total ways = $11! \times 2!$
$= 79\,833\,600$

Question 3

a. no repitition ${}^5P_5 = 120$

b. 2 runners tie ${}^5C_2 \times 4! = 240$

c. 2 groups of 2 runner tie ${}^5C_2 \times {}^3C_2 \times 3! = 180$

Question 4

a. repetition is allowed $C(n+r-1, r)$

$n = 6$ | $= C(6+12-1, 12)$

$r = 12$ | $= C(17, 12) = 6188$ ways

b. select 2 of each kind, $6 \times 2 = 12$

select remaining 12 $n = 6$ $r = 12$

repetition is allowed

$C(n+r-1, r)$

$= C(6+12-1, 12) \cdot C(17, 12) = 6188$ ways

c. select 5 chocolate croissants and 3 almond croissant

select remaining $24 - 5 - 3 = 16$ croissants; $n = 6$ $r = 16$

repetition is allowed

$C(n+r-1, r)$

$= C(6+16-1, 16) = C(21, 16) = 20349$ ways

No.: Date:

Question (5)

a) 2 wins in 4 Games → $C(4,2) = \frac{4!}{2!(4-2)!} = \frac{4!}{2!2!} = 6$

1 win in 3 Games → $C(3,1) = \frac{3!}{1!(3-1)!} = \frac{3!}{1!2!} = 3$

⇒ 2 wins and 1 ties/wins : $C(4,2) \cdot C(3,1) \cdot 2 = 36$

1 win and 3 ties/wins : $C(3,1) \cdot C(4,3) \cdot 2^3 = 96$

Finally to

Number of scenarios = $2 \cdot (36 + 96)$ (two teams)

$= 264$

b) we know that the game is not settled in the first 10 rounds

10 penalty kicks $= 2 \cdot 2 \cdots 2 = 2^{10} = 1024$

if 264 scenarios the game ends in the first round

then $1024 - 264 = 760$ scenarios where game doesn't end in first 10 rounds

then total scenarios = scenarios where game doesn't ends in first 10 kicks × scenarios where games end in 10 kicks

$= 760 \times 264 = 200640$

c) In sudden death, there are 3 options in each round:
Team A wins, Team B wins or a tie.
if a team wins, the game ends and if tied continues.

thus 2 ways the game ends in shoot out.

first round: 760 scenarios (game did not end)
second round: 760 scenarios (game did not end)
shoot out: 2+2+2+2+2 (five rounds) = 10 scenarios

total number = 760 · 760 · 10 = 5,776,000

[QUESTION 6]

A professor gives a multiple-choice quiz that has ten questions, each with four possible responses, a, b, c, d
what is the minimum number of students that must be in the professor's class in order to guarantee that at least three answer sheets must be identical? (assume no answers are left blank)

- each question have 4 choices
- The pigeonhole is the answer sheet
- we need to determine the number of pigeons for at least one pigeonhole to contain three pigeons
- The possible of distinct answer sheet

$4^{10} = 1048576$

- To guarantee at least three answer sheets are identical, there must be minimum of $2(1048576)+1 = 2097153$ students in the examination

[QUESTION 7]

In a secondary examination, 75% of the students have passed in history and 65% in Mathematics, while 50% passed both in history and mathematics. If 35 candidates failed in both the subjects, what is the number of candidates sit for the exam?

$n(H \cup M) = n(H) + n(M) - n(A \cap M)$

75% $H = 0.75$
65% $M = 0.65$
50% $H \cap M = 0.50$

since $n(H \cup M) = n(H) + n(M) - n(H \cap M)$

$\therefore n(H \cup M) = 0.75 + 0.65 - 0.50$
$= 0.9$

let N = total number of candidates

$N = \frac{n(H \cup M)}{0.90}$

$n(H \cup M) = N - n(H' \cap M')$
$= N - 35$

$N = \frac{N-35}{0.90}$

$N(0.90) = N - 35$

$0.90N - N = -35$
$-0.1N = -35$
$N = \frac{-35}{-0.1}$
$= 350$ candidates

Question 8

number from 300 through 780 that have ~~at le~~ 1 as at least one digit
= 92

~~and~~ total number from 300 through 780
= 480

probability the number chosen will have 1 as at least one digit
$= \frac{92}{480} = 0.19$

[QUESTION 9]

a) $10C6 \times \frac{6!}{2!4!} = 210 \times 15$

6 parking lots

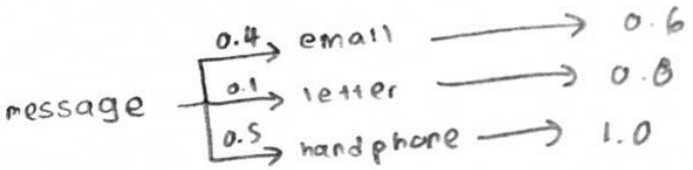
$= 3150$ ways

b) $7C6 \times 15 \text{ ways} = 105 \text{ ways}$

6 cars and one group of empty parking lots

Probability $= \frac{105}{3150} = 0.03333$

[QUESTION 10]



a) Propability of trainee receives the message $= (0.4 \times 0.6) + (0.1 \times 0.8) + (0.5 \times 1.0)$

$= 0.24 + 0.08 + 0.5$

$= 0.82$

b) $P\left(\frac{\text{prob received by email}}{\text{prob. message receives}}\right) = \frac{(0.4 \times 0.6)}{0.82} = \frac{0.24}{0.82}$

$= 0.293$

$= 12/41$

Question 11

events

C - cars

T - light truck

F - fatal accident

N - Not fatal accident

$P(F|C) = \frac{20}{100000}$

$P(F|T) = \frac{25}{100000}$

$P(T) = 0.4$

$P(C) = 1 - P(T) = 1 - 0.4 = 0.6$

$$P(T|F) = \frac{P(T \cap F)}{P(F)} = \frac{P(F|T) \cdot P(T)}{P(F|T)P(T) + P(F|C)P(C)}$$

$$= \frac{(0.00025)(0.4)}{(0.00025)(0.4) + (0.0002)(0.6)}$$

$$= 0.4545$$

Question 12

No. of letters = 9

No. of boxes = 4

We have 9 different coloured letters and 4 choices of where to put them, without restrictions $4^9 = 262144$

Using the principle of inclusion - exclusion, if the letters are put into two boxes we have four choices for which boxes the letters get put into and 2 choices per letter

So

$4 \times 3^9 = 78732$ disallowed ways

for allowed ways $262144 - 78732 + 4 = 183416$ //